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Digital computational tools in school mathematics: ecological, epistemological and existential challenges

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This paper examines three areas of challenge for the integration of digital computational tools into school mathematics. The ecological challenge involves adapting everyday practices of school mathematics in response to the introduction of such tools. The epistemological challenge involves developing disciplinary and didactical knowledge to guide the use of such tools in school mathematics and the associated evolution of the subject. The existential challenge involves understanding how social representations, values and identities shape the use (and non-use) of such tools.

Introduction

Responding to the conference theme – *ICT in mathematics education: the future and the realities* – this paper examines three areas of challenge relating to the integration of digital computational tools into school mathematics:

- **Ecological:** adapting the everyday practice of school mathematics to make use of such tools within the operative constraints of time, space and infrastructure.
- **Epistemological:** developing disciplinary and didactical knowledge to inform the use of such tools in school mathematics and the associated evolution of the subject.
- **Existential:** understanding how (collective and individual) representations, values and identities relating to school mathematics mediate the use (and non-use) of such tools.

Ecological challenges

The introduction of digital computational tools into school mathematics involves change in the range of material resources available and sometimes in the physical environment in which teaching and learning take place. These changes bring perturbations to, and adaptations of, established relations between teacher, students, subject and tools which I have tried to capture in my Structuring Features of Classroom Practice framework (Ruthven, 2009). This framework was developed by taking disparate ideas originally developed to understand such issues prior to the arrival of digital computational tools, and drawing them together in the light of early studies of the introduction of such tools.

The use of digital resources often involves changes in the *working environment* of lessons in terms of room location, physical layout and class organisation, requiring modification of the classroom routines which enable lessons to flow smoothly (Jenson & Rose, 2006). Equally, while new technologies broaden the range of tools and resources available to support school mathematics, they present the challenge of building a coherent *resource system* of compatible elements that function in a complementary manner and which participants are capable of using effectively (Amarel, 1983). Likewise, innovation may call for adaptation of the established repertoire of activity formats that frame the action and interaction of participants during particular types of classroom episode, combining to create prototypical *activity structures* for particular styles of lesson (Leinhardt, Weidman & Hammond, 1987).

Moreover, incorporating new tools and resources into lessons requires teachers to develop their *curriculum script* for a mathematical topic, the cognitive structure which informs their planning of lesson agendas, and enables them to teach in a flexible and responsive way. This structure covers variant expectancies of events and alternative courses of action, forming a loosely ordered model of goals, resources and actions for teaching the topic. It interweaves mathematical ideas to be developed, appropriate topic-related tasks to be undertaken, suitable activity formats to be used, and potential student difficulties to be anticipated (Leinhardt, Putnam, Stein & Baxter, 1991). Finally, teachers operate within a *time economy* in which they seek to optimise the 'rate' at which the physical time available for classroom activity is converted into a 'didactic time' measured in terms of the advance of knowledge (Assude, 2005).

Let me illustrate this framework through the particular case of a teacher developing his teaching practice to make use of dynamic geometry (Ruthven, 2010). In terms of *working environment*, each session started in the teacher's normal classroom and then moved to a nearby computer suite. This movement between rooms allowed the teacher to follow an activity cycle in which working environment was shifted to match changing activity format. Starting sessions in the classroom avoided disrupting established routines for launching lessons, providing an environment more conducive to maintaining student attention "without the distraction of computers in front of each of them." In the recently set up computer suite the teacher was still establishing start-up routines with students for opening a workstation, logging on to the school network, using shortcuts to access resources, and maximising the document window. The teacher was also developing shut-down routines. Near the end of each session, he prompted students to save their files and print out their work, reminding them to give their file a name indicating its contents, and to put their name on their document to make it findable amongst output from the shared printer.

In terms of *resource system*, this teacher saw work with dynamic software as complementing the established construction work with classical manual tools which preceded it, by strengthening attention to geometric properties. Nevertheless, he felt that old and new tools lacked congruence, because certain manual techniques appeared to lack computer counterparts. Accordingly, he saw dynamic software as involving different methods and having a distinct function: "I don't think there's a great deal of connection. I don't think it's a way of teaching constructions, it's a way of exploring the geometry." The teacher was also concerned that students were spending too much time on cosmetic aspects of presentation. He was trying out a new lesson segment which involved showing students an example illustrating to what degree, and for what purpose, it was legitimate to "slightly adjust the font and change the colours a little bit, to emphasise the maths, not to make it just look pretty", so establishing socio-mathematical norms for using the new tool.

