epiSTEMe Teaching Notes

Fractions, Ratios and Proportions

Overview of the Teaching Notes

INTRODUCTION

This module on 'Fractions, Ratios and Proportions' is one of four topic-specific modules that have been developed as part of the epiSTEMe project (Effecting Principled Improvement in STEM Education). The Appendix outlines the epiSTEMe teaching model, and the way in which its principles have been encapsulated across the four modules.

The module that follows covers those aspects of fractions, decimals, percentages, ratios and proportions that are suitable for the start of the Key Stage 3 curriculum. The topic is identified in UK policy documents as the most important aspect of elementary number after calculation, yet research in many countries (including the UK) has shown it to be one of the most challenging areas for students. This module attempts to rise to the challenge by applying the messages for teaching from contemporary theory and research.

Distinctive features of the module include: 1) emphasis on equivalences (and differences) across fractions, decimals, percentages and ratios; 2) opportunities to use fractions, decimals, percentages and ratios in the context of proportional reasoning; 3) tasks that make explicit links with parallel material in Key Stage 3 science (e.g. trampolines, stopping distance); 4) opportunities to discuss relevance for real-life events; 5) a dialogic approach to teaching, through whole-class discussions and collaborative group work amongst students.

Structure of the module

The main sequence of activities is set out in these Teaching Notes, and is supported with a Study Booklet for students and a set of Projection Slides for classroom use. The sequence has been organised into Lessons, notionally of one hour, but with only 50 minutes of activity actually scheduled, leaving 10 minutes slack. To help in planning how to fit Lessons into sessions of a different length, or in adapting to unplanned circumstances, each Lesson has been chunked into shorter Parts. Most (but not all) Lessons end with optional homework to be used at your discretion. In case you do not wish students to take Study Booklets out of school, homework exercises have been copied onto separate sheets as well as into the Booklets.

Implementing the module

The module comprises seven Lessons. The core material appears in Lessons 1, 2, 3, 4 and 5 and we ask that all students cover this material. Lessons 6 and 7 address more challenging topics, and should be included if your students cope reasonably well with Lessons 1 to 5. So there is a five-lesson version of the module and a seven-lesson version.

Whichever version you choose, you should feel free to translate the plans into a form that will work for your particular class. This includes whether or not you exactly follow our proposed division into whole-class, small-group and individual activity. Our main request is that you translate in a way that seeks to maximise the students' understanding of key concepts.

Use your discretion to decide whether certain activities require more or less emphasis and more or less time than suggested. Consequently, a Lesson may not fit neatly into a single class session. This is no cause for concern as long as you follow the sequence of activities as they are described in the Notes. In general, devoting more than five or seven sessions to the activities is preferable to rushed and scattered teaching within the current session structure. Recognising the need for flexibility over timing, we have not provided Projection Slides that summarise the *aims* of each Lesson. However, we appreciate that you may wish to add such slides and/or begin each session with a review of material covered in the previous session and an overview (including aims) of the new material to follow.

There are aspects of the Lessons, which we would ask you not to alter. A key feature of the module is that it includes a large amount of student thinking and talking. Try to ensure that, as far as possible, time for this thinking and talking is preserved. Likewise, these activities have been designed to allow students to formulate their own ideas about topics. Research shows that this will facilitate effective talk and thinking, and so aid progress in understanding. So while the Lessons should guide students in developing a 'mathematical' view, we ask you (and any assistants working with you) to help them to test and refine their own ideas rather than giving them ideas yourself. The point is to lead or support student activity rather than to proceed immediately to 'correct solutions'.

The tasks that we have designed for collaborative activity in small-groups should be effective regardless of ability or gender composition, or the existing social relations between group members (e.g. friendships). With the exception of the small-group task described on Page 7, they should, within limits, also be effective regardless of group size. However, when groups become very large, students can sometimes experience difficulties with managing the dialogue. For instance, some students can get left out or groups can split into subgroups. For that reason, we ask that the students normally work in groups of two, three or four.

LESSON 1: PROPORTIONAL REASONING

Overview

Lesson 1 is detailed on Pages 7 to 9. The lesson starts by using real-world examples to introduce students to the concept of proportionality and concludes by outlining the basic elements of proportional reasoning.

Aims

Lesson 1 aims to teach students that:

- The concept of proportionality is concerned with the relations between the parts of a whole and the whole itself, and between one part and another part
- Scaling up 'in proportion' involves multiplying by some constant, not adding the constant as many students believe

Structure

The lesson is in three parts and uses a mixture of whole-class and small-group teaching.

- *Part 1* (whole-class and small-group): Triggers recognition of familiar language and ideas related to proportionality and establishes a working definition of proportion
- *Part 2* (small-groups and whole-class): Introduces proportional reasoning
- *Part 3* (whole-class): Reinforces the multiplicative nature of proportional reasoning

Resources

- PowerPoint Slides 1-3

LESSON 1: PART 1

Objective
To trigger recognition of familiar language and ideas related to proportionality and to establish a working definition of proportion

Time
25 minutes

Resource
- Slide 1: Spot the proportionality words

Activity
Whole-class activity
- Explain that the whole module is about proportions, and how we describe them.
- As a whole-class activity, ask the students to identify words or phrases *in the headlines* that are related to proportionality.
- Build on the students' responses and ask them to propose what 'proportion' means. Get several students to make proposals and invite comparisons.
- The class should converge on a working definition of 'proportion'. Such a definition may be that proportions refer to parts of wholes.

