Mathematical Technologies as a Vehicle for Intuition and Experiment: A Foundational Theme of the International Commission on Mathematical Instruction, and a Continuing Preoccupation

Kenneth Ruthven
University of Cambridge

Abstract

At the 5th International Congress of Mathematicians in 1912, one of the first reports of the newly founded International Commission on the Teaching of Mathematics (later redesignated the International Commission on Mathematical Instruction [ICMI]) focused on attempts to give a more central place to intuition and experiment in school mathematics. The main areas singled out as part of this trend towards "practical mathematics" and creation of the "mathematical laboratory" were geometrical drawing, graphical methods, practical measuring and numerical computation. Whereas many of the diverse technologies associated with this reform movement faded away, one in particular—squared paper—became implanted in school mathematics. This success was due to four factors: disciplinary congruence *with an influential contemporary trend in scholarly mathematics;* external currency *in wider mathematical practice beyond the school;* adoptive facility *of incorporation into existing classroom practice; and* educational advantage *of perceived benefits outweighing costs and concerns. Parallels are drawn with more recent ICMI attention to computer-based technologies as providing support for intuition and experiment in school mathematics.*

A Foundational Theme

The International Commission on the Teaching of Mathematics (later redesignated the International Commission on Mathematical Instruction [ICMI]) was formed in 1908, by resolution of the 4th International Congress of Mathematicians, as a means of promoting international exchange of ideas. At that time, there were significant movements for the reform of school mathematics teaching within educational systems across Western Europe and North America (Howson, 1984), reflecting wider pressure for educational change towards practical approaches that would give a more active role to the learner through concrete experience relevant to life (Brock & Price, 1980). At the 5th International Congress of Mathematicians in 1912, the section on Didactics received ICMI reports examining "the current state and modern trends" of mathematics teaching (Fehr, 1913, p. 595). In particular, the report of Subcommission A [Mathematics in Secondary Education] focused on "Intuition and experiment in mathematical teaching in secondary schools" (Smith, 1913); it discussed contemporary developments aimed at providing an "intuitive," "perceptual," "experiential," and "experimental" base for the subject (p. 611), through "applying mathematics seriously to the problems of life, and … visualizing the work" (p. 615). This, then, represented a foundational theme for the ICMI.

Introducing the Subcommission's report, Smith (1913) described a "spirit of unrest" (p. 613) in secondary education, "in particular with respect to this whole question of intuition and experiment in mathematics" (p. 614). His judgment was that "more progress towards the recognition of the role of intuition and experiment in secondary mathematics seems to have been made of late in Austria, Germany and Switzerland than in France, England and the United States" (p. 615), and that a key difference of approach lay in "the plan of the Teutonic countries to mix the intuitional and deductive work from the outset, while in France, and now in England, the plan is to let an inductive cycle precede a deductive one [with] the United States [showing the beginnings of a] tendency towards the Anglo-French plan" (p. 615). Indeed, this latter plan seems to have been indicative of a more qualified view of the place of such work: The report noted that, in England, "the prevalent view … is that the proper field for methods of intuition and experiment is in the middle and lower classes rather than the upper" (p. 613), and that "many teachers consent to the postponement of abstract methods during the earlier teaching only on the understanding that the abstract character of the higher teaching shall be preserved" (p. 614).

Some Substantive Manifestations

The main substantive areas, which the Subcommission singled out in relation to this trend to give a more central place to intuition and experiment in the mathematics curriculum were geometrical drawing, graphical methods, practical measuring and numerical computation.

Geometrical Drawing

The report described the situation of geometrical drawing as still evolving, and its definition as variable:

> In the matter of geometric drawing and graphic representation of solids, the various countries seemed to be in a transition stage between the period in which this was considered part of the duties of the art teacher and that in which it was to be taken over by the department of mathematics. The tendency is general to consider this work part of mathematics. The nature of the work is not, however, at all settled; even the term "descriptive geometry" has no well-defined meaning (Smith, 1913, p. 614).

In some countries, there was a very clear linkage to material traditionally taught in art or craft classes or in technical subjects:

> In the curriculum of Bern the course includes work in geometrical ornament, such as parquetry flooring; orthogonal projection of geometric solids; drawing of conics and other plane curves; drawing of machine models; shadow constructions; axonometry; polar perspective; solids of rotation; and plans and elevations (Smith, 1913, p. 621).

