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Linking researching with teaching: Towards synergy of scholarly and craft knowledge

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This paper will focus on relationships between researching and teaching; between researchers and teachers working to develop a knowledge-base for mathematics teaching. In doing so, it will treat 'researching' and 'teaching', 'researcher' and 'teacher', simply as convenient typifications, recognising the possibility that institutions and individuals may participate in both practices and take on both roles. Moreover, as the last sentence illustrates, this paper will reserve 'practice' for use in the sense of social practice; it will employ the more direct terms 'teacher' and 'teaching' to refer to what some sources will speak of as 'practitioner' and 'practice'. The knowledge-base for teaching will be seen as drawing both on scholarly knowledge created within the practice of researching, and on craft knowledge created within the practice of teaching. A particular concern will be with how greater synergy can be fostered between these distinctive practices, their characteristic forms of knowledge, and the associated processes of knowledge creation.

Mathematics education as a research domain: between academic acceptance and pedagogical pertinence

A recent study, under the auspices of the International Commission on Mathematical Instruction (ICMI), suggests that 'mathematics education as a research domain' is still engaged in 'a search for identity' (Sierpinska & Kilpatrick, 1998). The study reveals a community held together not by a common idea of research, but by research as a common ideal. While most of the contributors identify the development of knowledge and resources capable of supporting the teaching and learning of mathematics as an important goal for the field, there is disappointment about the achievement which has been demonstrated on this score.

Invited to reflect on the deliberations of the study within the ICMI publication itself, Bishop (1998) expresses concern over 'researchers' difficulties of relating ideas from research with the practice of teaching and learning mathematics' (p. 33). While noting 'some signs that research and researchers are relating more closely to the ideas of reform in mathematics teaching' (p. 35), he argues that 'researchers need to engage more with practitioners' knowledge, perspectives, work and activity situation, with actual materials and actual constraints, and within actual social and institutional contexts' (p. 36). Reviewing the ICMI publication, King and McLeod (1999) find 'surprisingly little... about the implications of well-developed research areas for classroom practice' (p. 232), and suggest that: 'At the same time that researchers in mathematics education have been shifting paradigms, other researchers with more traditional views have been busy... arguing against educational reform' (p. 231). More trenchantly, Steen (1999) concludes: 'From this thicket of meaning-challenged words and related disputations about goals, aims, methods, criteria, and results emerges a single irony that seems to enjoy widespread assent: Research has had essentially no impact on the practice of mathematics education' (p. 240).

The ICMI study expresses a trend which can readily be seen by examining how, over the last 30 years, the concerns of international congresses in mathematics education have shifted, and the contents of leading international journals have changed (Bishop, 1992; Boero & Szendrei, 1998). There has been a move from *inter*-national exchanges about the teaching of mathematics towards *supra*-national dialogues about research in mathematics education. Hence, while there remains a very real sense in which, as Sekiguchi (1998) puts it, 'educational research is essentially local practice, the major part of which consists of practical studies in socially and culturally bounded places and communities' (p. 395), the emerging international research community has become an important audience for -and potential influence on- such studies. It has created its own distinctive culture of research, detached from the more pragmatic and locally contextualised concerns of national teaching communities.

As Silver and Kilpatrick (1994) have noted, this international community has 'tended to take a path of least resistance, focusing on topics that are relatively easy to discuss internationally' (p. 749) so that 'conversations about the learning of specific mathematics content and processes or about theoretical issues have tended to occur at a level that allows members of the community to bypass important aspects of the conditions and traditions of educational practice within countries' (p. 750). Consequently, 'important research questions may be largely ignored within the international community because they do not relate readily to abstractions or universals, requiring instead attention to the nuances of local educational settings' (p. 750).

Yet, such nuances -and their significance- may be too readily taken for granted. Gouldner (quoted by Hargreaves, 1999) contrasts conceptions of the applied social scientist as technician and as clinician; suggesting that the technician tends to take problems at face value, as formulated by the client, whereas the clinician makes his own independent assessment of the client's problem: 'Not only does the clinician assume that the client may have some difficulty in formulating his own problems, but he assumes, further, that such an inability may in some sense be motivated and that the client is not entirely willing to have these problems explored and remedied'. Therefore, the clinician 'does not take his client's formulations at their face value', argues Gouldner, 'but he does use them as points of departure in locating the client's latent problems' (pp. 241-242). The formation of an international research community has created new possibilities of recognising, examining, challenging and suspending local assumptions.

The development of this international community has been encouraged by the widespread trend for universities to play a greater part in teacher education, and the corresponding drive to gain recognition for education as a research field. This has created a new generation of career mathematics educators with a much stronger identity as academics, influenced by dominant representations of research and valorisations of it. Silver and Kilpatrick point to a 'prevalent tendency to emphasize the connection of one's scholarly work to the academic disciplines rather than to educational practice'. This they attribute to the 'important pragmatic concern [of] many mathematics educators... to establish... the academic quality and rigor of their research' (p. 739).

The often precarious status of mathematics education as a research subspecialism -sometimes located within mathematics, more commonly within education, but consistently on the margins of the host field- has heightened aspirations for academic acceptance. Institutional location has also shaped the terms of such acceptance, influenced by established models of knowledge and enquiry in the host field. Against this background, Valero and Vithal (1998) suggest that researchers may be prone to choose methodological integrity over educational relevance when these appear to conflict. Specifically, researchers may prefer to work in less problematic research environments, and to address topics of only marginal relevance to the mainstream of mathematics teaching, rather than risk having their research considered methodologically poor.