Small-group activity
- Divide the class into groups of four (or 5 or 6 when class size requires this) [NB This is the only task where group size matters – see Page 5]
- Write around four suitable proportion questions on the board, e.g. What proportion of the group walks to school? What proportion of the group likes coca-cola? What proportion of the group has blue eyes?
- Ask the groups to discuss the questions and answer them.

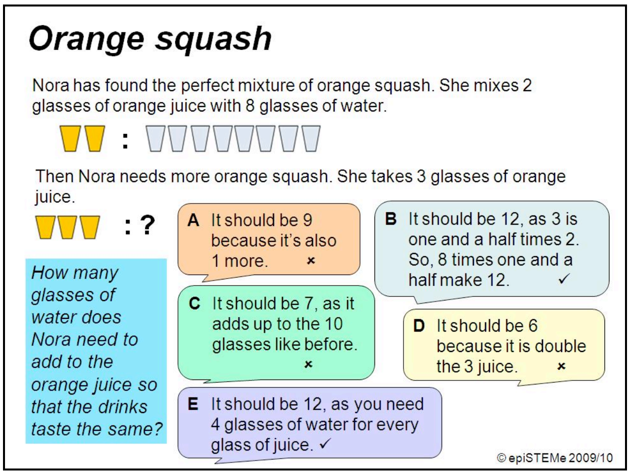
Whole-class activity
- Group answers can be compared in whole-class mode.

LESSON 1: PART 2

Objective
To introduce proportional reasoning

Time
15 minutes

Resource
- Slide 2: Orange squash



Activities
Before working in groups, the students should indicate in their Study Booklet whether they agree or disagree with each statement in the *Orange Squash* task.

Small-group activity
- In groups, the students should work on the *Orange Squash* task. Remind the students about the Ground Rules developed in the Introductory Module, especially that they should try to come to an agreement in their group about which statement(s) they think is/are correct and explain and justify their answers.
- Circulate to gain information about the thinking within different groups. Respond to any requests for clarification, but without suggesting or providing a 'correct answer' to any element of the task. Intervene only where necessary to ensure that appropriate discussion is taking place and that all students are involved, helping the students to exchange and examine their ideas, and work towards their own agreed conclusion.

Whole-class activity
- Ask the groups to hold up their answers and read out which statements have been suggested by the group as correct. Do not give correct answers at this stage but ask the students to explain and justify their answers in their own words. Try to give them enough time to express their ideas and thinking. Invite other students to comment.
- Switch to 'authoritative talk', if needed, to make sure that all students understand that scaling up the parts of a whole in proportion involves multiplying by a constant, and not adding (as in Statement A, which research suggests is a frequent student error).

LESSON 1: PART 3

Objective
To reinforce the multiplicative nature of proportional reasoning

Time
10 minutes

Resource
- Slide 3: Squash

Squash

For a new type of orange squash, you add 3 cups of water to 1 cup of orange juice.

How many cups of water do you need to add to the orange juice so that the drinks taste the same?

Cups of orange	Cups of water	
1	3	
2	6	
4	12	
5	15	
…	…	Right or wrong mixture?
10	13	× 30
15	45	✓
100	300	✓

Activity
Whole-class activity
- Consolidate the message of the previous task by working with the whole-class on the *Squash* task that involves easy ratios.
- Ask the students to work out how many parts of water are needed for each amount of orange, encouraging them to explain their reasoning (see whole-class dialogue in the Introductory Module). You can ask other students if they agree.
- Record the correct answers in the table (writing on the projected slide or editing in PowerPoint) and ask the students to write down the results into their Study Booklet.
- Draw attention to the equivalences – and how to establish equivalences. Then get the students to decide whether large relations, e.g. 100 orange to 300 parts water, are or are not equivalent to the 1:3 starting place.
- By responding to what the students say, make sure that, by the end of the lesson, all students have understood the mathematical strategy of multiplying by a constant.

END OF LESSON 1

LESSON 2: PROPORTIONAL REPRESENTATIONS

Overview

Lesson 2 is detailed on Pages 11 to 15. It starts by practising proportional reasoning, as introduced in Lesson 1. It ends by introducing the different proportional representations, i.e. ratios, fractions, decimals and percentages.