In a paper presented at the previous Congress, Godfrey had portrayed this (re)appropriation of "geometrical drawing" to the mathematics curriculum in broader educational terms:

> Under the old system the instruction in geometrical drawing had been divorced from theoretical geometry, to the detriment of both studies. It was frequently taught as a branch of the fine arts rather than as mathematics. One result of this has been an exaggerated respect for artistic finish and "inking in"; but the worse feature is that "geometrical drawing" came to be identified with a vast collection of special and unrelated rules; the educational value of the subject had sunk to zero (Godfrey, 1909. p. 455).

Equally, however, Godfrey linked the new place of geometrical drawing in the mathematics curriculum to the idea that more formal treatment of geometric properties needed to rest on an experiential base:

> The reforming party held that a more vivid realization of the shapes and properties of geometric figures was necessary before these properties could profitably be made the subject of strict logical treatment (Godfrey, 1909, pp. 453–4).

In particular, this "more vivid realization" depended on students developing a better grasp of the operation of geometrical instruments and their mathematical functionality through direct use of them:

> To experiment in geometry, a child must learn to measure and draw with sufficient accuracy … For another reason, too, these instruments must be provided. Problems of construction are unmeaning unless it is specified what instruments are allowed … Problems of construction, then, cannot be undertaken intelligently unless the learner understands these instrumental restrictions; and he is not likely to understand the restrictions unless he actually handles and uses the legitimate instruments (Godfrey, 1909, pp. 454–5).

At the same time, Godfrey portrayed such "bodily activity" as essential for pupils, promoting an active attitude on their part, and improving the quality of their thinking:

> Geometrical instruments satisfy the child's need for bodily activity. He thinks better if he is using his fingers. Ideas are suggested to him by the activity of drawing figures. His attitude becomes active rather than passive (Godfrey, 1909, pp. 455).

Graphical Methods

The widespread use of graph paper in scientific and technical work developed during the nineteenth century. In the course of an 1833 paper, the astronomer-mathematician Herschel recommended the use of such paper in terms **that** make it clear that he expected it to be unfamiliar to his readers, since he described both its design and manner of use with care. As interest in this new technology grew among scientists and engineers, many manufacturers entered the market. Over the second half of the nineteenth century, its price fell by two orders of magnitude and its uptake and use increased enormously (Brock & Price, 1980). By the early twentieth century, the use of squared (including graph) paper had spread to education, as the Subcommission reported:

> Graphic methods of one form or another are now found in the courses in mathematics … in all countries, having gradually made their way from engineering, through thermodynamics and general physics, to pure mathematics (Smith, 1913, p. 622).

The report indicated that such uptake, while only recent, was widespread; indeed the report suggested that so enthusiastic was adoption of this new technology that it was in danger of overuse:

> Of the value of squared millimetre paper there is no question anywhere, but it seems equally true that its use has been abused by the over-extensive treatment of equations and by its application to proving the obvious (Smith, 1913, p. 614).

Thus, although linked particularly to the treatment of equations and functions, the use of graphical methods extended to many other topics:

> In England about 90% of the schools state that the graphical study of statistics is given … The graphical representation of functions is taught in all… secondary schools. The work is concerned with the plotting of equations and with the approximation of the roots … The use of vectors is found in a large majority of the schools, in connection with mechanics (velocities, acceleration, forces), this latter subject being part of the mathematical course in England … Graphical statics is taught generally. Areas are estimated by squared paper in most schools, but the planimeter is rarely used (Smith, 1913, p. 622).

Again, in his contribution to the earlier Congress, Godfrey had illuminated a pedagogical motivation for the adoption of such methods, in terms of giving greater emphasis to mathematical rationales as well as rules, and of recruiting the interest of students:

> Many teachers and examiners held that the teaching had laid too great stress on manipulative skill, at the expense of intelligent study of the why and wherefore… There was a kind of rebellion against this practice, and teachers sought to lighten the "toil" by introducing graphs, logarithms tables and other interesting matters at a comparatively early stage. All this had a decidedly stimulating effect. It is a revelation to a boy to learn that a function of a variable may be associated with a curve; that he can solve equations, extract roots, etc. by graphical methods (Godfrey, 1909, pp. 457–8).