Here, there are echoes of Schön's wider critique of technical rationality as a basis for professional activity:

> On the high ground [of professional practice], manageable problems lend themselves to solution through the application of research-based theory and technique. In the swampy lowland, messy, confusing problems defy technical solution. The irony of the situation is that the problems of the high ground tend to be relatively unimportant to individuals or society at large, however great their technical interest may be, while in the swamp lie the problems of greatest human concern. The practitioner must choose. Shall he remain on the high ground where he can solve relatively unimportant problems according to prevailing standards of rigor, or shall he descend to the swamp of important problems and nonrigorous enquiry? (Schön, 1987: p. 3)

Nevertheless, pedagogical pertinence remains a prominent concern of mathematics education researchers, not least because most also work concurrently as mathematics teacher educators and/or as teachers of mathematics. This provides motivation to establish a persuasive and productive relationship between their research and teaching activities; a motivation likely to be strengthened by the expectations they encounter, as researchers, in forming working relationships with teachers.

Relationships between researchers and teachers: purposes, perspectives and power

Relationships between researchers and teachers can be characterised in terms of three ideal types, starting with traditional data-extraction agreements, and shifting towards more intensive and reciprocal collaboration in the form of clinical partnerships or co-learning agreements (Wagner, 1997). In data-extraction agreements, collaboration extends only as far as negotiation between researchers -as seekers of data- and teachers -as sources of data, or gate-keepers to it- regarding reasonable terms of access. While collaboration on this basis may offer useful insights into the practice of teaching, it provides little opportunity for interaction between the thinking of researchers and teachers. In a clinical partnership, collaboration extends to give teachers a part in formulating and conducting investigations: however, researchers retain responsibility for the process of enquiry, and it is the practice of teaching which is the subject of analysis and reform. In a co-learning agreement, teachers become more active counterparts in the process of enquiry, and the practice of researching also become subject to analysis and reform.

Closer collaboration and deeper interaction between researchers and teachers is liable to lay bare important differences of perspective, calling for sensitive management and constructive dialogue. In exemplifying and examining such differences, this section will draw on two sources: a refreshingly candid account by Wiske (1995) of a collaborative programme in which researchers worked with teachers to find ways of using information technologies to teach mathematics, science and computing more effectively at the high-school level; and the reflections of Newman, Griffin and Cole (1989) and their collaborating teachers as they worked together to develop curriculum modules aimed at supporting cognitive change. Episodes have been chosen as illuminating breakdowns in collaboration; they should not be taken as typifying the projects concerned, both of which were largely successful in building cordial and productive working partnerships.

In one vignette, Wiske describes the tensions emerging within one project group where teachers were concerned that students often had no idea of what operation to use in solving word problems, tending to rely on ritual manoeuvres rather than problem analysis. The researcher in the group construed the situation differently, as symptomatic of a more fundamental lack of understanding of 'intensive quantities' on the part of students.

> The teachers defined the [issue] in terms of the types of problems, taken from their texts, tests, and workbooks, that students frequently failed. The professor... defined the [issue] in terms of an underlying mathematical concept, described with language that was unfamiliar to most of the teachers... [The professor] recalls the early conversations with teachers... as full of conflict. He and the teachers became 'polarized' over the way they defined the important questions worth investigating... [His analysis] 'was largely construed as abstruse and theoretical and without purpose'. (Wiske, 1995: p. 193)

Here, teachers are intent on addressing students' needs, exploiting the resources available to them; resources which help to define these needs and the means by which they can be addressed. Prime amongst these resources are texts and tests; as well as pedagogical approaches developed in the course of, and in relation to, working with these resources. This is characteristic of the way in which skilled practical thinking incorporates the task environment, and exploits setting-specific knowledge (Scribner, 1986). Equally, the researcher is intent on analysing students' difficulties, as defined within his task environment, exploiting the resources available to him.

Probing deeper, then, we see that, far from collaborating within a common task environment, researchers and teachers are seeking to co-ordinate their distinctive practices, and to co-operate within them. Another vignette draws this out further.

> [A teacher] participated in pilot testing some lessons with small groups of students... and recommended several significant alterations to make the lessons more practical with whole classes... Her recommendations were based on two concerns. First, the 'flow of ideas' was not sufficiently clear and, second, the lessons involved too much telling by the teacher... 'You don't just tell the students to do this... you have a discussion so they

> understand what they're doing'. When she was asked a few months later to teach the revised experimental unit in her class, she found that 'some of the things we had said absolutely would not work had been put back into the lessons'. (Wiske, 1995: p. 197)

Here, the teacher was drawing on her craft knowledge in adapting the lesson designs of the researchers, grounded in cognitive theories, to the conditions of class teaching and the characteristics of students. But, because such issues were not salient in the researchers' theoretical frame, the teacher's advice did not register. Eventually, the researchers came to recognise how this teacher could help them to revise lessons to make them more feasible for classroom use. But, it appears that this happened only when they themselves started to engage with the teacher's task environment through co-teaching. As one research assistant put it: 'When the teachers say it's not possible you think, well, they just don't understand... But the fact is you don't know until you do it... Having to think what you want to do and do it at the same time is not easy' (p. 198); And, in the words of another research assistant: 'You have an idea on paper, but when you try to chunk it up into classes, the connections get lost... Watching one kid [in a clinical study], you forget how much of the problem comes from the constant distractions in a class' (p. 198). Even here, however, classroom experience seems to be being construed unfavourably against research norms.

Another vignette brings out the different purposes and perspectives of teachers and researchers. Wiske describes the reaction of one teacher to conducting a series of clinical interviews jointly with a researcher, in which students were expected to puzzle over a complex system, even to the point of frustration.

> The teacher regarded the clinical interview as an educational experience for the student. She wanted the child to be treated as the teacher would have treated her in class, not allowed to feel stupid or discouraged by a prolonged period of ignorance unlike anything the teacher would willingly sustain in class. (Wiske, 1995: p. 203)

Not only is this episode illustrative of 'struggles between researchers eager to understand how children's minds work and teachers who felt pressed to educate these minds' (p. 195); one also senses a teacher expressing a concern to respect and nurture students; to set a moral example as well as an intellectual one.