Aims

Lesson 2 aims to teach students that:

- Multiplicative strategies are required for working out proportional relations
- There are different ways of representing proportions: fractions, decimals, percentages and ratios

Structure

The lesson is in three parts and uses a mixture of whole-class, small-group and individual teaching:

- *Part 1* (small-group and whole-class): Consolidates Lesson 1 teaching on scaling up, using everyday examples
- *Part 2* (individual work and whole-class): Individual practice in proportional reasoning
- *Part 3* (whole-class): Introduces how ratios, fractions, decimals and percentages can be used to represent proportional relations

Resources

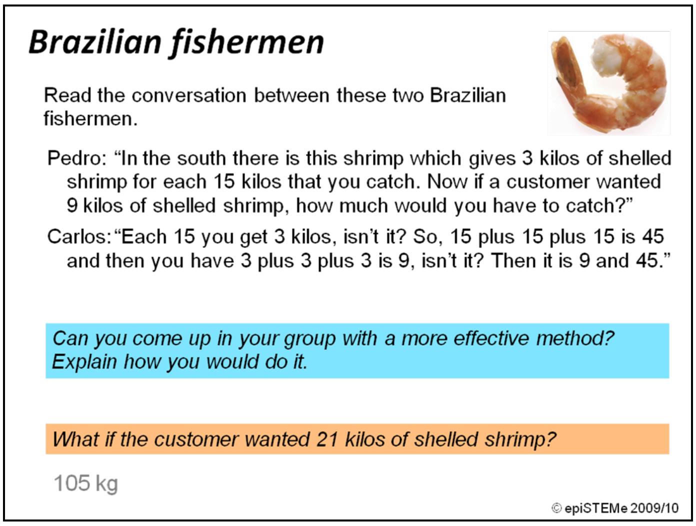
- PowerPoint Slides 4-10 (Possible homework is on Slide 11)

LESSON 2: PART 1

Objective
To consolidate proportional reasoning

Time
20 minutes

Resources
- Slide 4: Brazilian fishermen
- Slide 5: Mortar
- Slide 6: Bradley Stoke



Activities
Introduce the *Brazilian fishermen* task, telling the students that the text is genuine dialogue from a genuine research project.

Small-group activity
- In groups, the students should work on the *Brazilian fishermen* task.
- Remind the students that they should listen to each other's views, and try to come to an agreement (as in the Ground Rules) about a more effective method.

Whole-class activity
- Collect the students' answers and let them explain their reasoning. Record the different strategies on a board or flipchart.
- Make sure that the students understand the need for multiplicative strategies in proportionality and that they know how to use them.

Note
This task highlights the effectiveness of using multiplicative strategies with large numbers, as opposed to adding the same amount several times, which may induce calculation mistakes.

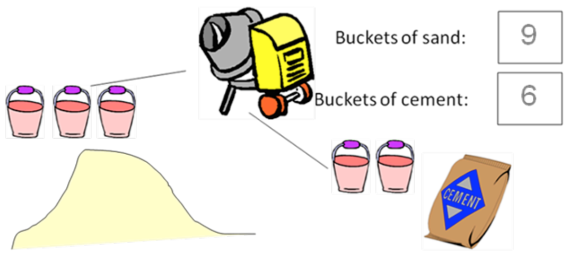
LESSON 2: PART 1 CONTINUED

Mortar

Mortar is a mixture of cement and sand (and water). It is crucial to get the proportions of cement and sand just right. If you use too much cement it gets too hard; and with too much sand it crumbles.

To make 5 buckets of mortar you have to mix 3 buckets of sand with every 2 buckets of cement.

How many buckets of sand and cement do you need to mix 15 buckets of mortar?



Bradley Stoke

"[...] developer Lovell Homes was found to have used the wrong mix in its mortar. Seventeen homes were declared unsafe. In one piece of unforgettable publicity, MP John Cope was shown pushing over a garden wall with his bare hands. The homeowners were eventually moved out and the houses rebuilt."

http://www.flickr.com/photos/20654194@N07/3352941007/

Activities

Whole-class activity

- Work with the students in whole-class dialogic mode on the *Mortar* task.
- Then present the *Bradley Stoke* example, explaining that in real-life people can make mistakes with sand and cement mixtures, sometimes with serious consequences.

You may wish to point out that the error was caused by scaling up additively rather than by using multiplication!

LESSON 2: PART 2

Objective

To provide individual practice in proportional reasoning

Time

10 minutes

Resources

- Slide 7: Hot pink paint
- Slide 8: Thinking distance

Hot pink paint

Andrew mixes 6 tins of red paint with 2 tins of white paint to produce 'hot pink' paint.

He needs to paint the walls of a large disco, so he buys all the red paint that the shop had in stock. This gives him 15 tins of red paint.

How many tins of white paint will he need to make the same hot pink paint?

Tins of red paint	Tins of white paint
3	1
6	2
12	4
15	5
21	7

Thinking distance

A driver's reaction time influences how quickly the driver can stop a car.

Reaction time is the difference between the moment the driver sees an obstacle and the time that the driver pushes the brakes. The distance travelled in that time is called *Thinking Distance*.

At a speed of 30 mph, the thinking distance is 15 metres for a driver who has a reaction time of 1.0 seconds.

What are the thinking distances for the other reaction times?

Reaction times	0.5 seconds	1.0 seconds	1.5 seconds	2.0 seconds
Thinking distance	7.5	15 metres	22.5	30

Activities

Individual work

- The students should work individually on the *Hot pink paint* and *Thinking distance* tasks to consolidate their proportional reasoning.

Whole-class activity

- Collect the students' answers and let them explain their reasoning. Different approaches should be recorded on a board or flipchart for comparison.
- Make sure that the students understand the need for multiplicative strategies in these two tasks and that they know how to use these strategies before you move on to the next part.