Practical Mensuration

The ICMI Subcommission reported widespread interest in activities such as practical surveying as means of bringing measurement and estimation to life; an example was:

> In the Prussian schools the theodolite is usually found, and… along with it are seen simple instruments for angle measure, angle mirrors and prisms, measuring rods, and the like. Simple instruments are often made by the pupils, particularly instruments for the measure of angles. Much is made of out-of-door work in the classes in geometry and trigonometry, in the measuring of heights and distances [by] the pupils making use of the instruments (Smith, 1913, p. 617).

While the report only referred in passing to laboratory work, Godfrey's account of recent trends in England, delivered at the preceding Congress, suggested that practical measurement, estimation and computation were closely linked to it:

> Many schools now arrange that boys of 13–15 shall take, as part of their mathematics, a course of experimental work in the laboratory. During this course they are taught to measure and weigh (incidentally learning to realize the advantages of the decimal notation), to determine the surfaces and volumes of actual objects, to determine densities and specific gravities, to discover the simpler laws of hydrostatics, etc. (Godfrey, 1909, p. 453)

Such laboratory work formed part of a wider movement to develop more "heuristic" approaches across mathematics and the sciences as a whole:

> The development of "practical" or "experimental" mathematics as an approach to teaching the subject involving the use of apparatus of various kinds… and perhaps a special room—a mathematical laboratory … —has obvious parallels with the somewhat earlier emergence of practical science teaching… The growth of both physics and chemistry teaching, as well as demanding the use of various materials and apparatus, was accompanied by the rise of a new approach to teaching, "heurism," or the method of discovery (Brock & Price, 1980, p. 370).

In the United States, the most prominent advocate of a "laboratory method" was Moore, a leading mathematician of the time (Roberts, 2001). Writing in the American School Review, Moore's Chicago colleague, Myers elaborated an educational rationale for the mathematical laboratory, bringing out the link to the wider heuristic movement:

> Fundamentally, laboratory method means work … on the pupil's part or, better still, method of getting work done *by* the pupil on his own initiative, under the impulse of his natural interests, *and largely under the guidance of his own intelligence* (Myers, 1903, pp. 730–731).

As with Godfrey, the ideas of illuminating the mathematical rationale of problems and recruiting pupils' interest to them were prominent in Myers' argument:

> An advantage that can hardly be overestimated of the laboratory procedure with mathematical classes is that pupils *sense* the difficulties to be overcome *as real* and *natural*, actually needing to be resolved and demanding a knowledge of the mathematical tool as a means of their resolution. In short, it recognizes the educational importance of letting the student know both ***how*** and ***why*** he must use the mathematical tool to get on well in any line of study (Myers, 1903, p. 732).

Myers also appealed to the ideas of linking school mathematics both to practical needs beyond the school, and to the development of particular intellectual "faculties":

> Laboratory work with real problems, in the formulation and handling of which the pupil habituates himself to the transition from the concrete to the abstract, trains the faculties of analysis and abstraction, teaches him to make his own mathematical problems, to grow his own mathematics, and goes far toward supplementing the too isolated and too abstract teaching of secondary mathematics of today (Myers, 1903, pp. 735–736).

His listing of "a fairly complete equipment for a mathematical laboratory" was extensive, and indicated the range of applications envisaged, including:

> Set of drawing instruments, drawing board, T-square, and 30°- and 45°-triangles for each pupil … Carpenters' tapes, surveyors' tapes, and architects' scales … Three-, five-, and seven-place logarithmic tables … Logarithmic slide rules and computing machines … A surveyor's compass, a transit, and level, and leveling rods and flagpoles … A surveyor's plane table and a sextant … Weighing apparatus, as steelyards, balances, etc.; pendulums, barometers, and thermometers … Force appliances, such as cords, pulleys, etc., and the simple machines … Spherical blackboards, both concave and convex … Three plane blackboards for projective and descriptive work in geometry … Gyroscope taps … (Myers, 1903, pp. 737–738).

Patterns of Uptake

The uptake of the technologies associated with these developments was, of course, affected by considerations of financial and educational cost, as the Subcommission's report noted in relation to slide rules:

> The slide rule has not yet found general acceptance in the secondary schools. The reason for this is thought to be the expense of the instrument, the cheaper ones not being accurate enough to be of value; but the question of time to acquire the necessary facility is also a serious one. In the upper classes the numerical computation is performed almost exclusively by the aid of logarithms (Smith, 1913, p. 624).