In terms of *activity structure*, the teacher's account of his lesson pointed to a combination of activity formats: "a bit of whole class, a bit of individual work and some exploration"; a structure that the teacher wanted "to pursue because it was the first time [he]'d done something that involved all those different aspects." The teacher also highlighted how arrangements had not worked as well as he would have liked in fostering discussion during student activity. He would be giving more thought to how best to organise this. The teacher noted ways in which use of the software helped to structure and support his exchanges with students, creating three-way formats of interaction between student, computer and teacher. Such opportunities arose from helping students to identify and resolve bugs in their dynamic geometry constructions. The use of text-boxes created conditions under which students could be more easily persuaded to revise their written comments.

In terms of *curriculum script*, the teacher reported that he was developing new knowledge of "unusual" and "awkward" aspects of software operation liable to "cause a bit of confusion" amongst students, as well as of how to turn such difficulties to advantage in helping students to develop the target mathematical ideas for this lesson. In the first session of the lesson, after an opening demonstration by the teacher, students themselves used the software to construct a dynamic figure consisting of a triangle and the perpendicular bisectors of its edges, and were then asked to investigate this construction. The teacher was developing strategies for helping students appreciate that concurrence of perpendicular bisectors was geometrically significant, by getting them to drag the dynamic figure: "I don't think anybody got that without some sort of prompting. It's not that they didn't notice it, but they didn't see it as a significant thing to look for." In the second session, using the dynamic figure that they had constructed the previous day, students investigated how the position of the point of concurrence of the perpendicular bisectors was affected by dragging vertices to change the shape of the triangle (see Figure 1). During this session the teacher asked the class about the position of this 'centre' when the triangle was dragged to become right angled. Afterwards, he commented that he "was just expecting them to say it was on the line" and that he had not anticipated what a student pointed out: "I don't know why it hadn't occurred to me, but it wasn't something I'd focused on in terms of the learning idea, but the point would actually be on the mid point." One can reasonably infer that the teacher's curriculum script developed to encompass this new variant as a direct result of this episode.

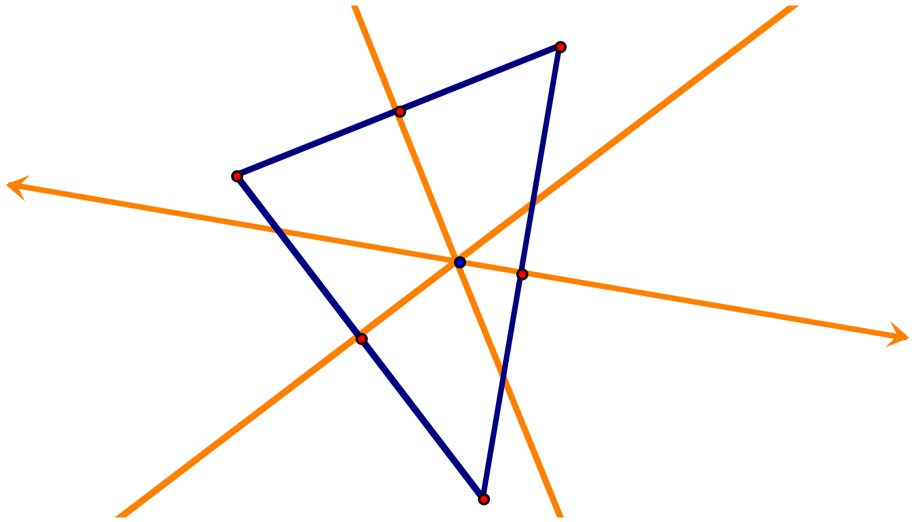

Figure 1: Example of a dynamic figure under investigation in the lesson

In terms of *time economy*, this teacher linked his overall management of time to key stages of investigation – "the process of exploring something, then discussing it in a quite focused way as a group, and then writing it up" – in which students moved from being "vaguely aware of different properties" to being able to "actually write down what they think they've learned." Because he viewed the software as a way of engaging students in disciplined interaction with a geometric system, he was willing to spend time to make them aware of the construction process underlying dynamic figures by "actually putting it together in front of the students so they can see where it's coming from." Equally, he was willing to invest time in having students learn to use the software.

This example of a teacher's developing craft knowledge relating to working environment, resource system, activity structure, curriculum script and time economy conveys the extent of the professional learning involved in adapting everyday teaching practice to the changing ecology created by the introduction to school mathematics of a digital mathematical tool such as dynamic geometry.

Epistemological challenges

Many types of digital computational tool are still at a relatively early stage in their evolution, with significant differences of design between alternative tools of similar type, and between successive generations of a particular tool. This adds to the complexity of establishing stable mathematical didactical analyses. For example, Mackrell (2011) found considerable diversity in basic features of the most commonly used dynamic geometry packages:

- Different repertoires of tools and organisation of them.
- Different styles of interface and modes of interaction.
- Different and inconsistent order of selecting action and object.
- Differing modes of behaviour of figures under dragging.

She comments that "This diversity is an indication that creating [a] program is not simply a matter of representing the conventions of static Euclidean geometry on a screen, but is dependent on the epistemology of the designer and is influenced by both cultural conventions and pedagogical considerations" (p. 384). Mackrell also comments on the limited volume of research on the impact of such design decisions, even for well-known ones such as selection order and the draggability of objects.

Moreover, the mathematical representations and actions provided by digital computational tools may diverge in important respects from those associated with traditional written inscription. This calls for mathematical didactical analysis to establish a coherent intellectual framework covering the digital and the traditional, and to establish appropriate curricular sequences. For example, the idea of dragging has developed in ways quite unanticipated when the first dynamic geometry software was created. Arzarello et al. (2002) have identified a wide variety of ways in which dragging may be used with dynamic figures, including:

- Wandering dragging: of points without a plan in order to discover configurations or regularities in the drawing.
- Bound dragging: of a point already linked to an object.
- Guided dragging: of the basic points of a drawing in order to give it a particular shape.
- Dummy locus dragging: of a basic point so that the drawing keeps a property.
- Line dragging: drawing new points along a line in order to keep the regularity of the figure.
- Linked dragging: of a point attached to an object.

But we still await development of a full mathematical theorisation of dragging. Equally, little of the didactical analysis necessary to incorporate dragging into the curriculum and to underpin a systematic development of curricular sequences has yet been undertaken.

Such curricular sequences must acknowledge the expansion of mathematical concepts and techniques which digital computational tools make available. While it is relatively straightforward to bring such tools to bear on familiar tasks, ultimately a renewed curriculum must incorporate tasks which would be inconceivable without the mediation of digital computational tools, and this, of course, calls for a corresponding shift in mathematical thinking. Laborde (2001), reflecting on a multi-year project working with teachers, has identified a progression in types of curricular scenario employing dynamic software (which I exemplify here in relation to the geometrical topic already discussed):

- Facilitates material aspects of familiar task: e.g. construction of a diagram consisting of a triangle and its perpendicular bisectors.
- Assists mathematical analysis of a familiar task: e.g. through dragging the triangle to identify the concurrence of perpendicular bisectors as an invariant property.
- Substantively modifies a familiar task: e.g. dragging the triangle to identify a variable characteristic which correlates with the internal or external positioning of the circumcentre.
- Creates a task which could not be posed without dynamic software: e.g. a task in which three circles have been constructed with a common free centre, each circle passing through a different vertex of the triangle; dragging of the 'free' centre is then used to identify the conditions under which two or all three of the circles coincide (see Figure 2).

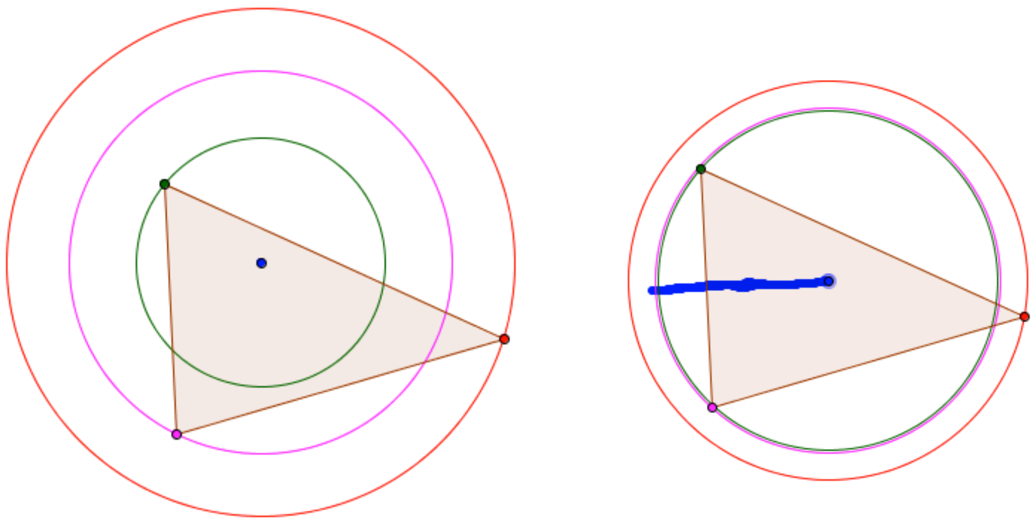

Figure 2: Construction for investigating where to position a common centre so that circles, each passing through one triangle vertex, coincide

But tools differ in complexity. Dynamic geometry systems are relatively complex tools which radically augment available mathematical representations and actions, and which have not yet achieved stability in design. By contrast, the arithmetic calculator is much less complex, employs broadly familiar mathematical representations and actions, and has achieved relative stability in design. Its use in primary mathematics was the subject of extensive developmental work through the 'calculator aware' number project which influenced the English National Curriculum established in 1989 (Shuard et al., 1992). In particular, this led to the inclusion in that curriculum of a detailed section on calculator methods – alongside more extensive sections on mental and written methods – which evolved over time.

Official guidance recognised that, despite its congruence with established mathematical representations and actions, the introduction of the arithmetical calculator to primary school mathematics had a number of implications for curricular sequences. In particular, the availability of a calculator made it possible for children to tackle problems using real data relating to familiar situations from an early stage; use of, and experimentation with, the calculator led to children encountering negative numbers and decimal fractions much earlier than in the traditional curriculum; the ease of computation with a calculator made experimental methods of problem solving based on trial and improvement much more feasible.

However, the English National Curriculum has never specified any standard calculator methods of computation to mirror standard written methods. In the cases of addition, subtraction and multiplication, of course, the relatively straightforward way in which such operations are performed on the calculator means that there can hardly be said to be a distinctive calculator method as such. However, the case of division is less straightforward, because the calculator carries out a particular form of division, meaning that it can be necessary to interpret the calculator result and translate it into an alternative form. Had the curriculum made explicit the need for a calculator-based method of quotient and remainder division, this would have provided a publicly visible capstone for the 'calculator aware' curriculum. Such a capstone could stand alongside – if not replace – the long division algorithm, the culmination of the traditional written arithmetic curriculum, which, for many members of the public, is a totemic mathematical achievement.

Existential challenges

The introduction of digital computational tools has a potential to modify and to be perceived to call into question certain established features of school mathematics. To the extent that such features are highly valued, particularly by powerful and influential groups, the introduction of these tools, or development of their use beyond a certain point, is likely to encounter reluctance or more active resistance. This process is mediated by the social representations in circulation: the simplifying models through which people make sense of a new and unfamiliar phenomenon by relating it to more established and familiar ones.

It is the calculator which has become the popular archetype around which prevalent social representations of digital computational tools in school mathematics are formed. In England, at least, this is reflected in a shift in policy that has taken place at all levels of schooling towards a curriculum that is more 'calculator beware' than 'calculator aware'. In 2011, when the English government announced a review of the National Curriculum, this extract from a press release by one of the politicians who was to serve as schools minister during the period of the review conveys the government's sentiments on this matter:

> We should ensure that schools equip children with the mathematical basics that allow them to succeed in life. We are in danger of producing a 'Sat-Nav' generation of students overly reliant on technology. (Truss, 2011)

Discussion on an online comment board – from *The Guardian*, a newspaper with a broadly liberal readership – provided me with an opportunity to gauge popular opinion on this matter and to analyse the associated social representations (Ruthven, 2013). Many of the comments depict the use of calculators by pupils as antagonistic to thought and subversive of intelligence:

> One of the most important things that a child learns is the ability to think. If you give them a tool that discourages that at such a young age, that aspect of their thinking will be stunted.

> A child's mind needs exercise just as their body does.

> Nothing clever about using a calculator to work out numbers.

Some comments portray use of calculators not only as developmentally debilitating but as a morally iniquitous avoidance of effort:

> Using a calculator… rots the brain, not to mention the poor ethic it instils… if they don't work out the answers with hard graft.

> Going straight for the answer is the easy, cheap and wrong way to go.

Where contributions concede that using a calculator does involve a degree of expertise, this tends to be presented as distinct from mathematics itself:

> Learning to work a calculator is only learning to work a calculator, not learning how to do maths.

A common suggestion is that access to calculators should be granted pupils only once they have become confident with number and proficient in mental or written calculation:

> They should learn how basic arithmetic works first, which means doing it either in their head or on paper.

Some comments are salutary in showing how opposition to calculators is embedded in contributors' sense of personal worth, grounded in their own educational experiences. One such identity narrative from a contributor conveys a sense of personal accomplishment associated with mastery of mental and written calculation, expressed in a continuing proud refusal of the calculator:

> I learned arithmetic the old-fashioned way, using a sums book, following the methods demonstrated by the teacher on the blackboard. By six, I could add and subtract up to a hundred, by eight I had long division and multiplication, and all the tables to ten… My mathematical skills took me all the way through A level into degree-level statistics, and then a (boring) first job in Health Service data analysis. I have never owned a stand-alone calculator, and I don't use the one in MacOs X.

Another (atypical) identity narrative conjures up a very different type of personal history, offering a sense of how, for some pupils at least, calculators serve as a catalyst for developing interest and capability with numbers:

> I am really good at mental arithmetic, but as a child abhorred rote learning of times tables, couldn't see the point as I could work them out in an instant. It almost alienated me completely from maths, luckily playing with calculators… rekindled my interest in number games. So when I was older and scientific calculators starting coming in… I used to play with it, especially the functions that worked out means and standard deviations. That set me up well for the types of maths I used in later life, inferential statistics.

This narrative indicates that there are some threads of popular opinion which are more positive about calculator use in school mathematics. Nevertheless, it seems that the currently dominant strands of popular thought tend to devalorise the use of calculators in particular, and of digital computational tools more generally, through the following polarised associations:

- Cognitive self-sufficiency: thinking 'independent' of digital tools versus unthinking 'dependence' or 'over-reliance' on such tools.
- Mathematical essence: 'purely' mathematical mental/written methods versus (wholly/partially) 'non-mathematical' use of digital tools.
- Moral virtue: 'effortful' use of 'rigorous' mental/written methods versus 'lazy' recourse to 'slipshod' use of digital tools.
- Epistemic value: use of mental/written methods taken as exercising intelligence and developing understanding versus use of digital tools taken as doing neither.

Transforming such representations, many of which are shared, at least in part, by educators themselves, represents a considerable challenge for the field.

Conclusion

In the light of these – still not well understood – ecological, epistemological and existential dimensions, we should not be surprised at limited progress to date in integrating digital computational tools into school mathematics. At the same time, this paper has tried to show how research has started to give us a better understanding of these issues, and has the potential to develop new knowledge which can help to tackle these challenges.

References

Amarel, M. (1983). Classrooms and computers as instructional settings. *Theory into Practice,* 22, 260-266.

Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practises in Cabri environments. *ZDM,* 34(3), 66-72.

Assude. T. (2005). Time management in the work economy of a class. A case study: Integration of Cabri in primary school mathematics teaching. *Educational Studies in Mathematics,* 59(2), 183-203.

Jenson, J., & Rose, C. B. (2006). Finding space for technology: Pedagogical observations on the organization of computers in school environments. *Canadian Journal of Learning and Technology* 32(1). Accessed at http://www.cjlt.ca/content/vol32.1/jenson.html

Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning,* 6(3), 283-317.

Leinhardt, G., Putnam T., Stein, M. K., & Baxter, J. (1991). Where subject knowledge matters. *Advances in Research in Teaching*, 2, 87-113.

Leinhardt, G., Weidman, C., & Hammond, K. M. (1987). Introduction and integration of classroom routines by expert teachers. *Curriculum Inquiry,* 17(2), 135-176.

Mackrell, K. (2011). Design decisions in interactive geometry software. *ZDM,* 43(3), 373-387.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Education & Didactique,* 3(1), 131-149.

Ruthven, K. (2010). An investigative lesson with dynamic geometry: A case study of key structuring features of technology integration in classroom practice. Proceedings of the 6th. Congress of the European Society for Research in Mathematics Education [CERME 6] pp. 1369-1378.

Ruthven, K. (2013). System error? Debating digital technologies in mathematics education. In D. Leslie & H. Mendick (Eds.) *Debates in Mathematics Education* (pp. 83-95). London UK: Routledge.

Shuard, H., Walsh, A., Goodwin, J., & Worcester, V. (1991). *Calculators, children and mathematics.* London UK: Simon & Schuster.

Truss, E. (2011). Elizabeth leads debate on calculators. Accessed at http://www.elizabethtruss.com/news/elizabeth-leads-debate-calculators