LESSON 2: PART 3

Objective

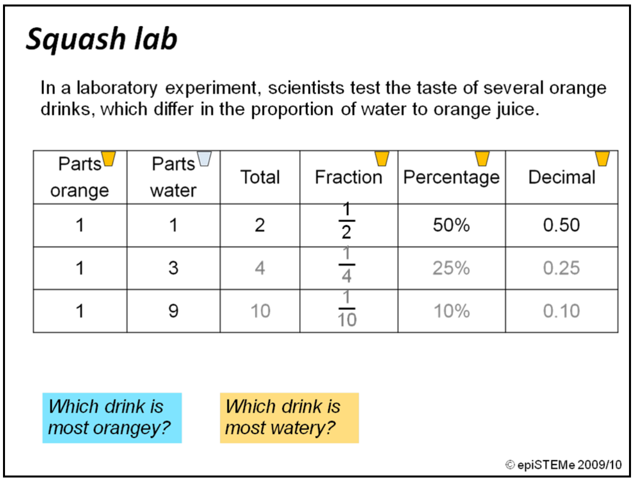
To show how ratios, fractions, decimals and percentages can be used to represent proportional relations

Time

20 minutes

Resources

- Slide 9: Squash lab
- Slide 10: Proportions in real life



Activities

Whole-class activity

- In whole-class mode, use the *Squash lab* task to highlight ratios, which will have been implicit in all problems used so far. Enter the answers by writing or typing.
- Then lead a discussion of how fractions, percentages and decimals might also have been used, making sure that the part-part nature of ratios and the part-whole nature of the other representations are emphasised (see Note below).
- Show the students *Proportions in real life* and encourage them to identify the forms of representation used in a series of real-world examples.

Note

- Ratios represent part-part relationships. Fractions, percentages and decimals represent part-whole relationships. Therefore the students need to understand that they have to establish the 'whole' first. This is shown in the 'Total' column.
- The ratios were chosen so that they convert into familiar fractions of 1/2, 1/4 and 1/10, which some of the students may be able to convert into percentages without calculations, i.e. some students may already know that 1/4 is the same as 25% or that 1/10 is 10%.

LESSON 2 HOMEWORK (OPTIONAL)

Trampolines

Tina gets a trampoline for her birthday. Her brother Brian measures how much the trampoline sinks when Tina stands on the trampoline. Under Tina's mass of 50 kg, the trampoline sags 10 cm.

How much would the trampoline sag for each member of Tina's family? Complete the table below.

	Cat Garfield (10 kg)	Tina (50 kg)	Brian (60 kg)	Mom Helen (75 kg)	Dad Greg (100 kg)
Trampoline sagging	2 cm	10 cm	12 cm	15 cm	20 cm

How much would the trampoline sag if all five jumped on together? 59 cm

Note

To consolidate material covered in Lessons 1 and 2, the homework exercise provides practice in scaling up and down in proportion.

END OF LESSON 2

LESSON 3: CALCULATING REPRESENTATIONS

Overview

Lesson 3 is detailed on Pages 17 to 22. Most of the lesson is devoted to the calculation of different proportional representations. At the end, there is a discussion about preferences for one proportional representation over another.

Aims

Lesson 3 aims to teach students:

- How proportions can be represented and calculated
- Some representations are preferred in some contexts, and others are preferred in other contexts, depending on mathematical and social factors

Structure

The lesson is in three parts and uses a mixture of whole-class, small-group and individual teaching.

- *Part 1* (small-group and whole-class): Teaches students to calculate the different forms of proportional representation
- *Part 2* (whole-class and individual work): Provides practice in calculation over an extended and integrated series of tasks
- *Part 3* (whole-class): Considers the contexts in which the different forms are used, now and historically

Resources

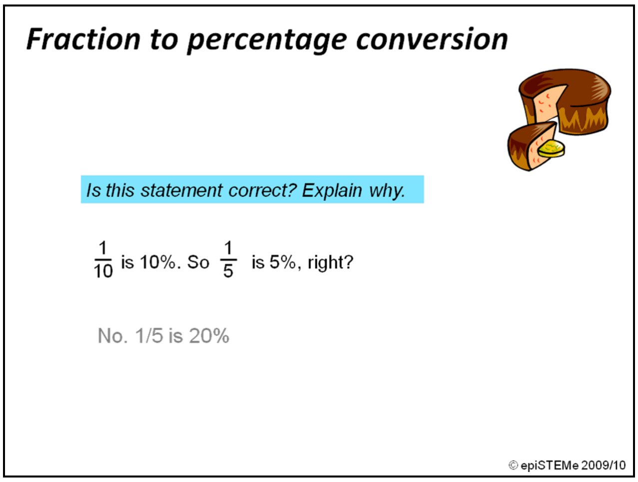
- PowerPoint Slides 12-23 (Possible homework is on Slides 24-27)

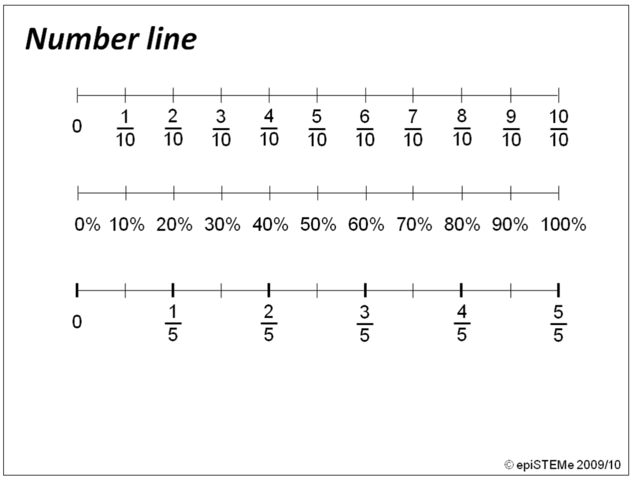
LESSON 3: PART 1

Objective
To calculate ratios, fractions, decimals and percentages for use as proportional representations

Time
20 minutes

Resources
- Slide 12: Fraction to percentage conversion
- Slide 13: Number line
- Slide 14: Pancake
- Slide 15: Vinaigrette





Activities
Before working in groups, the students should indicate in their Study Booklet whether they agree or disagree with the *Fraction to percentage conversion* statement.

Small-group activity
- In small-groups, the students should discuss the *Fraction to percentage conversion* statement, and agree a group response.

Whole-class activity
- Collect the students' answers and let them explain their reasoning. If many students agree with the statement or find it difficult to explain their reasoning, ask them if 1/2 would be 2%. Most students will be familiar with the fraction 1/2 and the equivalent proportional representation of 50%. Then let them revisit the original statement.
- Switch to 'authoritative talk' and use the *Number line* to show equivalences between fractions and percentages for the *Fraction to percentage conversion* task.

LESSON 3: PART 1 CONTINUED

Pancake

Aunt Maggie bakes the best pancakes in the family. For the mixture, she uses 6 cups of milk and 4 cups of flour.

Use ratios, fractions, decimals, and percentages to show the proportions of milk and flour in the mixture.

	Milk		Flour		Pancake mix
Ratio	6 cups	to	4 cups		10 cups
Fraction	$\frac{6}{10}$ of the mixture		$\frac{4}{10}$ of the mixture		$\frac{10}{10}$
Decimal	0.6 of the mixture		0.4 of the mixture		1.0
Percentage	60% of the mixture		40% of the mixture		100%

Vinaigrette

Uncle Sam has the best recipe for salad vinaigrette. He mixes 9 spoons of olive oil with 3 spoons of vinegar.

Use ratios, fractions, decimals, and percentages to show the proportions of olive oil and vinegar in the mixture.

	Olive oil		Vinegar		Vinaigrette
Ratio	9 spoons	to	3 spoons		12 spoons
Fraction	$\frac{9}{12}$ of the mixture		$\frac{3}{12}$ of the mixture		$\frac{12}{12}$
Decimal	0.75 of the mixture		0.25 of the mixture		1.0
Percentage	75% of the mixture		25% of the mixture		100%

Activities

Small-group activity

- In small-groups, the students should work on the *Pancake* task.
- Emphasise that everyone should have an opportunity to contribute, and a group response should be agreed.
- As before, circulate to gain information about the thinking and respond only to requests for clarification.

Whole-class activity

- Do not collect the students' strategies for solving the *Pancake* task at this point (follows later) but instead complete the *Vinaigrette* task together as a whole-class exercise.
- One key step is to enter the total number of spoons in the ratio row of the vinaigrette column, and emphasise that this should be the denominator (bottom number) in the fractions. Decimals and percentages can be calculated by dividing the denominator into the numerator (top number). It is crucial that the students understand this, so demonstrate on a board or flipchart using 'authoritative talk'.

Small-group activity

- Once the *Vinaigrette* task is complete, get the students to revisit the *Pancake* task in groups (as some groups may need to revise their previous strategies).

Note

Simplification of representations can be covered if it crops up naturally (e.g., in Pancake, 6/10 into 3/5), but this is not essential (since it will be covered in Lesson 5). It may also confuse some students unless explicit links are made between simplified and non-simplified representations, so be wary of accepting 3/5 (etc) without relating this to 6/10 (etc).

LESSON 3: PART 2

Objective

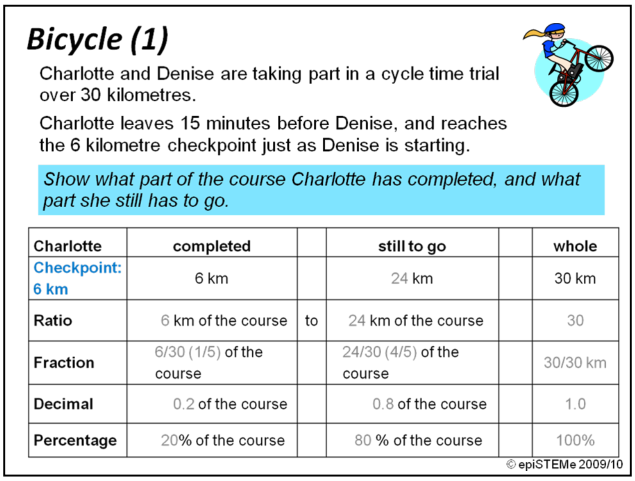
To practise the calculation of different proportional representations over an extended and integrated series of tasks

Time

20 minutes

Resources

- Slides 16-21: Bicycle (1) – (6)



Bicycle (1)

Charlotte and Denise are taking part in a cycle time trial over 30 kilometres.

Charlotte leaves 15 minutes before Denise, and reaches the 6 kilometre checkpoint just as Denise is starting.

Show what part of the course Charlotte has completed, and what part she still has to go.

Charlotte	completed		still to go		whole
Checkpoint: 6 km	6 km		24 km		30 km
Ratio	6 km of the course	to	24 km of the course		30
Fraction	6/30 (1/5) of the course		24/30 (4/5) of the course		30/30 km
Decimal	0.2 of the course		0.8 of the course		1.0
Percentage	20% of the course		80 % of the course		100%

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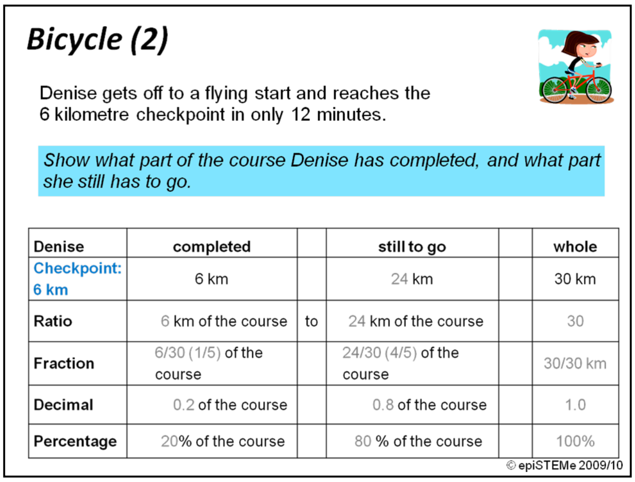

Bicycle (2)

Denise gets off to a flying start and reaches the 6 kilometre checkpoint in only 12 minutes.

Show what part of the course Denise has completed, and what part she still has to go.

Denise	completed		still to go		whole
Checkpoint: 6 km	6 km		24 km		30 km
Ratio	6 km of the course	to	24 km of the course		30
Fraction	6/30 (1/5) of the course		24/30 (4/5) of the course		30/30 km
Decimal	0.2 of the course		0.8 of the course		1.0
Percentage	20% of the course		80 % of the course		100%

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Activities

Tell the students that the next task is about time trials. If they are unfamiliar with time trials, you could let them perform a time trial in the courtyard (running around a track) or show a short video clip from the Tour de France, e.g. http://www.youtube.com/watch?v=OjJ60Kx2j8I.

Whole-class activity

- Start the *Bicycle* task in whole-class discussion using *Bicycle (1)* and *Bicycle (2)*. *Bicycle (2)* has exactly the same results as *Bicycle (1)*.
- The only difference between the two tasks is that Charlotte reaches the 6km checkpoint after 15 minutes and Denise after 12 minutes. The time does not play any role in finding the correct answers to this task.

LESSON 3: PART 2 CONTINUED

Bicycle (3)

But Charlotte has had the benefit of a downhill stretch, so that she is still 6 kilometres ahead when Denise reaches the 6 kilometre checkpoint.

Show how Charlotte is getting on.

Charlotte	completed		still to go	whole
Checkpoint: 12 km	12 km		18 km	30 km
Ratio	12 km of the course	to	18 km of the course	30
Fraction	12/30 (2/5) of the course		18/30 (3/5) of the course	30/30 km
Decimal	0.4 of the course		0.6 of the course	1.0
Percentage	40% of the course		60 % of the course	100%

Bicycle (4)

Denise is now on the downhill stretch. She manages to cover 9 more kilometres in the next 15 minutes.

Show how Denise is getting on.

Denise	completed		still to go	whole
Checkpoint: 15 km	15 km		15 km	30 km
Ratio	15 km of the course	to	15 km of the course	30
Fraction	15/30 (1/2) of the course		15/30 (1/2) of the course	30/30 km
Decimal	0.5 of the course		0.5 of the course	1.0
Percentage	50% of the course		50 % of the course	100%

Bicycle (5)

Meanwhile Charlotte has finished the downhill stretch, and begun to climb a gently uphill section of the course.

In the same 15 minutes, she has managed to cover only another 6 kilometres.

Show how Charlotte is getting on.

Charlotte	completed		still to go	whole
Checkpoint: 18 km	18 km		12 km	30 km
Ratio	18 km of the course	to	12 km of the course	30
Fraction	18/30 (3/5) of the course		12/30 (2/5) of the course	30/30 km
Decimal	0.6 of the course		0.4 of the course	1.0
Percentage	60% of the course		40 % of the course	100%

Bicycle (6)

How does the story end? Write in the box and complete the table.

Charlotte	completed		still to go	whole
Checkpoint: km	km		km	30 km
Ratio	km of the course	to	km of the course	
Fraction	of the course		of the course	
Decimal	of the course		of the course	
Percentage	% of the course		% of the course	

Activities

Individual work

- Let the students complete some of *Bicycles (3)* to *Bicycles (6)* individually.
- Use your discretion when to hand over the task to them, depending on the ability of the students.

LESSON 3: PART 3

Objectives

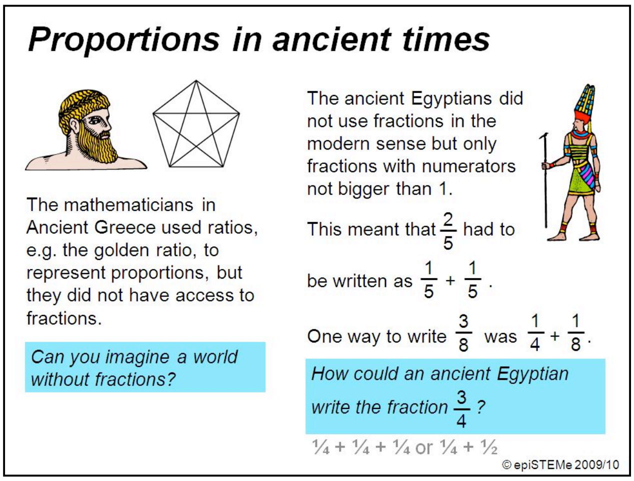
To discuss preferences for one proportional representation over another, depending on the context

Time

10 minutes

Resources

- Slide 22: Proportions in real life (optional)
- Slide 23: Proportions in ancient times



Activities

Whole-class activity

- Encourage the students to give examples of where each form of representation (fractions, decimals, percentages and ratios) is typically used in everyday life. The *Proportions in real life* slide may stimulate ideas, but it may not be necessary.
- Discuss why one form of representation is preferred when often any could have been used. The reasons are a mixture of mathematical factors (e.g. part-part vs. part-whole) and social factors (e.g. use % in opinion polls to down-play sample size and as the form closest to whole numbers which makes comparisons easier for most readers).
- Further discussion could take place about the use of *Proportions in ancient times*.
- Given the emphasis on fractions in mathematics curricula, it may surprise students to learn that ancient societies used ratios (which are actually more versatile than fractions). “In fact, philosophers also had something against fractions in those days (something along the lines of `the unit being indivisible', since if you break up your measuring unit into smaller ones you are effectively replacing it by a smaller measuring unit!); this was a prejudice that mathematicians got around (by looking at ratios instead of fractions), but practical people happily used fractions anyway.” http://www.msri.org/activities/pastprojects/jir/bwachtel/CountonNumberOne.html

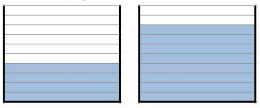
LESSON 3 HOMEWORK (OPTIONAL)

Watering flowers

The two pictures below show the proportion of two cups filled with water (shaded) that is needed to water a pot of flowers on *cold* and *hot* days.

What proportion of a cup-full is needed on cold and hot days?

cold days hot days



On cold days, you need 4/10 of a cup-full to water a pot of flowers.

On hot days, you need 8/10 of a cup-full to water a pot of flowers.

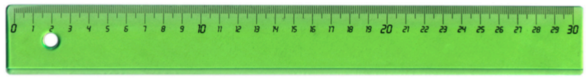
Did you use a fraction, decimal, percentage or ratio? Fraction

Write a sentence explaining why you used this type of number rather than the other three types.

____

Measurement

Describe the size of the strawberry.



4.5 cm

____

Did you use a fraction, decimal, percentage or ratio? Decimal

Write a sentence explaining why you used this type of number rather than the other three types.

____

Survey

Two schools took part in a survey to find out whether students like their sports lessons.
In Seaside School, 250 students participated in the survey. 50 students said they liked their sports lessons.
In Woodland School, where 280 students participated in the survey, 70 students liked their sports lessons.

Compare the proportion of students who like sports lessons in the two schools.

20% in Seaside School liked sports lesson compared with 25% in Woodland School.

____

Did you use a fraction, decimal, percentage or ratio? Percentage

Write a sentence explaining why you used this type of number rather than the other three types.

____

Mixture

This is a short instruction for boiling rice.
•Take 1 cup of rice and put the rice in a pot.
•Add 2 cups of water and let the water boil.
•Switch off the heat and let the pot stand for 10 minutes.

Describe the relation between rice and water in the pot.

1:2

____

Did you use a fraction, decimal, percentage or ratio? Ratio

Write a sentence explaining why you used this type of number rather than the other three types.

____

Note

It is unlikely that the students will use the same form of representation for each problem, e.g. decimals are likely to feature in 'Measurement' and ratios will probably be favoured in 'Mixture'. Therefore, the homework exercise not only provides practice in calculating proportional representations, but also underlines the everyday contexts in which particular forms are preferred.

END OF LESSON 3

LESSON 4: CONVERTING REPRESENTATIONS

Overview

Lesson 4 is detailed on Pages 24 to 29. The lesson starts by converting between different proportional representations, and ends with conversion from integers and proportional representations to other integers.

Aim

Lesson 4 aims to teach students:

- How to convert between different proportional representations and between proportional representations and integers

Structure

The lesson is in three parts and uses a mixture of whole-class, small-group and individual teaching.

- *Part 1* (individual work and whole-class): Introduces direct conversion between different proportional representations.
- *Part 2* (small-groups and whole-class): Provides practice in conversion across extended and integrated problems.
- *Part 3* (individual work and whole-class): Problem solving that includes conversion between proportional representations and integers.

Resources

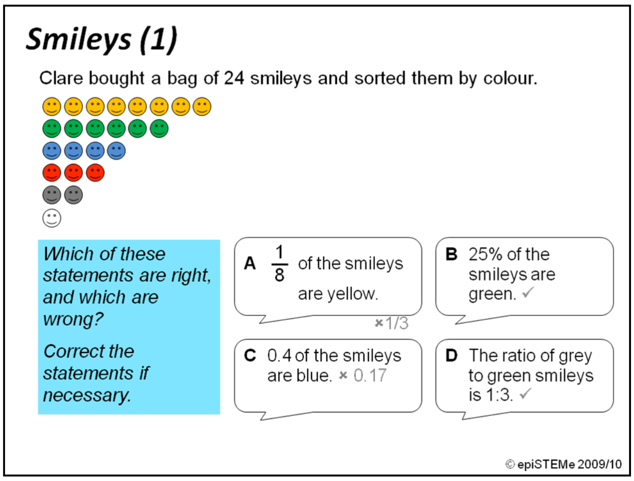
- PowerPoint Slides 28-36 (Possible homework is on Slide 37)

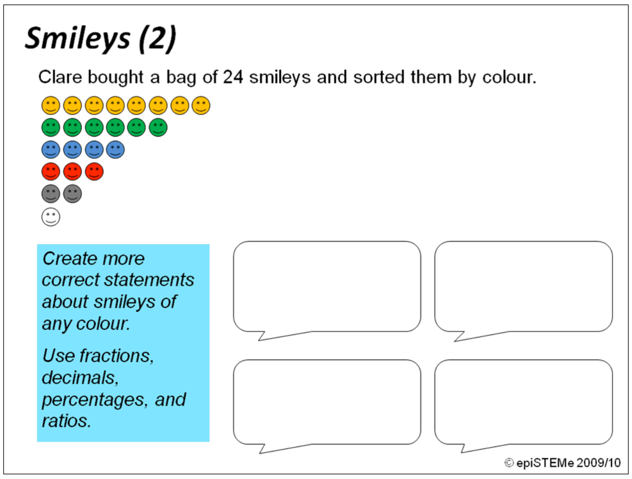
LESSON 4: PART 1

Objective
To convert between different proportional representations

Time
10 minutes

Resources
- Slide 28: Smileys (1)
- Slide 29: Smileys (2)





Activities
Individual work
- Let the students work individually on *Smileys (1)* to consolidate some of the messages from the previous lesson.

Whole-class activity
- Debrief in whole-class, encouraging the students to provide reasons and answers. In other words, follow Introductory Module guidelines for whole-class dialogue.
- Then, use *Smileys (2)* and ask a few students to make up other correct statements. Ask the other students to comment on whether these statements are right or wrong.

LESSON 4: PART 2

Objectives

To practise conversion across extended and integrated problems

Time

20 minutes

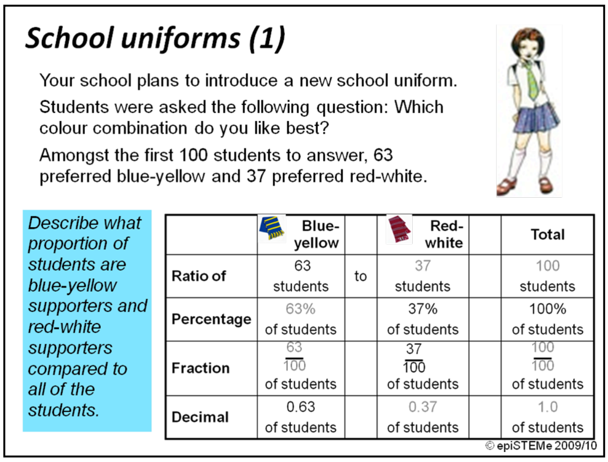
Resources

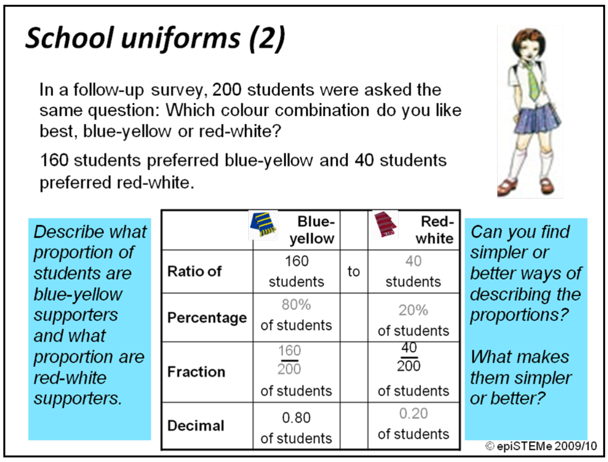
- