

NEW PERSPECTIVES ON RESEARCH IN MATHEMATICS EDUCATION

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DEVELOPING RESEARCH IN MATHEMATICS EDUCATION

Twenty Years of Communication, Cooperation
and Collaboration in Europe

Edited by
Tommy Dreyfus
Michele Artigue
Despina Potari
Susanne Prediger
Kenneth Ruthven

ROUTLEDGE


DEVELOPING RESEARCH IN MATHEMATICS EDUCATION

Developing Research in Mathematics Education is the first book in the series *New Perspectives on Research in Mathematics Education*, to be produced in association with the European Society for Research in Mathematics Education (ERME). This inaugural volume sets out broad advances in research in mathematics education which have accumulated over the last 20 years through the sustained exchange of ideas and collaboration between researchers in the field.

An impressive range of contributors provide specifically European and complementary global perspectives on major areas of research in the field on topics that include:

- the content domains of arithmetic, geometry, algebra, statistics and probability;
- the mathematical processes of proving and modelling;
- teaching and learning at specific age levels from early years to university;
- teacher education, teaching and classroom practices;
- special aspects of teaching and learning mathematics such as creativity, affect, diversity, technology and history;
- theoretical perspectives and comparative approaches in mathematics education research.

This book is a fascinating compendium of state-of-the-art knowledge for all mathematics education researchers, graduate students, teacher educators and curriculum developers worldwide.

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New Perspectives on Research in Mathematics Education – ERME series

ERME, the European Society for Research in Mathematics Education, is a growing society of about 900 researchers from all over Europe and beyond.

The ERME series documents the growing body of substantial research on mathematics education within the context of ERME. Volumes in the ERME series can be monographs or collections growing out of the collaboration of European researchers in mathematics education.

The volumes are written by and for European researchers, but also by and for researchers from all over the world. An international advisory board guarantees that ERME stays globally connected. A rigorous and constructive review procedure guarantees a high quality of the series.

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in Europe

*Edited by Tommy Dreyfus, Michèle Artigue,
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Kenneth Ruthven*

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SERIES FOREWORD

New perspectives on research in mathematics education – ERME series

ERME, the European Society for Research in Mathematics Education, is a growing society of about 900 researchers from all over Europe and beyond. Between 1998 and 2018, ten biannual Congresses of ERME (CERME), nine summer schools (YESS) and six ERME Topic Conferences (ETC) have taken place.

The ERME series documents the growing body of substantial research on mathematics education within the context of ERME. Volumes in the ERME series can be monographs or collections growing out of the collaboration of European researchers in mathematics education; for example, post-conference publications of selected contributions to ETCs or research in EU-funded projects.

The volumes are written by and for European researchers, but also by and for researchers from all over the world. An international advisory board guarantees that ERME stays well connected to the rest of the world and includes results of non-European research. A rigorous and constructive review procedure guarantees the high quality of the series.

The inaugural volume of the ERME Series is titled *Developing Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration in Europe*. We thank all involved editors, authors and reviewers for the joint work. It is a highly interesting start of the series which will – hopefully – fuel the international communication, cooperation and collaboration.

The series editors: Viviane Durand-Guerrier,
Konrad Krainer, Susanne Prediger, Nad'a Vondrová

PREFACE

The aim of this book is to present, on the occasion of the 20th anniversary of the European Society for Research in Mathematics Education (ERME), the most important directions and trends of European research in mathematics education. The book reports on the main lines of development in ERME over the course of these 20 years, showing the ERME spirit in the process: a spirit of communication between different sub-areas and different countries, a spirit of cooperation between different theoretical approaches and research paradigms, a spirit of collaboration between established and developing researchers. Prior to the creation of ERME, several research traditions had already developed within Europe, each with its own identity reflecting cultural and educational particularities of its birthplace. With this book, we hope to establish shared understandings in which to ground future European research in mathematics education as well as to showcase the specific character of European research traditions for audiences inside and outside Europe.

At ERME conferences (CERMEs) participants spend most of the time in thematic working groups (TWGs). New working groups are added at each conference, typically two or three, following an open call for proposals and then selection by the International Program Committee for the conference. Equally, working groups are discontinued when they no longer attract enough interest.

Given how central the TWGs are to the scientific interaction that takes place within ERME, it was natural that the core of this book should reflect the work and achievements of these working groups. All groups that have been operational over sufficiently many CERMEs to warrant a substantial chapter have been included. The authors of each chapter have been selected from among those who have recently been active in leading the work of the relevant groups.

The chapters in the book have been ordered so as to begin with those focusing on specific content domains, age levels, and mathematical processes (such as proving and modelling). The following chapters then examine aspects of teaching and learning that range across contents and processes (including learning environments, affect and research about teachers). The final chapters deal with linguistic, cultural and social aspects of learning and teaching mathematics, as well as with the role of theories in mathematics education research. These 18 core chapters of the book are preceded by an introduction that provides an account of the evolution of ERME as a society, written by its former presidents.

The book concludes with two commentary chapters contributed by eminent researchers from outside Europe. Together these commentary chapters situate the research done in ERME and presented in the book in the wider context of mathematics education research worldwide. At the same time, they show how ERME might, in the future, inspire and be inspired by developments outside Europe.

Nowadays it is increasingly hard for researchers even to read all the relevant work that has been published on their topic, let alone to take comprehensive account of it in conducting and reporting their own research. There is a pressing need for greater coordination of research efforts; in particular, in the form of synthesis of frameworks and findings. Thus, this book seeks to identify cumulative achievements in research and to highlight starting points for further development within and beyond ERME. The book does not, therefore, aim to summarise 20 years of highly diverse research by hundreds of researchers but, rather, aims to display the patterns and threads that have the potential to unify these research efforts into a cumulative and coherent body of knowledge providing a view from the past into the future.

In this sense, the authors of the core chapters were asked to focus on the sustained development of ideas in the working groups over several years and ERME conferences. Also stressed were ideas that arose within a group and have become relevant for other groups, ideas that arose within ERME and initiated or influenced developments in mathematics education research beyond ERME, and ideas that originated in one geographical area, were brought into ERME and, as a consequence, were taken up more widely.

The book was initiated by the board of ERME in early 2015. When the board invited us to edit the book, we designed a process according to which initial chapter drafts would be available prior to CERME 10 (February 2017), enabling them to be discussed in the TWGs at the conference. Thus, the CERME spirit of communication and cooperation came to bear also in the process of writing the book. Chapters were then finalised, taking into account the feedback from discussions at CERME 10 as well as new developments reported at that conference. Chapter lengths were tailored to the size of the group (or, in a few cases, of groups that had recently split) and the number of years that the group had been working. We are looking forward to launching the book at CERME 11 (February 2019).

Given the nature of the book, it contains many references to CERME proceedings. In order to save space, a shortened format has been used for references to

these proceedings in the chapters. The full references of all proceedings are listed below this preface.

In conclusion, some general patterns seem to appear across the different core chapters. In many groups, work in the earlier years of CERME was characterised by members' difficulties in understanding each other, due to the very diverse research traditions and theoretical frameworks in use within Europe. Through comparing and contrasting these, mutual understandings of the different lines of research have been reached. Even if most researchers kept on working within their respective traditions, they succeeded in finding commonalities, and understanding differences. Moreover, efforts of networking theories have been described, not only for purposes of comparing and contrasting but for coordinating or combining theoretical approaches. Indeed, in quite a number of cases where researchers have started to combine theoretical frameworks, this has led to a widening of perspectives from rather narrow research topics to more comprehensive foci that require more complex approaches.

It will, of course, be for the reader of this book to form an opinion as to what extent ERME has succeeded in showing the existence of a dynamic community which, over 20 years, has been able to create an identity for European research in mathematics education, structured around a common spirit, while remaining respectful of diversity and open to outside influences; to create original ways of collaborative research and to support the acculturation and integration of young researchers; and to contribute substantially to the advance of international research on a wide range of issues, while playing a pioneering role in some areas.

The editors: Tommy Dreyfus, Michèle Artigue,
Despina Potari, Susanne Prediger, Kenneth Ruthven

References of CERME proceedings

Note: The complete references of the CERME proceedings are listed here. All CERME proceedings are accessible via <http://www.mathematik.uni-dortmund.de/~erme/index.php?slab=proceedings>

CERME 1: Schwank, I. (Ed.) (1999). *European Research in Mathematics Education I: Proceedings of the First Conference of the European Society for Research in Mathematics Education* (CERME 1, August 1998). Osnabrück, Germany: Forschungsinstitut für Mathematikdidaktik and ERME.

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CERME 10: Dooley, T., & Gueudet, G. (Eds.) (2017). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME 10, February 1–5, 2017). Dublin, Ireland: DCU Institute of Education and ERME.

THE EUROPEAN SOCIETY FOR RESEARCH IN MATHEMATICS EDUCATION

Introduction by its former presidents

*Ferdinando Arzarello, Paolo Boero, Viviane
Durand-Guerrier and Barbara Jaworski*

1 Introduction

During the weekend of 2–4 May 1997, 16 representatives from European countries met in Osnabrück, Germany, to establish a new society, the European Society for Research in Mathematics Education, in short ERME, to promote Communication, Cooperation and Collaboration (the three Cs) in mathematics education research in Europe. The three Cs came, over the years, to characterize the “ERME spirit”.

The full foundation of this society took place at CERME 1, the first conference of the society, in August 1998, also in Osnabrück. The first president of ERME was Jean-Philippe Drouhard (France, 1997–2001). Jean-Philippe had accepted to join us in writing this chapter, but he passed away suddenly in November 2015. We miss him. Paolo Boero (Italy), who was an active member of the group of scholars who initiated the process of establishing the new society and was a member of the first Board, became the second president of ERME (2001–2005), and Barbara Jaworski (United Kingdom), also a founding member, was the third (2005–2009). They were followed by Ferdinando Arzarello (Italy, 2009–2013) and Viviane Durand-Guerrier (France, 2013–2017). From February 2017, the ERME president is Susanne Prediger (Germany). The society has been governed from the beginning by an elected board with representation criteria from the regions of Europe.

In what follows, we first present the motivations for *establishing the new society* 20 years ago and the steps through which the original aims were achieved. In Section 3, we present a brief summary of a significant reflection that took place on *quality and inclusion* in the CERME conferences. In the last section, we report on further developments aimed at the *ongoing strengthening of the society*.

2 Establishing the new society

After CERME 1, three main challenges had to be met by the ERME Board elected in Osnabrück: (1) to take initiatives to prepare a new generation of European researchers in mathematics education according to the ERME spirit; (2) to involve a sufficiently large number of European researchers in mathematics education, in order for CERME to become the representative forum of European research in the field; and (3) to ensure stability to the new-born society through a legal status anchored in the laws of one of the European countries. At the same time, the society and its initiatives had to be opened to countries beyond Europe, in order to promote worldwide scientific exchanges in the field.

From the very beginning, the society has been linked to the organisation of the CERME conferences with a wide spectrum of themes and orientations to profit from the rich diversity in European research, as will be demonstrated in the various chapters of this book. The conferences have been structured around a number of Thematic Working Groups (TWGs), each with a designated research focus. Participants have particularly appreciated the concrete possibility, offered by the TWGs, to develop their personal research through systematic, collaborative work with other researchers engaged in the same area, and to get constructive feedback from them on their papers (before, during and after the conference). Since CERME 10 there has been an open call for new TWGs: the current call can be found on the ERME website (www.mathematik.uni-dortmund.de/~erme/). The International Programme Committee (IPC) of each CERME is elected by the ERME Board, as is the IPC chair. The choice of TWGs and their leaders is discussed in the IPC and approved by the ERME Board. The decision to include a new group is taken according to the nature, focus and relevance of the research, its potential to attract participants, and its distinction from existing groups. The choice of group leaders and co-leaders is an important lever for ensuring the quality of work in the group, for planning activities to include all those who wish to participate and for opening the society to the diversity of research in Europe.

From CERME 1, the tasks of drawing up a constitution and establishing a legal status were undertaken by the ERME Board mainly thanks to the extraordinary work performed by Board member Graham Littler (UK). On behalf of ERME, he approached the UK Charity Commission to request charitable status in the UK, and dealt with the very complicated paperwork involved. In CERME 3 a draft was presented and voted on, as a basis for the formal establishment of the society. This was proposed in the General Meeting at CERME 4 and ratified in CERME 5.

An important issue of ERME policy consists of supporting and educating new/young researchers in mathematical education. The Board elected at CERME 2 in 2001 gave the task of designing a Summer School to Paolo Boero, Barbara Jaworski and Konrad Krainer. It was to be held every two years in alternate years from the CERMEs. The main target population consisted of PhD students in mathematics education. By analogy with CERME, the Summer School was conceived as a working place for students. TWGs (led by “expert” researchers – about 60% of

the whole time) offered students a unique opportunity to present the current status of their research (be it initial, or near to the conclusion), to receive constructive feedback from the “expert” and from the other participants and to establish links with other young researchers interested in the same subject.

The first Summer School, held in Klagenfurt in 2002, showed that the design of the school was suitable to meet the students' expectations. Gradually the number of applicants rose from 40 in Klagenfurt to more than 100 (resulting in 72 participants) for the last four schools, including students from other continents. In parallel with the design and the implementation of the school, the Board helped to set up an informal branch of the society, YERME (Young European Researchers in Mathematics Education), to be involved in the preparation of the school and in other initiatives of interest to young researchers. The decision was formally taken in the General Meeting in CERME 8 when the first two representatives of young researchers in the ERME Board were elected. From the institution of YERME, the summer schools took on the abbreviation of YESS – YERME Summer School. YESS 9 is being prepared as this book goes to press.

Another important issue, from the beginning of ERME, CERME and YESS, was the encouragement of mathematics education researchers from Eastern Europe to join in ERME activities. For this purpose, ERME designated funds to contribute to travel and accommodation where financial hardship was demonstrated. When, in 2009, very tragically, Graham Littler died, the ERME Board decided to name this fund the Graham Littler Fund. Since then, this fund has been topped up regularly and used to provide financial support for participants to CERME and YERME where a need has been identified.

During the years 2001–2005, when it was important to ensure the representativeness of the new society, the Board worked hard to establish relationships with several research groups and existing national societies in the field of mathematics education; also researchers from other continents were invited to join ERME initiatives. As a result, the number of participants in CERME doubled at each conference until CERMEs 5, 6 and 7, when it stabilized at about 450–500 participants. It increased anew up to about 700 participants in CERMEs 9 and 10.

3 Scientific quality and inclusion in CERME conferences

From the very beginning, the issues of quality and inclusion in CERME conferences were main concerns of the society. Quality refers to scientific standards relating to papers presented and published, and to activity in the TWGs. Inclusion refers to ways in which participants are included in activities in the groups, and in presentations and published papers. CERME's policy of encouraging presentation (after two rounds of revision) of as many papers as possible was sometimes seen to act against high scientific standards. Seeking a balance between quality and inclusion was seriously problematic.

In CERME 6, held in Lyon, France, members of the ERME Board collected data in several ways from delegates at the conference concerning issues related

to quality and inclusion in CERME conferences. In response, one group leader wrote: “Being all-inclusive and academically qualitative are *a priori* incompatible.” While this is an individual view, it nevertheless flags a tension between inclusivity and scientific quality. Participants acknowledged that newcomers are drawn quickly into the activity of the group. There were almost no comments that suggested that group work was not friendly and welcoming, that participants were not (overtly) encouraged to take part and join in the discussions. In scientific terms, we can see inclusion to be facilitated through the review process in which a critical review can be helpful and supportive and enable the improvement of a paper. However, to quote Gates and Jorgensen (2009, p. 164), we were aware that “the field . . . in which the participants engage recognizes and conveys power to those whose habitus is represented and privileged in the field”. Thus, and according to Atweh, Boero, Jurdak, Nebres and Valero (2008, p. 445), “collaboration between educators with varying backgrounds, interests and resources may lead to domination of the voice of the more able and marginalization of the less powerful”.

Such considerations led Jaworski, da Ponte and Mariotti (2011) to start characterising inclusion and quality and to relate the characterisation to the specific aims of ERME in terms of Communication, Cooperation and Collaboration, in contributing to the ERME spirit. Inclusion is characterised in affective and scientific terms. Quality is characterised scientifically through “key ideas” and their development. The key ideas need to be there for scientific quality to exist at all; they need to be engaged with for scientific quality to be recognisable in the activity of a group. We present below how it works in CERME.

Starting communication: Participants have read the accepted papers; they come together with friendliness and the sincere desire to work inclusively. There are key ideas in the accepted papers to which the review process draws attention. Activity and discussion begin to encourage communication related to the ideas where the objective is to know each other’s ideas and relate them to each other.

Developing cooperation in engaging with debate: Group organisation enables a focus on the key ideas. Friendly and considerate interaction, with attention to language enables participants to start to engage with the ideas. The emphasis is on including all people attending a TWG, including those who do not present accepted papers, in order to get reactions, questions and comments. This group activity may contribute, on the one hand, to improving the accepted papers (in terms of their revision for publication in the final proceedings), and on the other hand, to improving the quality of all participants’ research work.

Developing cooperation in recognising ideas: Group leaders create activity to encourage a focus on getting participants engaged with the key ideas which are recognised. The emphasis is on reaching a quality of interaction relating to scientific ideas.

Enabling collaboration: Here we see an enabling of critical inquiry into the essences of the ideas; deep engagement of a scientific quality with deep probing of ideas and corresponding critical debate. From here, collaboration can begin.

It seems clear that, for the last point to be possible, both the second and third have to be achieved. This means dealing with the organisational challenges raised

in the CERME 6 survey, which are far from trivial. However, it could be that a theoretical perspective of this sort, of what is involved in achieving inclusion and quality in group work in CERME, can act as a basis for thinking about dealing with the challenges and conceptualising in practical terms what we are aiming for in CERME.

4 Carrying on strengthening the society

From 2009, the main activities of ERME, in particular CERME and YESS, continued and expanded further. We stress below some of the main ERME activities contributing to the development and visibility of the society.

During this period, the number of participants from developing countries was increasing, due to the growing effort of ERME to support people from those countries through the Graham Littler Fund. Two decisions, taken during the General Meeting in CERME 7, contributed to expanding ERME beyond the strict boundaries of Europe. The first concerned countries from North Africa. While these countries were living the exciting period of the so-called “Arab Spring”, some researchers were attending CERME 7 and claimed that the possibility for them to take part in events like CERME was important also for the progress of democracy in their own countries. Thus it was decided that researchers from these countries could apply for financial support from the Graham Littler Fund. The second was the decision of the ERME Board to organize the first CERME outside the European Union, namely CERME 8 in Turkey.

Since 2011 ERME has become an affiliated society of ICMI (International Commission on Mathematical Instruction). As such, ERME has the opportunity to present a quadrennial report to the General Assembly of ICMI, so that ERME activities can be known by the whole world community working in mathematics education. More generally, an important concern was to encourage the European research community to develop relationships with mathematicians and to take part in the ICMI activities. ICME-13 in Europe in July 2016 was a wonderful opportunity to improve the international visibility of ERME. As an ICMI affiliate organisation, ERME organised two sessions, one a general presentation of ERME with a focus on teacher education research, the other on YERME.

A further action of ERME in recent years concerned the issue of rating of publication outlets. As is well known, it is very difficult to escape the influence of the ranking and grading of scientific journals for the development of researchers' careers. However, the current systems are often based on crude bibliometric analyses that have little to do with scientific quality. This represents a disadvantage for researchers in the field of mathematics education. For this reason, ERME, together with the Education Committee of the European Mathematical Society (EMS), and supported by the ICMI, decided in 2011 to appoint a joint commission, with the aim to propose a grading of research journals in mathematics education based on expert judgment. The result of their careful and joint analysis, based on consultation with 91 experts from 42 countries, was a document rating the 17 most

important journals in mathematics education on a scale with four grades (Törner & Arzarello, 2012). We understand that this document has been used by some official boards charged with assessing the work and the consequent promotion of mathematics educators.

Another very important issue is the publication of the proceedings for each CERME conference. All the proceedings are available online on the ERME website and since CERME 9, thanks to Nad'a Vondrovà and Konrad Krainer, CERME Proceedings are also available on the open Archiv HAL, managed by the French CNRS (Centre national de la recherche scientifique). A detailed list appears at the end of the Preface of this book.

A further decision concerned the ERME Topic Conferences (ETC), which are conferences organised on a specific research theme or themes related to the work of ERME as presented in associated working groups at CERME conferences. Their aim is to extend the work of the group or groups in specific directions with clear value to the mathematics education research community. These initiatives take place during the year in which CERME does not take place. After careful preparatory work, in the General Meeting in Prague (2015), the rules for ETCs were approved. Three ETCs were held in 2016, and three were approved in 2017 for appearance in 2018.

5 Conclusion

A main objective of this Introduction is to present the evolution of the spirit and the activities of ERME to the reader of this book as background for the core chapters of the book (Chapters 1–18), which are based on the work of the TWGs at and between the conferences of the society. Another objective is that present and future members of the society, who have not taken part in its evolution and its activities, should know how choices have been made, and then re-thought and deepened (according to accumulated experience). In doing so, we wish to stress that the future of the society is in the hands of the members and not in the paragraphs of the Constitution.

In addition, we wish to illustrate how ideas about the society and its activities have progressively matured and how emerging needs have been taken into account. We hope that ERME's "historical" evolution will give a concrete idea of the openness of the society and of the deep reasons why CERME, YESS and the other ERME initiatives are organized in the present way.

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1 Chapter 1: From geometrical thinking to geometrical working competencies

Space and shape

Transformations

Methodology

of research

better models Bridging practical + theoretical approaches
Need of theory, paradigm and model

FIGURE 1.1 Main themes or issues of the group

allow the connection between theoretical and empirical aspects. Beside refer

ences to general psychological (Piaget's or Vygotski's works) or didactical theories

(Brousseau's or Chevallard's theories), specific theoretical and methodological

developments were provided during the meetings. The van Hiele levels, Fischbein's

figural concept and Duval's registers were among the most relevant cognitive and

semiotic approaches that were adopted. Regarding the epistemological and didacti

cal approaches, a decisive importance was granted to the geometrical paradigms

and the Geometrical Working Spaces, and more recently to a lesser extent, on the

notion of geometrical competencies. A great number of researches focused on differ

ent aspects of geometrical understanding, such as figural apprehension, visualisation

and the effort of conceptualisation in geometry using different methodologies and

frameworks. Among the numerous papers presented in the

Working Group on

Geometry, we can distinguish some recurrent topics that we present below in parallel

with the history of the group.

The first working group specifically dedicated to geometry was created in

CERME 3. Since this meeting various names were given to this group including

some nuances, from thinking of researching through teaching and learning, ending at

the last edition (CERME 10) with the sole name Geometry. On Figure 1.1, which

is drawn from the synthesis of CERME 7, themes and issues generated by the con

tributions are organised in a conceptual tree.

On Figure 1.2, some unifying characteristics related to geometric competencies

that have been developed during CERME 8 are presented.

Further on, we will illustrate the evolution of the group with a short overview

of the papers presented during these conferences, intending to show the richness

of the themes developed throughout these meetings.

2.1 Before the official creation of the group

In the first two sessions, even if the group did not officially exist, some communica

tions were already centred on geometry with a diversity of viewpoints. Some reflexive

and the reform of mathematics examination at a time when social and professional

mobility was a major issue. Other contributions insisted on the need of metaphors

and images (Parzysz, 2002) or focused on epistemological and cultural approaches

about geometrical knowledge (e.g. Arzarello, Dorier, Hefendehl-Hebeke, & Turnau,

1999; Burton, 1999). Mostly descriptive, the studies addressed the geometric content and often men

tioned some general mathematical skills or cognitive and instrumental dimensions

in geometric activity. They also indicated some “frictions” with other mathemati

cal domains like algebra, and some typical interactions in the use of technologies

with digital geometric software (Dreyfus, Hillel & Sierpinska, 1999). In addition, the natural relationship of geometry to proof and proving was regu

larly examined, both in theoretical and philosophical essays and in reports of studies

involving empirical research. They link proving, arguing, modelling and discover

ing processes. They also deal with the students’ way for proving and their use of

a logical discourse, figural signs and technology when they are faced with proof

exercises in geometry. Attention was also paid to the links between the discovery

process, visualisation and instrumentation processes with software (Jones, Lagrange

& Lemut, 2002). There were also studies concerned with the teacher education and professional

development in geometry and the organisation of courses (Houdement & Kuzniak,

2002). Despite the fact that several works deal with mathematical proofs, visualisation

and dynamic geometry, surprisingly, other natural links with geometry are virtually

missing during the first congresses. In fact, few researches focus on modelling of

physical phenomena using geometrical tools. Visual • observation • exploration Operational • instrumentation • instrumentalization Reasoning • argumentation • deductive Figural • to model • conjecture • define

FIGURE 1.2 Geometrical competencies

2.2 Thematic from CERME 3 to CERME 5

During these conferences, the new geometry group starts the study of geometrical

thinking using and developing the key concepts of paradigms, developmental stages

and generalisation in space (Houdement & Kuzniak, 2004; Gueudet-Chartier,

2004; Houdement, 2007). At the core of many researches, visualisation is often

directly connected to Duval's registers of representation (Perrin-Glorian, 2004;

Kurina, 2004; Pittalis, Mousoulides & Christou, 2007). In an original study,

the visualisation issue is raised for sighted and blind students (Kohanová, 2007).

Other papers focus on the notion of instrumentation, both with classical drawing

tools (Vighi, 2006; Bulf, 2007; Kospentaris & Spyrou, 2007) or digital tools (Rolet,

2004). Education for future teachers (Kuzniak & Rauscher, 2006) and reasoning

(Ding, Fujita & Jones, 2006; Markopoulos & Potari, 2006) appear as topics of

continuity, while concepts and conceptions remain key themes in many studies

(e.g. Modestou, Elia, Gagatsis & Spanoudes, 2007; Marchini & Rinaldi, 2006).

2.3 Thematic from CERME 6 to CERME 7

The theoretical and methodological dimensions of research in geometry remain

prominent topics, especially in further development of the notions of geometrical

work and Geometrical Working Spaces. Many points were considered as a com

mon background, as they were developed during former sessions. These points

were related to the use of geometrical figures and diagrams (Deliyianni et al., 2011)

and to the understanding and use of concepts and proof in geometry (Gagatsis,

Michael, Deliyianni, Monoyiou & Kuzniak, 2011; Fujita, Jones, Kunimune,

Kumakura & Matsumoto, 2011). For an epistemological and didactical approach,

researchers used the geometrical paradigms and geometrical work spaces. Attention

is also paid to 3D geometry forms of representation through the possible use of

digital tools (Mithalal, 2010; Hattermann, 2010; Steinwandel & Ludwig, 2011).

In addition to the usual geometrical topics, special attention is paid to general

or cross-cutting aspects, such as educational goals and curriculum in geometry

(Girnat, 2011; Kuzniak, 2011), communication and language (Bulf, Mathé &

Mithalal, 2011), the teaching, the thinking and the learning processes in geom

etry. Moreover, Mackrell (2011) questions the interrelations between numbers, algebra and geometry, especially in digital environments.

2.4 Restructuring in CERME 8 to CERME 10

More recently, the working group sought to revisit and extend the issue of geo-

metrical thinking and geometrical work by reformulating it in terms of four

geometrical competencies (reasoning, figural, operational and visual) organised

in a tetrahedron (see Figure 1.2). Each geometric competence constitutes a pole

of geometrical thought and it is the study of the link between these competences

that makes it possible to better understand the global functioning of geometri-

cal thought (Maschietto, Mithalal, Richard & Swoboda, 2013). Further on, at

CERME 9, there were more specific contributions about the way geometry is,

or should be, taught and the four competencies were used as a general way of

describing the geometrical activity and for creating links between different points

of view (theoretical and empirical). In this group the discussions were related to

geometry teaching and learning (Douaire & Emprin, 2015; Kuzniak & Nechache,

2015) and issues such as teaching practices and task design (Mithalal, 2015; Pytlak,

2015). Furthermore, cultural and educational contexts modifying the geometry

curricula were also discussed, introducing a new issue about the role of language

and social interactions in the teaching and learning of geometry. In CERME 10,

the four competencies were used to describe geometrical thinking: reasoning,

figural, operational and figural. The group took these dimensions as a background

that was very helpful to understand each other and to compare our approaches to

the issue of what is at stake in the teaching and learning of geometry. Three main issues were addressed during the working group: The role of

material activity in the construction of mathematical concepts, including using

instruments, manipulation, investigation, modelling . . . ; Visualisation and spa

tial skills; Language, proof and argumentation. In comparison to the previous

CERME, this time psychological points of view, among others, were represented.

This raised new questions, often with very different theoretical and methodologi

cal backgrounds sometimes far from mathematics.

3 Studying the teaching and learning of geometry

through a common lens

Since the creation of the group, the development of shared theoretical frameworks

was central to ground collaboration between participants. This point is particularly

evident in CERME 3, where the main theoretical concerns of the group were

summarised by Dorier, Gutiérrez and Strässer (2004). Their

classification is used to

highlight some theoretical approaches which have been reinforced by their pres

entation and discussion during CERME conferences.

3.1 Geometrical paradigms

The history of geometry shows two contradictory trends. First, geometry is used as a

tool to deal with situations in real life but, on the other hand, geometry for more than

two thousand years was considered the prototype of logical, mathematical thinking

and writing after the publication of Euclid's "Elements". These contradictory per

spectives are taken into account in geometry education by Houdement and Kuzniak

(2004) with geometrical paradigms. Three paradigms are distinguished. "Geometry I:

natural geometry" is intimately related to reality; experiments and deduction

grounded on material objects. "Geometry II: natural/axiomatic geometry" is based

on hypothetical deductive laws as the source of validation with a set of axioms as

close as possible to intuition and may be incomplete. "Geometry III: formal axio

matic geometry" is formal, with axioms that are no more based on the sensory reality.

These various paradigms are not organised in a hierarchy making one preferable to

another, but their work horizons are different and the choice of a path toward the

solution is determined by the purpose of the problem and the researcher's viewpoint.

Useful to provide a method to classify geometrical thinking, geometrical para

digms have also been helpful to interpret tasks eventually given to students and

future teachers and can be used to classify the students' productions, offering an

orientation for the teacher of geometry. In contrast to van Hiele's theory, this

approach is not dependent on the general thinking and reasoning development,

but relates more to an epistemological viewpoint.

3.2 Development stages

The so-called "van Hiele levels" of geometrical reasoning are among the most

used theoretical frameworks for organising the teaching and learning of geom

etry according to development stages. The description of the "van Hiele levels"

already gives hints to fundamental links between these levels and the model sug

gested by Houdement and Kuzniak (2004). There was some discussion whether

geometrical knowledge progresses through sequences of stages. Some papers

presented (Braconne-Michoux, 2011) appear as contradictory to the traditional

"van Hiele levels". This is especially true if the levels are linked to clearly identified

and fixed ages or if individual persons are thought to necessarily follow the order

of the respective levels. Positioning itself on ideas of stages of development and/or

learning is obviously important for constructing and discussing geometric tasks or

research projects. The stage levels can help to find and further develop appropriate

tasks, and they are obviously helpful for exploratory activities.

3.3 Registers of representation

Semiotic consideration has always been important in geometry and the distinction

of figure, in the sense of the most general object of geometry (either 2D or 3D) ver

sus its material representation, is classic in the field. Three main groups of semiotic

representation can be distinguished: material representation (in paper, cardboard,

wood, plaster, etc.), a drawing (made either with paper and pencil, or on a com

puter screen), and a discursive representation (a description with words using a

mixture of natural and formal languages). Each register bears its own internal func

tioning, with rules more or less explicit. Moreover, students have to move from

one register to another, sometimes explicitly, sometimes implicitly, sometimes

back and forth. Questions about registers of semiotic representation and cogni

tive processes have been studied in depth by Duval (1993). He defines "semiotic

representations as productions made by use of signs belonging to a system of rep

resentation which has its own constraints of meaning and functioning". Semiotic

representations are necessary to mathematical activity, because its objects cannot be

directly perceived and must, therefore, be represented.

3.4 Instrumentation with artefacts and digital tools

There is a broad consensus in the community of mathematics teachers and educators

that learning geometry is much more effective if concepts, properties, relationships,

etc. are presented to students materialised by means of instruments modelling their

characteristics and properties. Furthermore, the use of didactic instruments is very

convenient, if not necessary, in primary and lower secondary grades. There is a

huge pile of literature reporting the continuous efforts devoted by mathematics

educators since long ago to explore the teaching and learning of geometry with the

help of manipulative, computers, and other tools. Unfortunately, many researchers

interested in this topic have generally preferred to participate in the CERME group

on technology and thus have deprived, in some part, the group of discussions on

this subject.

3.5 On Geometrical Working Spaces

In relation to semiotic, instrumental and discursive proof, the idea of global think

ing on geometric work was first introduced in CERME 4 (Kuzniak & Rauscher,

2006) and was based on the model of Geometrical Working Spaces (GWS) pre

sented during a plenary lecture at CERME 8 (Kuzniak, 2013).
The basic idea

behind this approach is that some real geometrical work
appears only when a

student's activity is both coherent and complex to develop
a reasoning using intui

tion, experimentation and deduction. The GWS is conceived
as a dynamic abstract

place that is organised to foster the work of people
solving geometry problems.

The model articulates two main concerns, the one of
epistemological nature, in a

close relationship with mathematical content of the studied
area, and the other of

cognitive nature, related to the thinking of the person
solving mathematical tasks.

This complex organisation is generally summarised in the
diagrams in Figure 1.3

(for details, see Kuzniak, Tanguay & Elia, 2016).

Visualisation Representamen Referential Epistemological
plane Cognitive plane i n s t r u m e n t a l g e n e s i s
Construction Proving [Sem-Ins] [Sem-Dis] [Ins-Dis]
Artefacts R V P T A C s e m i o t i c g e n e s i s d i s c
u r s i v e g e n e s i s

FIGURE 1.3 The Geometrical Working Space diagrams

For describing the work in its epistemological dimension,
there are three com

ponents in interaction, organised according to purely
mathematical criteria: a set

of concrete and tangible objects, the term sign or
representamen is used to sum

marise this component; a set of artefacts such as drawing
instruments or software;

and a theoretical system of reference based on definitions,
properties and theorems.

In close relation to the components of the epistemological level, three cogni

tive components are introduced as follows: visualisation related to deciphering and

interpreting signs; construction depending on the used artefacts and the associated

techniques; and proving conveyed through processes producing validations, and

based on the theoretical frame of reference.

The process of bridging the epistemological plane and the cognitive plane can

be identified through the lens of GWSs as genesis related to a specific dimension

in the model: semiotic, instrumental and discursive geneses. In order to understand

this complex process of interrelationships, the three vertical planes of the diagram

are useful and can be identified by the genesis which they implement [Sem-Ins],

[Ins-Dis] and [Sem-Dis] (Figure 1.3).

To the coordination of the geneses in the GWS model, there are three inter

nal fibrations that focus on the role of tools, controls and representations in the

mathematical work which allow us to describe the complexity of managing

the relationships between all the components involved in geometrical activity.

Richard, Oller and Meavilla (2016) have introduced three internal fibrations that

focus on the role of tools, controls and representations in the mathematical work.

This model was briefly discussed in CERME 10.

3.6 Within and beyond the model of GWS

The GWS model results largely from the work done in the group of geometry

where its use led to understanding that the model can easily be adapted to relate

with other theoretical approaches. This means that, depending on the issues and

problems dealt with, it can be complementarily associated with other theories:

(a) to increase its contextual perspective, such as the Mathematics Teachers'

Specialised Knowledge (MTSK), the Action, Process, Object and Schema (APOS)

theory, the Activity Theory (AT) or the Theory of Didactical Situations in

Mathematics (TDSM); (b) to refine it according to the needs of the studies, such as

Peirce's semiotics (1931), Rabardel's cognitive ergonomics (1995) or Richard and

Sierpinska's (2004) functional-structural approach.

Using the GWS model makes it possible to question in a didactic and scientific

way - mostly non-ideological - the teaching and learning of geometry, straddling

the mathematical education and the didactics of the field. For example, we know

that curricula change frequently or that the meanings of the words or concepts of

reference vary from one region to another, which is a problem from a research

point of view.

4 Building and developing a spirit of communication

and collaboration

As previously underlined, a strong point of the group is participants' emphasis on

relating theoretical and empirical aspects of research in geometry education. The

continuity of this trend was evident through the meetings of the group so far, as the

participants were mainly discussing results of empirical or developmental research

studies and theoretical reports about the teaching and learning of geometry. The

need for a common framework related to geometry education appeared necessary

in the working group in order to stimulate the discussions among members and to

allow the capitalisation of knowledge in the domain. Due to collaborations initiated during CERME meetings with colleagues from

France, Cyprus, Spain, Canada, Mexico and Chile, it has been made possible to

develop a joint theoretical framework. The framework should be dedicated to study

the teaching and learning of geometry, space and shape on the whole educational

system and should be neutral in the sense that it can be used to facilitate exchanges

in different countries and institutions. According to this need, the framework of

geometrical paradigms, as explained above, was introduced in the CERME 3 con

ference. In the pursuit of results at the CERME 3, the geometry working group of

CERME 4 and 5 continued by looking into geometrical paradigms. In particular, in CERME 4, Kuzniak and Rauscher (2006) analysed pre-service

schoolteachers' geometrical approaches, based on the notion of geometrical para

digms and levels of argumentation. They found that students' levels of understanding

and memorisation of the bases of the elementary geometry differ greatly and that

they keep the practical use of geometry. Although their study was conducted with a

particular population, their results can be useful for evaluating the long-term effects

of education in geometry. Moving forward, in CERME 5, during discussions about the possible uses of

geometrical paradigms, new participants of the group initiated a discussion about

the real benefit of this approach. Perspectives in these directions were given by

Houdement's (2007) and Bulf's (2007) papers. In fact, Houdement highlighted the

uses of this approach for comparing curricula in different countries and for reflect

ing on the necessity to teach Geometry II and the proper way to introduce it. In

Bulf's effort to examine the link between geometrical knowledge and the reality in

relation to the concept of symmetry, this approach was useful for tracing a double

play between the Geometry I and Geometry II on one side and Reality/Theory on

the other side. Furthermore, Kospentaris and Spyrou (2007) used the approach of

paradigms in teachers' education, as done in CERME 4 by

Kuzniak and Rauscher

(2006). Their results were in line with previous results presented at CERMEs

3 and 4 about pre-service teachers' geometrical thinking. They actually found

that visual strategies or measurement using tools are used by students at the end of

secondary school, interpreting it not by a developmental approach, but based on

the geometrical paradigms. The discussion of the aforementioned papers gave a

future perspective for the group in order to make precise the sufficiency of the so

far existing theoretical tools for determining the nature and the construction of the

GWS used by students and teachers. In fact, no paper was traced in the following

CERMEs that challenged these new theoretical tools.

Following up the spirit of the previous years, the participants in CERME 6,

who came from both Europe and America, have extended and enriched the results

obtained so far. Until then a common background was built and known by expe

rienced participants, thus the participants worked within the continuity of the

former sessions of CERME and their discussions were effectively facilitated by this

common culture. In fact, the participants came to two main conclusions regarding

the use of theory in research: (1) theory can serve as a starting point for initiating

a research study and (2) theory can act as a lens to look

into the data. An example

of such research is that of Kuzniak and Vivier (2010) who examined the Greek

Geometrical Work at secondary level from the French viewpoint, using a theo

retical frame based on paradigms and GWSs. Also, Panaoura and Gagatsis (2010)

compared the geometrical reasoning of primary and secondary school students,

based on the way students confronted and solved specific geometrical tasks, find

ing difficulties and phenomena related to the transition from Natural Geometry

to Natural Axiomatic Geometry. Therefore, a perspective for future research on

geometry theories and their articulation for the group was the use of geometrical

paradigms as a tool for analysing existing curricula and students' behaviour.

The creation of a common spirit of communication has also built ideas of collab

oration between participants through the discussions of the group. This was evident

regarding the focus of research in specific educational levels. Actually, at the first

discussions of the working group the attention was given to primary education, as

many of the papers were about young students' geometrical concepts (Marchetti,

Medici, Vighi & Zaccomer, 2006; Marchini & Rinaldi, 2006) and the role of spe

cific tools for the teaching and learning of geometry at that age level (Vighi, 2006).

However, in subsequent meetings of the group (in CERME 5), collaborations

were envisaged about the transition from a lower to a higher educational level and

also the adaptation of a common framework to work out such kinds of studies as

paradigms, GMS, spatial abilities and conceptions about the figure.

This was succeeded in the next meeting of the group in CERME 6 as, among

the research presented in the group, the dimension of the students' transition from

primary to secondary school was also taken into account. For example, Deliyianni,

Elia, Gagatsis, Monoyiou, and Panaoura (2010) investigated the role of various

aspects of figural apprehension in geometrical reasoning in relation to the students'

transition from primary to secondary education, revealing differences between the

two groups of students' performance and strategies in solving geometrical tasks.

In a similar sense, Panaoura and Gagatsis (2010) compared primary and secondary

school students' solutions of geometrical tasks and stressed the need for helping

students progressively move from the geometry of observation to the geometry of

deduction as they transit to a higher educational level.

Finally, in an effort to build a spirit of communication and collaboration

in the group, collaborations between experienced and new researchers were

accomplished (e.g. the work of Deliyianni et al. (2011) and Gagatsis et al.

(2011)). These common works facilitated not only the communication between

old and new research, but also the collaboration between researchers from dif

ferent countries (e.g. France and Cyprus).

5 Toward a common research agenda?

Even if exchange within the group is still very rich and exciting, the geometry

group seems to have gradually forsaken some of its initial ambitions, because of the

existence of various groups specifically interested in technology use, proof, teacher

education, semiotic aspects, etc. It has been partially disembodied and deprived of

what has always been the strength of geometry: its transdisciplinary contribution

to human thinking. Another challenge faced by the group comes from its difficulty to capitalise its

results and findings because of two kinds of volatility. The first is natural and comes

from the renewal of participants who are younger and younger, and sometimes

beginners in the field. Experienced researchers were attracted by other groups,

developing topics closer to their own researches. On the other hand, this move

ment is also positive in the sense that new participants might give new ideas and

perspectives to these groups. Another reason for the difficulty to capitalise in the field is the constant cur

riculum changes in geometry. This can be illustrated by the

erratic presence of

geometric transformations. Moreover, and fundamentally, geometrical activity

seems more and more oriented to other mathematical fields through modelling

activity based on geometrical support, such as physics, geography, etc. To overcome some of these problems, we suggest that the viewpoint on geomet

rical work could help to shape a common research agenda aiming at understanding

better the competencies involved in geometrical work through and beyond the

whole education. It requires coordination between cognitive, epistemological and

sensible approaches, structured around three complementary dimensions which

relate to visual, experimental and reasoning competencies.

5.1 On semiotic work and visual competency

Geometry is traditionally viewed as a work on geometrical configurations which

are both tangible signs and abstract mathematical objects. This difference is clearly

identified under the classical opposition drawing versus figure, which focuses on

strong interactions and differences between semiotic and discursive dimensions.

The semiotic dimension, especially worked through the visualisation process, is

at the centre of Duval's research, which developed very powerful tools, such as

registers of semiotic representation, to explore the question. In his view, a real

understanding of mathematical objects requires interplay

between different reg

isters which are the sole tangible and visible representations of the mathematical

objects. Within a multi-modal approach on semiotic resources, it is possible to

study the entanglement of mathematical objects and their various semiotic repre

sentations. One of the main issues will be to see how these links are formed and

reformed and can hide the very nature of geometrical objects to students.

5.2 On instrumental work and experimental competency

Geometry could not exist without instrumented activities or drawing tools, and the

study of their different uses allows identification of two types of geometry which

are well described by Geometry I and Geometry II paradigms. Some construc

tions are possible or impossible depending on tools or milieus. Thus, the trisection

of the angle is impossible with ruler and compass, but quite possible with origami

or mechanical instruments. The same is true for duplicating the cube. In general,

it is possible to understand how, and justify why, constructions with drawing tools

are effective, by questioning the structure of the mathematical objects, especially

the nature of numbers, involved in these constructions. From precise draw

ings based on mathematically wrong constructions (like the Dürer pentagon) to

mathe matically correct but imprecise, in the sense of

their measure, constructions

(like Euclid's pentagon), it is possible to see all the epistemic conflicts opposing

constructions based on an approximation to constructions based on purely deduc

tive arguments. The tension between precise and correct constructions has been

renewed with the appearance of dynamic geometry software (DGS). As Strässer

(2002) suggested, we need to think more about the nature of geometry embedded

in tools, and reconsider the traditional opposition between practical and theoretical

aspects of geometry.

5.3 On the work around proof and reasoning competency

Since antiquity, demonstration work in its demonstrative form has always been

emphasised in geometry, considered as a kind of ideal of rational thought grasped

in its most intuitive and visible form. But in education, this idealised form of

advanced mathematical work is not so obvious to discover and to implement,

because it is hidden behind the play between practical and formal geometric para

digms. Moreover, thanks to its graphical precision, the software finally convinces

users of the validity of the results. Proofs are no longer only formal and the modes

of argumentation are enriched by experiments that give new meaning to the clas

sical epistemological distinctions between the iconic and non-iconic reasoning

and relate to discursive-graphic reasoning (Richard, 2004).
Increasingly, human

reasoning is supported on representation and processing
delegated to a machine

(Richard et al., 2016). It remains to understand how to
animate the figures (instru

mental aspect) by coordinating better the semiotic aspects
associated with figures

and the discourse of proof in natural language.

5.4 On modelling competency in geometrical work

Being as old as the first forms represented on walls of
prehistoric caves, the very

constitution of the geometric model is certainly an
incarnation of what is the

modelling of space and forms. Unfortunately, modelling is
not widely practised in

compulsory education, and problem solving in geometry
classes too often relies on

classical and not open tasks. In several European
curricula, emphasis is now placed on the notion of meas

urement in order to understand certain geometric
properties. The passage from

synthetic geometry to arithmetic and algebra is thus
favoured. However, the geo

metric model remains necessary to develop the visual
discovery of new properties,

but also of proofs without recourse to measurement and
algebra. Geometry is

always one of the ingredients of the discovery of the
beauty of mathematics and is

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12 Chapter 12: Classroom practice and teachers' knowledge, beliefs and identity

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