Moderation was a guiding precept for Smith (Donoghue, 2008), evident where the report refers to "an extreme laboratory method with a minimum of mathematics" (Smith, 1913, p. 614) as part of the spectrum of innovation under investigation in the United States. Indeed, the laboratory itself remained rare, and practical methods were generally subject to a more restrained classroom implementation. Writing soon after, in the British *Mathematical Gazette*, Fawdry (1915, p. 36) noted that the form of "practical mathematics" actually taking root in schools was much more modest than envisaged by its advocates, focusing on topics such as "numerical evaluation of algebraic expressions, accurate construction of geometrical problems, plotting of curves, graphical solutions, [and] use of logarithms in computation," which could "be conducted in a classroom without the use of further apparatus than a box of instruments, some squared paper, and a table of logarithms." In his appraisal of the failure of the laboratory movement in the United States, Roberts points to the impact of a range of broader social factors on educational developments:

> With regard to the school environment, Moore, like many other educators, largely failed to foresee the consequences of changing demographics. In the face of the surge of students into the schools, calls for educational efficiency that had emerged during the last half of the nineteenth century became much more insistent and attractive. The efficiency advocates claimed to offer means to control the flood of students by carefully circumscribing requirements in terms of time and effort. In contrast Moore's "mathematical laboratory," which called for such extravagances as performing all demonstrations in two different ways and for blurring of subject-matter boundaries, could well be seen as a prescription for waste and confusion. Moreover, at the very time that Moore was proposing to justify mathematics education primarily as an aid to science and engineering, the population of high school students was exploding with students, most of whom were not aiming to become scientists or engineers (Roberts, 2001, p. 694).

It seems that graphical methods, particularly in algebra, were the most conspicuous success of these reform movements in terms of pervasiveness and permanence:

> The use of graphical methods in elementary algebra teachings is universal and entirely a twentieth century development. Other aspects of the same movement are the adoption of descriptive geometry by the mathematicians, the use of handy 4-figure tables, and of graphical methods in statics, and, though, in these cases, the victory is less complete than that of the "graph," it is remarkable and equally modern (Godfrey, 1913, p. 641).

Brock & Price argue that the adoption within school mathematics of the technology of squared (including graph) paper reflected the influence of new educational philosophies and wider pressure to strengthen scientific and technical education. But as well as these drivers external to mathematics itself, the internal motor of Klein's advocacy of "functional thinking" should not be underestimated. Such functional thinking was portrayed as a core mathematical process, integrating pure and applied mathematics, fusing arithmetic with geometry (Klein, 1908). Even if the Subcommission's report expresses some skepticism as to whether pupils were learning to think functionally through their use of squared paper, this concern does demonstrate the significance of the association:

> Graphic methods of representing functions have become universal in the last generation. From the idea of a line representing an equation the tendency is at present to that of graphic representation of a function. Just how much the pupil is acquiring the function concept seems often to be questioned, and the whole subject is in the experimental stage at present (Smith, 1913, p. 614).

Why was it, then, that squared paper prospered whereas many other technologies associated with this reform movement faded away? I suggest that its success depended on the conjunction of the following features:

- *Disciplinary congruence* with an influential contemporary trend in scholarly mathematics.
- *External currency* in wider mathematical practice beyond the school.
- *Adoptive facility* in terms of ease of incorporation into existing classroom practice.
- *Educational advantage* through perceived benefits of use considerably outweighing costs and concerns.

A Continuing Preoccupation

Modern parallels to this episode are intriguing. The commercialization and diffusion of new mathematical technologies during the nineteenth century for use within scientific and technical professions prefigures a rise of similar computerized technologies in the twentieth. Even more striking is the educational appropriation of both waves of new technology to aspirations for more authentic involvement of students in mathematical activity.

Such aspirations were evident at the ICMI Study Conference on technology held in 2006[1], not least in the opening address from a longstanding advocate **for** a stronger emphasis on intuition and experiment in school mathematics:

> The "Math Wars" … pit Reformers against an Unholy Alliance of Mathematicians and Back-to-Basics fundamentalists. The principled conflict is between rigor and engagement: the Reformers use slogans like engaged, child-centered and authentic; the Mathematicians accuse them of producing a "fuzzy math" stripped of the essence of mathematical thinking. Both sides are right in the values they assert. … Adopting principles of computation (including but not confined to programming) as the new Basics opens the possibility of creating a new math that is more

> engaging than what the Reformers propose and more rigorous than what the Alliance defends (Papert, 2006).