Newman, Griffin & Cole encountered similar issues of differing perspectives and purposes.

> One of the basic conflicts between teachers and researchers is in the fact that, for the teacher, it is important to find ways in which children can succeed as well as possible in their academic work. Yet... the researchers... were also interested in the ways and situations in which children were having difficulties with cognitive tasks... [One] teacher took it as her responsibility to make sure that lessons went as well as possible once the planning phase was over, no matter what the logic of the research demanded. Sometimes she would modify the lesson, using her intuitions about the needs of individual students. This complicated life for the researchers. It would have been convenient, from our viewpoint, for her lessons to be uniformly structured... But the changes eventually became part of the data... [R]esearch, as well as teaching, often needs to be modified as the process under observation unfolds. (Newman et al., 1989: pp. 145-146)

Quinsaat, another of the collaborating teachers explains her 'advocacy' for students during the course of the research: 'Research is intended to be a benefit for the children in the long run. But in the immediate circumstances, it is up to the teacher to protect the child from research situations which might violate their rights' (p. 143). She identifies similar issues affecting teachers.

> Many teachers I know assume that educational researchers end up exposing and criticizing the practitioner... It is easy to see how teachers might get this impression from the kind of research that is published about teachers and schools... Why, one might ask naively, should a competent teacher worry? If everything was going alright there would be nothing to hide. This point of view really *is* naive. I am willing to admit that things go wrong in my classroom more often than I would like, as would any honest professional... It would be extremely easy to find cases which could be embarrassing.

(Quinsaat, quoted by Newman et al., 1989: p. 144)

Wiske, too, acknowledges issues of status, influence and power, noting the way in which university-based researchers tend to be 'more equal' than school-based teachers, so that ' [w]hile most... participants recognized that the academics made a good faith effort to collaborate, school people found that the university people's world view tended to predominate in the design, conduct and interpretation of the research' (p. 206). Nonetheless, through what Wiske characterises as sustained commitment, reciprocal exchange and mutual education, the collaborative programme was able to progress beyond conflicting perspectives to arrive at cooperative purposes.

> When the [professor] shared readings that informed his conceptual framework [for understanding word problems], other members of the group were able to join him in further refining and applying this framework to their shared work. When a researcher and teacher sat down long enough to explain to each other their expectations about the appropriate way to conduct a clinical interview, they were able to invent a way of proceeding that made sense to both of them. When researchers traded roles of teacher and observer with collaborating teachers, their eyes were opened to insights previously invisible to them. (Wiske, 1995: pp. 208-209)

In terms, then, of the ideal types sketched at the start of this section, both of these projects seem to have been conceived originally as clinical partnerships, but to have moved to some degree towards co-learning agreements, as the assumptions and methods, not just of teachers but of researchers, came under scrutiny and became the subject of negotiation. These projects illustrate how the practices of teaching and researching each involve distinctive types of purpose and perspective. These differences were accommodated not so much by establishing common purposes and perspectives, as through finding ways in which the purposes and perspectives of the two practices could be co-ordinated. This made possible the co-operation of the practices of teaching and research. Such accommodation, co-ordination and co-operation was facilitated by some degree of engagement of each group in the practice of the other. Equally, there seems to have been a process of tacit negotiation through which each group regulated its degree of accommodation to the other, thus shaping the character of the co-operation. These issues of purpose and perspective, and of the exercise of power, emerge as central to understanding collaboration between researchers and teachers.

Role differentiation and role (dif)fusion: the examples of didactical engineering and research for innovation

An illuminating contrast in the conceptualisation of the respective roles of researchers and teachers in collaborative research has arisen between the French school of 'didactical engineering' and the Italian school of 'research for innovation'. The two approaches share a concern to develop teaching designs, identifying and analysing the didactical variables coming into play, and the related didactical strategies available. However, didactical engineering aims to develop highly precise designs which will be reproducible under suitably controlled classroom conditions; and to do so through systematic and exhaustive analysis of variables and strategies, framed in terms of an overarching didactical theory. By contrast, research for innovation aims to develop prototypical examples of designs in the expectation that they will be adapted to differing classroom circumstances; and to study these under the resulting conditions of natural variation, guided by a serendipitous theoretical eclecticism.

Bartolini Bussi (1994) suggests that, in comparing the two approaches, the role accorded to teachers 'acts as a litmus paper' (p. 123). She contrasts the sharp differentiation of teacher and researcher roles in didactical engineering, against their fusion in the role of teacher-researcher within research for innovation. This relates, in turn, to two further contrasts: between the responsibility of teachers, in the former, to implement precisely defined designs engineered by researchers, as against the expectation, in the latter, that teacher-researchers will not just contribute to the development of prototypical designs, but adapt them to their circumstances; and next, between the emphasis, in the former, on systematic observation of classroom activity by detached (albeit often teacher) observers, as against, in the latter, participant observation of their own classrooms by teacher-researchers in action. However different in character, each of these approaches has proved effective in sustaining longstanding research collaborations within its particular cultural setting.

Brousseau (in a text dating from 1975) outlines a basic experimental procedure for didactical engineering :

> [The research team] wishes to create a phenomenon in a precise, reproducible way, and to observe it... The development of the lesson is provided down to the smallest detail on the didactical proforma. This sheet is given to all observers before the lesson so that they can peruse it... At the end of the lesson, the children's work is collected; the written records and the observation grids are brought together. These documents are then examined during the working session which takes place after the observation. Everyone offers an opinion. It is necessary to determine in this way whether the predetermined objectives have been achieved. (Brousseau, 1997: pp. 277-278)

For the teachers involved in this process, Greslard and Salin (1999) suggest that establishing agreed written rules governing the collaboration, and institutional channels through which conflicts can be signalled and regulated, is very important. It allows teachers to be sure that, in the last resort, their professional prerogative will take precedence. Nevertheless, they add that 'this is true only if the reasons which make them refuse to do what the researcher proposes are based on the ordinary constraints of a teacher' (p. 31). This approach to didactical engineering, then, can be seen as a highly codified form of clinical partnership in which teachers agree to give researchers' purposes and perspectives an unusual degree of influence over the way in which they work.

Explaining the crucial points of contention which emerged between didactical engineering and research for innovation, Bartolini Bussi (1998) cites not only 'the limited role (if any) that the teacher had in the early elaboration of the theory', but the emphasis on detached observation which 'put the teachers under a lens directed by the university researchers [and] clashed against the tradition of a peer cooperation'. Yet there are further nuances to this issue of observation. The approach employed in didactical engineering implies an important depersonalisation of the teachers' actions. To the extent that a teacher is viewed as putting a predetermined design into action, it is that design which is under observation rather than the teacher, who is simply its agent. However: 'This is often difficult for the teacher. S/he must understand that the participants are speaking of his/her action as that of an actor caught in a network of constraints' (Greslard et al., 1999: p. 30).

Moreover, Malara (1999), while acknowledging the complexities of both taking the teaching role and decentring from it, offers a further argument for the shift from detached to participant observation within research for innovation: that external observers can never know the pupils as well as the teacher does, and that this limits their capacity for interpretation. Indeed, the proponents of reflective teacher research (Hatch & Shiu, 1998) and of 'researching from the inside' (Mason, 1998) would make a still stronger case for participant observation as a means of gaining access not just to richer contextual knowledge, but to the teacher's inner sense. Malara (1999) acknowledges the complexities that this introduces in conducting research for innovation 'since each participant has his/her point of view which contains implicit beliefs, expectations and even fears' (p. 50). She points to the way in which differences in the background of teachers, their teaching styles, their personal preferences, and the differing ways in which they construe the teacher-researcher role, all influence the teaching undertaken in their classrooms and the way in which it is researched. In attempting to bring together the teacher and researcher roles in this way, then, there is a danger of role diffusion.

In championing the role of teacher-researcher, and the method of participant observation by teacher-researchers, research for innovation can be seen as aspiring to a form of co-learning agreement. Nevertheless, here again the situation is more nuanced. While Malara (1999) describes the research collaborations as 'for finding a bottom-up solution to the teachers' real needs and for responding to their problems in the least academic and most practical way possible' (p. 39), the organisational structure and group dynamics of collaboration create a more familiar process of problem definition:

> Even though for a long time in our group, the problem of the passage from arithmetic to algebra ha[d] been seen as a possible object of research, the teachers were reluctant, either because of its width/complexity or because of [] disagreeing opinions about times and ways of initiating [] pupils to the use of letters... For a certain period of time the problem was put aside, but only apparently because… we university-researchers guided the discussion onto some ad-hoc articles [] which we suggested [] reading. This first phase of slow "underground" work was useful for the awakening of the teachers to the questions linked to this theme. Then the organization of a cycle of meetings with researchers who ha[d] already been working in this field for a long time stimulate[d] the teachers to a systematic study of the topic. (Malara, 1999: p. 44)

Hence it seems that expectations of leadership bring the university-based researchers to take the initiative -at least at the level of project formulation. As already indicated, however, the sometimes diverging purposes and perspectives of the teacher-researchers influence the teaching undertaken and the way in which it is researched -at the level of classroom implementation. While this gives a coherence to the work taking place within each classroom, Malara points to some loss of coherence across classrooms.

The way in which teachers shape the work which they carry out in their own classrooms raises the important question of the part played in the research by their craft knowledge of teaching, not just at the stage of implementation, but in the process of formulation. When Malara (1999) refers -in the quotation above- to the 'disagreeing opinions' of teachers, she is probably reporting the language which teachers themselves used to characterise their exchanges. However, it would be surprising if the ideas they presented were not based to some degree on knowledge gained through experience of teaching. More concretely, when she reports the task questions devised by one teacher as being 'immediately appreciated by the group for their originality and efficiency', and comments on the capacity of the teachers 'to assess potentialities and difficulties *a priori*' (p. 45), she is offering direct evidence of how craft knowledge, shaped by their experience of teaching, played a part in the development of the didactical designs.

The eclecticism characterising research for innovation suggests that it has a strong capacity to be receptive to such knowledge. The craft knowledge of teachers -sometimes seen as an expression of 'tacit' theories- might also be considered part of the 'kit of tools' to which Arzarello and Bartolini Bussi refer:

> Another feature of this trend of research is that no global coherent theoretical framework is assumed, because of its complexity and the number of interrelated variables to take into account. Rather, instruments are borrowed from various theoretical approaches or produced inside, and applied as elements of a kit of tools. Local coherence of the framework is necessary, but global coherence is considered impossible or at least irrelevant. (Arzarello & Bartolini Bussi, 1998: p. 250)

Recognising and valorising the contribution of craft knowledge is not simply a matter of acknowledging a distinctive contribution of teachers to the research collaboration, but of creating the conditions under which such knowledge can legitimately come under respectful forms of examination comparable to those applied to scholarly knowledge.

The part played by craft knowledge within didactical engineering is also interesting. Not only do informal mechanisms emerge through which craft knowledge can come into play, but also a form of participant observation. One mechanism is in the approaches that teachers adopt in the lessons between experimental sessions; another lies in the way in which teachers translate the researcher's didactical designs, however precisely stated, into classroom activity; and yet another arises through the post-lesson analysis of such sessions (again featuring as exchange of 'opinion' in Brousseau's earlier description). Greslard and Salin (1999) describe how the teacher is encouraged to reflect on the lesson, and to explain spontaneous decisions made during its course. Brousseau reports how insights from such sources provoked revision of the fundamental theory of didactical situations to take greater account of the role of the teacher:

> We once thought that we had envisaged all the possible classes of situations. But in the

> course of our studies... we saw that after a while the teachers needed some more space; they did not want to go on from one lesson to the next, wanting to stop so as to "review what they had done" before continuing; "some students are lost, we can't go on, something has to be done about it". It took us some time to realize that they really needed to do some things, for reasons that had to be understood... This is how we "discovered" (!) what all teachers do all along their courses but which our method of systematization had made unacknowledgeable. (Brousseau, 1997: p. 236)

Knowledge creation within teaching: the significance of craft knowledge

'Craft knowledge' refers to the professional knowledge which teachers use in their day-to-day classroom teaching; action-oriented knowledge which is not generally made explicit by teachers, which they may indeed find difficult to articulate, or which they may even be unaware of using.

> [C]raft knowledge describes the knowledge that arises from and, in turn, informs what teachers do. As such, this knowledge is to be distinguished from other forms of knowledge that are not linked to practice in this direct way. Craft knowledge is not, therefore, the kind of knowledge that teachers draw on when explaining the thinking underlying their ideal teaching practices. Neither is it knowledge drawn from theoretical sources. Professional craft knowledge can certainly be (and often is) informed by these sources, but it is of a far more practical nature than these knowledge forms. Professional craft knowledge is the knowledge that teachers develop through the processes of reflection and practical problem-solving that they engage in to carry out the demands of their jobs. (Cooper & McIntyre, 1996: p. 76)

There is, then, a process of knowledge creation within teaching. Through experimenting and problem-solving in the course of teaching, and through re-presenting teaching and reflecting on it, craft knowledge is developed. And this can also incorporate a process of knowledge conversion; by contextualising and activating scholarly knowledge within teaching, it can be brought to contribute to the development of craft knowledge.

> From a cognitive point of view, professional knowledge is developed as a product of professional action, and it establishes itself through work and performance in the profession, not merely through accumulation of theoretical knowledge, but through the integration, tuning and restructuring of theoretical knowledge to the demands of practical situations and constraints. (Bromme & Tillema, 1995: p. 262)

Moreover, knowledge conversion can proceed in the opposite direction, through eliciting craft knowledge and codifying it. Thus articulated through researching, craft knowledge can be brought to contribute to the further development of scholarly knowledge.

The following sections will exemplify these processes of knowledge conversion, by considering two projects concerned with the teaching of mathematics at the elementary level. The first illustrates an approach to eliciting and codifying craft knowledge; the second, conversely, an approach to contextualising and activating scholarly knowledge.

Eliciting and codifying craft knowledge: the example of expert direct instruction

A programme of research which has demonstrated the possibilities of eliciting and codifying the craft knowledge of teachers has been conducted by Leinhardt and her associates (1988a; 1989; 1991), employing concepts and methods drawn from a strand of cognitive science research which focuses on the analysis of expertise. Instruction was analysed by observing teachers in action in the classroom, and by interviewing them about their thinking. For example, teachers were invited to organise and classify mathematics problems (Leinhardt & Smith, 1985); or to give an account of their plan for a lesson prior to teaching, and of their handling of particular classroom episodes or lesson segments (as captured on video-recordings) after teaching (Leinhardt, 1989).

Teachers were identified as 'experts' on the basis of their consistency in producing both high gains in student achievement and high levels of final achievement. Compared with novice teachers, the instruction -and underlying cognition- of these expert teachers was characterised in the following terms:

> Expert teachers use many complex cognitive skills, weaving together elegant lessons that are made up of many smaller lesson segments. These segments, in turn, depend on small, socially scripted pieces of behaviour called routines, which teachers... use extensively. Expert teachers also have a rich repertoire of instructional scripts that are updated and revised throughout their personal history of teaching. Teachers are flexible, precise and parsimonious planners. That is, they plan what they need to but not what they already know and do automatically. Experts plan better than novices in the sense of efficiency and in terms of the mental outline from which they operate... From that more global plan... they select an agenda for a lesson... The agenda serves not only to set up and coordinate the lesson segments but also to lay out the strategy for actually explaining the mathematical topic under consideration. The ensuing explanations are developed from a system of goals and actions that the teacher has for ensuring that the students understand the particular piece of mathematics. (Leinhardt et al., 1991: p. 88)

As analysed here, then, the expertise of outstanding teachers is many-layered. Most readily articulated are the processes of deliberate analysis involved in the pre-active framing of a lesson agenda, in its inter-active accomplishment -and adaptation- within the lesson, and in post-active review. Most easily neglected are those largely reflex aspects of action and interaction, exemplified by the classroom routines through which the stability and predictability of classroom activity is produced. Leinhardt suggests that: '[the] importance [of routines] is often overlooked because spontaneity, flexibility and responsiveness are so highly valued in our culture, especially by educators' (1988a: p. 49). Equally, one could conjecture that routines receive less recognition precisely because they have become so reflex for expert teachers, in contrast to those aspects of teaching which command their deliberate attention and continue to exercise them.

This body of work analyses teachers' pedagogical knowledge and reasoning in terms of constructs of 'script', 'agenda' and 'explanation'. A teacher's 'script' for a particular curricular topic is viewed as a loosely ordered repertoire of goals, tasks and actions, continually developed and refined over time; it incorporates sequences of action and argumentation, relevant representations and explanations, and markers for anticipated student difficulties. The most important feature of a script is the way in which it acts as an organising structure, coordinating knowledge of subject and pedagogy with reasoning about actions and goals, hence underpinning the efficient and cohesive planning and development of lessons. Such a script provides a matrix of knowledge supporting the setting of a lesson 'agenda': a mental plan including lesson goals, actions through which these goals can be achieved, expectations about the sequencing of actions through the lesson, and important decision points within the lesson. The agendas of the expert teachers studied by Leinhardt showed more developed instructional logic and smoother flow; and they took more account of students' actions and reasoning, and sought more evidence of these. A crucial element of any script is its 'explanation' of each new idea. This involves a systematic organisation of students' experiences by the teacher intended to help them construct a meaningful understanding of the concept or procedure, including appropriate verbalisation and demonstration by the teacher -or the management of such contributions from students- in support of this goal. A model was developed of the different elements which contributed to the effectiveness of the explanations of expert teachers: anticipation of prerequisite ideas and skills; motivation of the new idea; specification of its conditions of use; principled legitimation of the new idea; integration of different elements of the explanation; completion of the explanation.

An unexpected finding of the studies concerned the way in which these expert teachers attended to the thinking of students.

> [Teachers] did build models, but in different ways that we had anticipated. Teachers seem to construct flags for themselves that signal material that will cause difficulty as it is being learned, and then they adjust their teaching of the topic in response to those flags or to past successes.. They seem to diagnose their teaching and its cycle rather than diagnosing the mental representation of a particular student. A major goal of teaching seems to be to move through a script, making only modest adjustments on line in response to unique student needs.' (Leinhardt, 1988a: pp. 51-52)

This and other characteristics of the teaching observed have led to the value of the model derived from these studies being questioned: '[O]n at least two points is this model lacking: the mathematics that students are being asked to learn and the lack of attention to individuals' (Fennema & Franke, 1992: p. 159). And these two points are seen as related: 'Although teachers may be able to achieve short-term computational goals without attending to students' knowledge, they may need to understand students' thinking to facilitate students' growth in understanding and problem solving' (Carpenter, Fennema, Peterson, Chiang & Loef, 1989: p. 502).

Leinhardt has acknowledged the need to study other forms of teaching:

> Although our experts have been shown to be responsive and supportive of student efforts to learn key concepts and procedures, the content, method, and direction of their lessons are situated primarily with the teacher. Cognitively based learning theories, however, suggest that it is pedagogically sound and cognitively necessary for students to have a role in determining the method and direction of their own learning... A key feature of [future] studies will be the distinction between the explanations that are essentially designed by teachers in advance, and those which students play an active role in constructing during classroom dialogue. (Leinhardt, 1991: p. 111)

However, conducting such studies is problematic if teachers have not developed pedagogical models compatible with such cognitively based learning theories. A major limitation inherent in simply studying expert teachers within an established pedagogical system is confinement to that system. The development of qualitatively new forms of pedagogy calls for intervention.

Contextualising and activating scholarly knowledge: the example of cognitively guided instruction

A programme of research into Cognitively Guided Instruction (CGI) (Carpenter et al., 1989; Peterson, Fennema & Carpenter, 1991; Fennema et al., 1996) has addressed this issue of how new forms of pedagogy might be developed through contextualising and activating scholarly knowledge. Its central hypothesis has been that:

> Research provides detailed knowledge about children's thinking and problem solving that, if available to teachers, might affect their knowledge of their own students and their planning of instruction. (Carpenter et al., 1989: p. 502)

This quotation signals the multiple senses of 'knowledge' about 'children's thinking' which it is important to distinguish in considering this body of work. First there is an important distinction between the conceptualisation of children's thinking in general, as against information about the thinking of particular children. There is then a further distinction between conceptualisations in general, as against the particular one adopted by the researchers. The researchers' evolving model classifies arithmetic word problems and the solution strategies adopted by students, describing progression in thinking in terms of the changing use of particular types of solution strategy in response to particular types of problem.

An early study examined what knowledge experienced teachers already had available to analyse such issues (Carpenter et al., 1988). Teachers were presented with tasks related to teaching, such as creating a word problem corresponding to a given number equation, assessing the relative difficulty of word problems, and -after viewing particular students solving problems- predicting how they would solve others. Most teachers proved relatively successful on such tasks, particularly those involving the types of problem commonly encountered at the grade level at which they taught. Although many teachers found it difficult to articulate the basis on which they made such judgements, they clearly had developed relevant knowledge. Moreover, the form that this knowledge took seems to have reflected the circumstances of their teaching. Teachers appear to have been oriented towards helping students to infer the computation expected through identifying features such as cue words within a problem statement; doubtless influenced by a curricular treatment of word problems in which prototypical situations or stereotypical verbalisations were associated with particular arithmetic operations or solution strategies.

By contrast, the CGI programme was based on the conjecture that organising classroom activity around less structured problem solving, and developing pedagogical strategies to focus attention on the solution strategies devised by students themselves, would prove beneficial to student learning. Consequently, the professional development programme associated with CGI aimed to familiarise teachers with the model, as a more powerful means of conceptualising problem and strategy types, and relating these to problem difficulty. Carefully chosen videotaped recordings of individual children solving problems were used as the stimulus for discussions aimed at highlighting key distinctions within the model, and at clarifying its use to characterise the mathematical thinking of particular children. Teachers were also encouraged to test out the model, by presenting agreed problems to children in their own class, recording their solutions for further discussion in workshops.

In addition, teachers were invited to reflect on how the model could be exploited in teaching, While the programme emphasised that it was teachers themselves who were best placed to make informed decisions about how the model should and could be used in their classrooms, the researchers acknowledge their influence on teachers' thinking about such matters:

> We do not believe that we did not influence directly what teachers did in classrooms. The mathematical content we showed and discussed with them was based almost exclusively on word problems. The videotapes were of individual interviewers asking a child to solve word problems, waiting while the child solved the problem, and asking questions such as 'How did you get that answer?' or 'Could you show me what you did?' Teachers were encouraged to ask children to solve word problems and ascertain how the problems were solved. We did not, however, directly prescribe either pedagogy or curriculum for teachers. (Fennema et al., 1996: pp. 408-409)

The double negative in the opening sentence of this quotation, the distinction between 'influence' at the start and 'prescribe' at the close, both signal the complexities and ambiguities of the line that the researchers were treading in their relationships with teachers.

Indeed, to study the impact of the programme on teachers, the researchers developed scales of 'cognitively guided beliefs' and 'cognitively guided instruction' (Fennema et al., 1996: pp. 412-413). At the lowest point, level 1, of the instruction scale, the teacher 'provides few, if any, opportunities for children to engage in problem solving or to share their thinking'; at the middle point in the scale, level 3, the teacher 'provides opportunities for children to solve problems and share their thinking' and is 'beginning to elicit and attend to what children share but doesn't use what is shared to make instructional decisions'; while, at the highest point, level 4-B, the teacher 'provides opportunities for children to be involved in a variety of problem-solving activities' and 'elicits children's thinking, attends to children sharing their thinking, and adapts instruction according to what is shared' such that 'instruction is driven by teacher's knowledge about individual children in the classroom'. While almost all of the participating teachers moved up these scales over the course of the study, the results from the final year show that the programme did not lead all teachers to the implied ideal: around half of the teachers lay above the middle point on the belief scale, and a third on the instruction scale. Challenging and changing teachers' beliefs is often portrayed as providing the impetus for them to rethink teaching approaches and develop new teaching skills. However, this study lends further support to previous investigations which have suggested that changes in pedagogy may be rather loosely coupled with -rather than directly induced by- changes in beliefs. The beliefs and instruction of a teacher were not always at the same level, and there was no overall pattern as to whether a teacher was at a higher level in beliefs or instruction. There was also no consistency in whether a change in beliefs preceded a change in instruction or vice versa.

Another study identified a subgroup of teachers who, while well disposed towards the ideas associated with CGI, reported difficulties in employing them in their classrooms (Knapp & Peterson, 1995). The barriers that these teachers cited included lack of time for planning; the absence of a curriculum package to support CGI; organisational difficulties in working with individual students or small groups; pressure to cover material in limited lesson time; characteristics of students; an emphasis on computational skills in standardised tests to be taken by students; expectations of teachers in the following grade. Although these teachers were working in similar circumstances to others participating in the project, they appear to have been less flexible in adapting to these circumstances. As Knapp and Peterson suggest, these teachers might have benefitted from greater opportunity for informal coaching through interaction with the researchers and other participating teachers so as to learn how to circumvent what they saw as obstacles. More specifically, such coaching might have given these teachers access to the craft knowledge through which other participants had found ways to manage similar circumstances.

A further study provides insight into how an exceptional teacher had created a classroom culture in which peer interaction supported high expectations and mathematical reflection, and how she managed the learning of students by varying student groupings, scaffolding problem solving and reflection, and monitoring individual progress. Indeed, the researchers were surprised by some features of her approach:

> Ms J did not use knowledge of children's thinking in the way we had anticipated... Because these problems were organized into a hierarchy of difficulty determined by the reasonably well-defined levels that children move through as they learn to solve the problems, we expected that teachers would... use the knowledge more or less as a template to assess what... students knew and then to systematically select more difficult problems for the children to solve... The hierarchy of problem types and solution strategies would be used systematically to make both daily and long-term instructional decisions. Ms J did not do what we had anticipated. Although at times she made use of the specifics of the hierarchy... we were unable to identify any systematic way in which she selected problems... Instead, she used the knowledge about problem types to dramatically broaden the scope of her curriculum and her expectations of children. She used all problem types from almost the first week of school, and children in her class had many opportunities to solve all types of problems using whatever solution strategy they chose. (Fennema et al., 1993: p. 578)

What emerges is a picture of a teacher who had already established a powerful social environment for learning in her classroom; and who had been able to contextualise and activate the cognitive model for her purposes in managing that environment: to strengthen her capacity to set challenges for her students, and to sharpen her understanding of their responses. This seems to have created a virtuous cycle in which success -judged from her perspective- strengthened motivation to use ideas from the CGI programme.

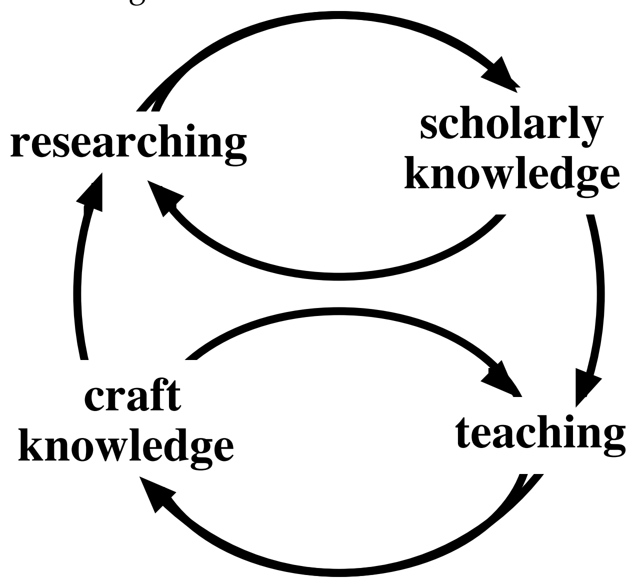
A case study of the CGI collaboration provides evidence that encounters with -and the advocacy of- teachers such as this one had an impact on researchers and on the programme. As this quotation indicates, assessments of that impact differ:

> [B]oth principal researchers noted changes due to their interactions with experienced practitioners -changes in the problems they posed, as well as in the classification strategies they used in math instruction- but they did not feel that these changes were significant... Nevertheless, that collaborating with teachers had an effect on the researchers' project is evident in that decisions such as teaching other operations like multiplication to younger pupils, working with older pupils against the 'better judgement' of the research team, or changing the nature of instructional designs all came from suggestions by collaborating teachers. (Huberman, 1999: pp. 297-298)

CGI provides a particularly fully researched example of a programme which has enabled teachers to contextualise and activate scholarly knowledge in their professional work, provoking a corresponding adaptation and development of their craft knowledge. It also offers a further illustration of the way in which interaction between researchers and teachers, between the practices of researching and teaching, can change both groups to some degree.

Establishing a dialogic cycle: coupling the construction and conversion of scholarly and craft knowledge

The ideas developed in the preceding sections point towards a dialogic cycle in which knowledge creation within the practices of researching and teaching become more co-ordinated, and knowledge conversion from one practice to the other is encouraged. In one phase of this cycle, scholarly knowledge is (re)contextualised and activated within teaching, stimulating (re)construction of craft knowledge. In the complementary phase, craft knowledge is elicited and codified through researching, stimulating (re)construction of scholarly knowledge. In both phases, conversion involves the filtering and reformulating of knowledge: only certain derivatives of scholarly knowledge will prove capable of being productively incorporated within craft knowledge; equally, only some derivatives of craft knowledge will prove able to be fruitfully appropriated as scholarly knowledge.



Huberman has pointed to some of the benefits to researchers and researching of 'sustained interaction' with teachers and teaching, 'in which researchers defend their findings and some practitioners dismiss them, transform them, or use them selectively and strategically in their own settings' (Huberman, 1993: p. 34). Reframing ideas in order to collaborate successfully with teachers appears to trigger a decentring process amongst researchers. In particular, it creates a need to address the counter-examples, qualifications and outright challenges which arise as ideas are tested out by teachers and within teaching. In doing so, researchers are obliged to go outside the study at hand, to marshall a broader range of scholarly thinking and research experience related to these ideas, and to bring them to bear on these claimed anomalies. Examples of this decentring -and the resulting learning- have been noted in passing within earlier sections.

The development of both researchers and teachers is supported by disruption of their taken-for-granted world. Huberman argues that:

> [O]nce they get beyond the initial discomfort of defining common meanings and of working out the social dynamics of their encounters, each party is bound to be surprised or annoyed or even shaken by some of the information and the reasoning put forth by the other party. Both bodies of knowledge are 'valid', albeit on different grounds, and both are contending for salience and prominence. Were the researchers and [teachers] to remain among themselves, there would probably be far fewer instances of cognitive shifts. (Huberman, 1993: p. 50)

Clearly, then, sustained interaction can also make an important contribution to the professional development of teachers. We need to learn more about approaches to professional development in which ideas, methods and findings from research are tested by teachers in their own classrooms in terms of the insight they provide into teaching and learning processes, and of the support they offer in improving the quality of these processes. Equally, however, without an appropriate renewal of craft knowledge, powerful factors act against change in pedagogy. Given that teachers already possess 'a highly efficient collection of heuristics... for the solution of very specific problems in teaching', resistance to change on their part 'should not... be perceived as a form of stubborn ignorance or authoritarian rigidity but as a response to the consistency of the total situation and a desire to continue to employ expert-like solutions' (Leinhardt, 1988b: p. 146). An essential component of the dialogic knowledge-creation cycle outlined above is development in the craft knowledge of teachers participating in the research. Eliciting and codifying this craft knowledge has the potential to improve the effectiveness with which coaching of other teachers can be undertaken, by providing more explicit frameworks for analysing teaching processes, for articulating mechanisms and functions, and for understanding adaptation to different conditions.

Summary

The emergence of mathematics education as an academic subspecialism has been accompanied by concerns that research in the field has not been as successful as many would wish in generating knowledge to illuminate the practice of teaching. Building more strongly reciprocal working relationships between researching and teaching, between researchers and teachers, is an important way of seeking to address this concern. However, closer collaboration and deeper interaction lay bare important differences of purpose and perspective. Success in such enterprises depends on developing an approach within which the distinctive practices of teaching and researching can accommodate to one another, through the co-operation of teachers and researchers, or through the co-ordination of teacher and researcher roles by teacher-researchers.

The tendency in such collaborations has been to highlight –and privilege- the creation of scholarly knowledge within the practice of researching, and its application within the practice of teaching. Yet, it transpires not only that the craft knowledge of teachers plays an important part in converting scholarly knowledge into actionable form, but that there is a significant –but largely tacit- process of knowledge creation within the practice of teaching. Equally, it transpires that research processes can play a valuable part in eliciting and systematising the craft knowledge of teachers. This chapter has argued that coupling the creation of scholarly knowledge within the practice of researching with the creation of craft knowledge within the practice of teaching makes possible approaches to collaboration between researchers and teachers which can contribute to building a more powerful and systematic knowledge-base for teaching. It has pointed to a dialogic cycle through which knowledge creation within the practices of researching and teaching can be co-ordinated, and knowledge conversion from one practice to the other supported.

This paper has viewed these issues primarily as they relate to the practice of research -as seems appropriate to this book. However, in other papers, the author has explored the complementary issues of how the practices of teacher education and of teaching could benefit from making fuller -and more critical- use of scholarly knowledge and research processes, through strategies of 'practical theorising' (Ruthven, forthcoming) and 'warranting practice' (Ruthven, 1998). Bringing these perspectives together points to the potential -and challenge- of developing a much higher degree of interactivity between the practices of educational research, classroom teaching, and teacher education than is currently typical.

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