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TOWARDS A NORMAL SCIENCE OF MATHEMATICS EDUCATION?

Abstract: This paper suggests that the first BaCoMET [Basic Components of Mathematics Education for Teachers] project (Christiansen, Howson and Otte 1986) can be seen as an important early attempt to sketch a 'disciplinary matrix' (Kuhn 1962, 1970) for the field of mathematics education. This project brought together representatives of different national traditions of research in mathematics education, with the aim of identifying fundamental ideas which should be given high priority in any teacher education programme. My analysis of the project draws on Kuhn's (1970) prioritisation of different senses of 'paradigm' in relation to the development of 'normal science', and consequently draws out the central part played by clusters of exemplary problems (and solutions) in mediating between symbolic generalisations and practical action.

Keywords: Didactical research; mathematics education; normal science and scientific paradigms; teacher education; theory-practice mediation.

THE IDEA OF BASIC COMPONENTS OF MATHEMATICS EDUCATION FOR TEACHERS

My first – indirect – contact with Michael Otte was as a newly appointed teacher educator, through reading his report on 'The education and professional life of mathematics teachers' (Otte 1979) in a volume prepared by the ICMI for a UNESCO series on New Trends in Mathematics Teaching (Christiansen and Steiner 1979). From the start – and entirely characteristically as I now recognise – Otte cautioned that "such a report can only… provide cues for orientation at a general level by furnishing a conceptual framework for the contextual analysis of problems" (p. 108). For Otte, "the special structural problem of the teaching profession" was "that it does not have a basic science such as law for the lawyer, medicine for the physician" (pp. 114-115), with the result that "the most central problems for teacher education are undoubtedly those of mediating between theory and practice, [and] between the subject matter and the social and educational sciences" (p. 126). In particular, he argued that "the present situation of the pedagogy of mathematics is characterized by great conceptual deficiencies" at a time when "we are faced with a host of practical problems which can no longer be handled by the conventional inventory of spontaneous principles gained from experience and transmitted by tradition" (p. 127).

Together with two fellow contributors to the UNESCO volume – Bent Christiansen and Geoffrey Howson – Otte set out to remedy this situation by initiating a project aimed at establishing what they termed 'Basic Components of Mathematics

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Education for Teachers'. The rationale for this project is briefly presented in the introduction to the resulting 'Perspectives on Mathematics Education' (Christiansen, Howson and Otte 1986). The animating idea was that "there were certain basic, fundamental… components of the didactics of mathematics which should be given high priority in any teacher education programme" (p. ix). A 'basic component' was taken as being an aspect of mathematics education which is:

> (i) fundamental in the sense that it plays a decisive part in the functioning of mathematics teachers; (ii) elementary in the sense that it is accessible to intending teachers of mathematics and of immediate interest and value to them; and (iii) exemplary in the sense that it exemplifies important didactical or practical functions of the teacher and their inter-relationships. (p. x)

Thus the project aimed to produce a text "to convey… such knowledge as would be useful to the teacher in carrying out his functions, i.e. *knowledge for action*" (p. xi). Nevertheless, in conceptualising the form that such 'knowledge for action' might take, the project group was exercised by "the relationships between, on the one hand, *theoretical knowledge* (scientific theories about subject matter and about didactical concerns), and, on the other, the *know-how* of the practitioner (i.e. the experienced teacher) who is operating and acting in appropriate ways in the classroom" (p. xi). Moreover, the multi-national project group faced a further challenge in synthesising and refining the diverse knowledge – both theoretical and practical – which individual members brought from different traditions of research, so as to build "knowledge which was in some form 'common' or 'shared' by the group" (p. xi).

PARADIGM AS METAPHYSICAL FRAME, DISCIPLINARY MATRIX OR EXEMPLARY PROBLEM

This first BaCoMET project was an important attempt to frame a shared perspective on the field across different traditions. A more recent ICMI study, entitled 'What is Research in Mathematics Education and What are Its Results?', has sought to explore – indeed, attempted to resolve – such differences of perspective within the field. Its main conclusion has been summarised by its leaders in the following terms: "[I]n spite of all the differences that divide mathematics education researchers (in terms of theoretical approaches, views on relations between theory and practice, philosophies of mathematics, etc.), they still constitute a community, and it is necessary to search for what constitutes its identity" (Sierpinska and Kilpatrick 1998, p. xi). The differences manifested in the course of the study were often characterised in terms of alternative – even conflicting – 'paradigms' for research in mathematics education. This issue is discussed most fully by Ernest (1998) who, following Habermas, talks of "multiple research paradigms, each with its own assumptions about knowledge and learning (epistemology), about the world and existence (ontology), and about how knowledge is obtained (methodology)" (p. 77). While this conception has been widely influential in methodological discussions in the social sciences, in my view, it represents an overly rationalistic and foundationalist approach.

I see the alternative perspective offered by Kuhn (1962, 1970) as a more fruitful one for appraising the situation of mathematics education. In the second edition of his work on 'The Structure of Scientific Revolutions', Kuhn (1970) added a substantial postscript in which he sought to tighten his loose central construct of 'paradigm'. In particular, he drew attention to two related – but ultimately distinct – senses of the term; the first more popular, the second more profound:

> On the one hand, ['paradigm'] stands for the entire constellation of beliefs, values, techniques, and so on shared by members of a given community. On the other, it denotes one sort of element in that constellation, the concrete puzzle-solutions which, employed as models or examples, can replace explicit rules as a basis for the solution of the remaining puzzles of normal science… Philosophically, at least, this second sense of 'paradigm' is the deeper of the two. (p. 175)

It is these shared models or examples which mediate between codified theory, expressed in what Kuhn terms 'symbolic generalisations', and practical tasks of framing and solving problems. This identification of the central part played in scientific thinking by a myriad of problem-solutions points to a much smaller granularity of scientific knowledge than do accounts which conceive such knowledge more exclusively in terms of symbolic generalisations. Kuhn's approach dissolves the idealised epistemological model of theory application, and highlights the crucial part that exemplars play in mediating theoretical constructs.

Accordingly, Kuhn draws attention to the central function of exemplary problems in the formation of disciplinary knowledge:

> The paradigm as shared example is the central element of what I now take to be the most novel and least understood aspect of this book… Philosophers of science have not ordinarily discussed the [exemplary] problems encountered by a student… for these are thought to supply only practice in the application of what the student already knows. He cannot, it is said, solve problems at all unless he has first learned the theory and some rules for applying it. Scientific knowledge is embedded in the theory and rules; problems are supplied to gain facility in their application.… [However] this localization of the cognitive content of science is wrong.… In the absence of such exemplars, the laws and theories [the student] has previously learned would have little empirical content. (pp. 187-8)

Working with exemplars not only gives substance to laws and theories but establishes a fine texture of largely tacit knowledge through which a wide range of situations can ultimately be related to a single idea:

> [The] ability to see a variety of situations as like each other, as subjects for… [some] symbolic generalization, is… the main thing a student acquires by doing exemplary problems.… After he has completed a certain number… he views the situations which confront him as a scientist in the same gestalt as other members of his specialists' group. For him they are no longer the same situations he had encountered when his training began. He has meanwhile assimilated a time-tested and group-licensed way of seeing. (p. 189)

Such shared disciplinary referents lie at the heart of Kuhn's account of a scientific community, the members of which "have undergone similar educations and professional initiations… absorb[ing] the same technical literature and draw[ing] many of the same lessons from it" (p. 177).

BUILDING A SHARED DISCIPLINARY MATRIX FOR MATHEMATICS EDUCATION

As a field, mathematics education has barely started to build a shared disciplinary matrix, codified in a standard literature. This involves not so much a search for grand overarching schemes as the development of many more modest and loosely coordinated analytic frameworks, clearly focused on issues of recognised significance, and closely associated with clusters of exemplary problem formulations and solutions. As I have argued elsewhere (Ruthven *et al.* 2002), taking replication and synthesis seriously could play a significant part in such development. From a technical perspective, replication of a study across varied sites not only makes it possible to address issues of generalisability and contextual influence more rigorously, but provides an important mechanism through which theoretical ideas and research tools can be sharpened and refined in action, particularly in response to the operational challenges and cultural differences which arise in translating them between educational sites, phases and systems and between research teams. From a social perspective, the diffusion of research design and instrumentation from one group to others through replication studies not only mediates the development of more strongly shared systems of language and method, but also directs attention to the degree to which carrying through such work calls for recontextualisation rather than straight replication, illuminating contextual influences and cultural differences which tend to be glossed over in current discussion, evaluation and synthesis of research in the field.

Likewise, critical reviews of research on particular topics, informed by appreciation of such contextual influences and cultural differences, could play an important part in development of the field. At present, the synthesis of research receives insufficient attention, perhaps on account of a popular perception of review studies as ‘secondary’ rather than ‘primary’ research, but also because of the challenges of carrying through such work rigorously and reflexively. The development of handbooks for the field is making some contribution in this respect, but their mode of production often limits their scope. Viewed in this light, the first BaCoMET project represented an unusually ambitious and sustained attempt at holistic synthesis across different traditions. Contemporary reviewers recognised this. One commented on how the book examines “aspects of mathematics education which are all acutely relevant to anyone engaged in initial or in-service teacher training” (Schwarzenberger 1987, p. 67); another considered that the book “makes an important contribution to the emerging consensus on what constitutes the discipline Mathematics Education and provides a very broad range of organising ideas and principles which a contemporary teacher education programme needs to address” (Booker 1988, p. 505); a third expressed the view – as did the others – that the book “will be a fundamental reference, for years to come, for the training of mathematics teachers” (Adda 1988, p. 108).

A feature which reviewers particularly appreciated in the BaCoMET text was the attention given to examples. Adda found the closing contribution on classroom organisation and dynamics "a revealing chapter based on many illuminating and stimulating examples" (p. 108), while Booker commented of the same chapter how "the fundamental ideas… are analysed in depth by the provision of a large number of well chosen classroom examples which are returned to on several occasions to reveal the full inter-related aspects" (p. 509). The opening chapter on social norms and external evaluation took a rather different approach:

> In this provision of illustrative lessons, the first chapter differs from much of the remainder of the book as it allows for the practical implementation and testing of the ideas being put forward. The inclusion of several sets of student-teacher directed questions provide for further investigation of these concepts and strategies (Booker, 1988: pp. 505-6)

This is the only case of exemplary tasks being proposed in the text, as opposed to exemplary analyses being offered. Equally, the sense conveyed – as in the text more generally – is of issues being raised for discussion rather than problems posed for solution. Reviewers noted how the style of the text is more discursive than conclusive; with Adda commenting that "this is not a book that deals with categorical assertions… it sets problems and puts its readers in the position of posing many more" (p. 108); and Booker concluding that the book "provides a very useful framework for organising the discussion of mathematics education with intending teachers and supplies a wealth of ideas on which practising teachers could well reflect" (p. 510). In this respect, then, the text does not employ generalisations and exemplars in the more definitive way implied by Kuhn's analysis of their function within normal science.

The fullest reviewer comment bearing on the relation between theoretical generalisations and exemplary analyses within the BaCoMET text was occasioned by a chapter on 'observing students at work':

> Using compelling, if familiar, examples [the authors] show how students frequently think about mathematical tasks in very personal ways; when this proves unexpectedly useful, the student is said to have unusual insight but when it leads to a misconception, the student is said to be in error. By highlighting the common basis to these two very different outcomes, the authors provide a meaningful context for the errors that students make in building and using their individual conceptions of the mathematics that the teacher is endeavouring to communicate. Practical means of investigating student conceptions are provided and these observations are related to an underlying theory of human information processing. (Booker 1988, p. 508)

Given the relatively extensive attention that this area had received within the mathematics education research community of the time, it not surprising to find this particular chapter being singled out. Significantly, some of the points made by Booker can be reformulated in terms of the emergence of critical features of a disciplinary matrix. For example, paradigmatic examples *should* be both compelling and familiar to those in the field; indeed these characteristics are related inasmuch as extensive public scrutiny plays a part in identifying particularly powerful examples. Equally, an important function of such exemplars is precisely to motivate and illustrate deeper theorisation of the issue, serving a bridging function between theory building and practical action. In this respect, Adda was more sceptical: while she considered that the chapter "provide[s] many good examples", she found "the argument of the theory stemming from cognitive science… a little unconvincing, especially as regards the complexity of the reported observations throughout this book" (p. 107). Once more, the comment is revealing: while the exemplars are found significant and persuasive, the capacity of the imported theory to enhance their analysis is questioned. Again, it is through wide and sustained public scrutiny of this type that scientific norms are established.

SUMMARY AND CONCLUSION

My argument has been not that issues of mathematics education can – or should – be treated wholly in scientific terms, but for the potential contribution of a normal science of mathematics education to the wider human enterprise. I have suggested that the first BaCoMET project can be seen as an important early attempt to sketch a disciplinary matrix for the field. My critical appreciation of this work has been guided by the different senses of 'paradigm' expounded by Kuhn, and notably by the central part that he identifies clusters of exemplary problems and solutions as playing in mediating between symbolic generalisations and practical action.

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