The most influential expression of these ideas over the last **forty** years has been Papert's (1980) conception of the "microworld" in general, and its specific instantiation in Logo. Following the publication of *Mindstorms* (where one can find particular echoes of Godfrey's emphasis, **seventy** years earlier, on the role of instrumented construction and bodily activity), Logo attracted widespread interest amongst educationalists and teachers during the 1980s, achieving some degree of curricular recognition by the early 1990s. Agalianos, Noss & Whitty (2001) have argued that the rise of Logo during this period was facilitated by an educational climate receptive to progressive educational ideas, just as the ensuing "conservative restoration" precipitated its decline. Nevertheless, even at the height of its influence, mainstream use of Logo largely involved selective assimilation to conventional practice:

> [With] Logo's introduction into mainstream US and UK schools in 1980 … the majority of classrooms took up Logo as part of an incremental view of educational change and were quick to absorb it into existing modes of work. Logo became a reinforcing agent of the traditional rather than a vehicle of the new (Agalianos, Noss & Whitty, 2001, p. 497).

Indeed, one could point to influential factors—similar to those operating a century earlier—**that** acted against the continuation even of this incremental use of Logo. In terms of *disciplinary congruence*, during the period of Logo's rise the "algorithmic thinking" associated with computer programming was being proposed as a modern equivalent of Klein's "functional thinking" (directly by Engel, 1977/1984; and, for example, in the revised publication arising from the earlier ICMI Study Conference on technology held in 1985, indirectly by Maurer, 1992, and by report in Graf, Fraser, Klingen, Stewart & Winkelmann, 1992): However, this position failed to achieve widespread acceptance, and lost ground as a wider range of software became available with new types of user interface which pushed programming into the background. This general trend also affected the *external currency* of programming; moreover, the commercialization of Logo led to it becoming associated with elementary education, and differentiated from the types of programming language and mathematical software used at later stages of education or outside education (Agalianos, Whitty & Noss, 2006). In terms of *adoptive facility*, like most computer-based resources over this period, the lack of a viable platform suited to conventional classroom use was an important barrier; it is notable, for example, that successive TIMSS surveys have shown much higher levels of access to and integration of calculators in secondary school mathematics compared to computers. Finally, in terms of *educational advantage*, the perceived value of Logo diminished as the place of more open and extended work in school mathematics was downplayed (due to the reductive trends noted by Agalianos et al.); equally, in this context, the lack of strong alignment between the expression of many mathematical ideas in Logo and their recognized curricular form became more acutely perceived as problematic.

In the light of similar disappointments from wider attempts to incorporate computer use into school mathematics, other contributions to the 2006 ICMI Study Conference struck a more reserved note than the opening address, commenting, for example, that:

> [W]ide scale attempts to insert digital technologies into Brazil's public schools system have tended to emphasize the computer as a catalyst for pedagogical change, without acknowledging the epistemological and cognitive dimensions associated with such change or the complexity associated with the appropriation of tools into mathematical and teaching practices (Healy, 2006, p. 79).

In addition to reports such as this, evidence from the most recent TIMSS international study (Mullis, Martin, Gonzalez & Chrostowski, 2004, pp. 294–295) shows that pervasive computer use is extremely rare in school mathematics, in line with a wider trend across the academic curriculum. Reviewing the educational reception of wave upon wave of new information and communication technologies over the last century, Cuban (1986, 2001) has suggested that a recurrent pattern of response can be found: a cycle in which initial exhilaration then scientific credibility give way to practical disappointment and consequent recrimination. He reports that while new technologies have broadened teachers' instructional repertoires to a degree, they remain relatively marginal to classroom practice, and are rarely used for more than a fraction of the school week. For scholars of school reform, this forms part of a much wider pattern of largely unsuccessful attempts to change the structures of curriculum, pedagogy and assessment at the heart of schooling. Nevertheless, as this paper has argued, it seems that some mathematical technologies—notably computational tables and squared paper a century ago, and their current counterparts in the form of arithmetical and graphical calculators—have been able to implant themselves successfully in secondary school mathematics, and that these successes can be explained in terms of a particular conjunction of conditions. Equally, however, the evidence from TIMSS and elsewhere is that these tools are used largely to effect or check arithmetical and graphical computation, extending only rarely to support intuition and experiment. Here again, then, there are important parallels between the limited way in which more widespread uptake of selectively adopted new technologies unfolded in the early twentieth and early twenty-first centuries.

Note

[1] Only the conference abstracts were publicly available at the time of writing.

References

Agalianos, Angelos, Noss, Richard, & Whitty, Geoff (2001). Logo in mainstream schools: the struggle over the soul of an educational innovation. *British Journal of Sociology of Education*, 22(4), 479–500.

Agalianos, Angelos, Whitty, Geoff, & Noss, Richard (2006). The social shaping of Logo. *Social Studies of Science*, 36, 241–267.

Brock, William H., & Price, Michael H. (1980). Squared paper in the nineteenth century: Instrument of science and engineering, and symbol of reform in mathematical education. *Educational Studies in Mathematics*, 11(4), 365–381.

Cuban, Larry (1986). *Teachers and Machines: The Classroom Use of Technology since 1920*. New York: Teachers College Press.

Cuban, Larry, Kirkpatrick, Heather, & Peck, Craig (2001). High access and low use of technologies in high school classrooms: Explaining an apparent paradox. *American Educational Research Journal*, 38(4), 813–834.

Donoghue, Eileen (2008). Personal communication, 8 March.

Engel, Arthur (1977/1984). *Elementary Mathematics from an Algorithmic Standpoint* (1984). Keele: Keele Mathematics Education Publications. English edition of *Elementarmathematik vom algorithmischen Standpunkt* (1977). Stuttgart: Klett.

Fawdry, R. C. (1915). Laboratory work in connection with mathematics. *The Mathematical Gazette*, 8(116), 36–39.

Fehr, Henry (1913). La Commission Internationale de l'Enseignement Mathématique de 1909 à 1912: Rapport présenté à la séance du vendredi 23 août 1912. *Proceedings of the Fifth International Congress of Mathematicians*, vol. II, pp. 591–597.

Godfrey, Charles (1909). The teaching of mathematics in English public schools for boys. *Proceedings of the Fourth International Congress of Mathematicians*, vol III, pp. 449–464.

Godfrey, Charles (1913). Methods of intuition and experiment in secondary schools. *Proceedings of the Fifth International Congress of Mathematicians*, vol. II, pp. 633–641.

Graf, Klaus-Dieter, Fraser, Rosemary, Klingen, Leo H., Stewart, Jan, & Winkelmann, Bernard (1992). *The effect of computers on the school mathematics curriculum*. In Bernard Cornu & Anthony Ralston (Eds.) The Influence of Computers and Informatics on Mathematics and Its Teaching (pp. 57–79). Paris: UNESCO.

Healy, Lulu (2006). A developing agenda for research into digital Technologies and mathematics education: A view from Brazil (abstract). Programme booklet for ICMI Study Conference on Technology Revisited (p. 79).

Howson, A. Geoffrey (1984). Seventy-five years of the International Commission for Mathematical Instruction. *Educational Studies in Mathematics*, 15(1), 75–93.

Klein, Felix (1908). *Elementary Mathematics from an Advanced Standpoint*. English translation undated. New York: Dover.

Maurer, Stephen B. (1992). What are algorithms? What is algorithmics? In Bernard Cornu & Anthony Ralston (Eds.) *The Influence of Computers and Informatics on Mathematics and Its Teaching* (pp. 39–50). Paris: UNESCO.

Mullis, Ina V. S., Martin, Michael O., Gonzalez, Eugenio J., & Chrostowski, Steven J. (2004). ***TIMSS** 2003 International Mathematics Report*. Boston: TIMSS & PIRLS International Study Center, Boston College.

Myers, G. W. (1903). The laboratory method in the secondary school, *The School Review*, 11(9), 727–741.

Papert, Seymour (1980). *Mindstorms: Children, computers and powerful ideas*. Brighton: Harvester.

Papert, Seymour (2006). From the Math Wars to the New New Math (abstract). Programme booklet for ICMI Study Conference on Technology Revisited (p. 17).

Roberts, David L. (2001). E. H. Moore's early twentieth-century program for reform in mathematics education. *The American Mathematical Monthly*, 108(8), 689–696.

Smith, David E. (1913). Intuition and experiment in mathematical teaching in the secondary schools. *Proceedings of the Fifth International Congress of Mathematicians*, vol. II, pp. 611–632.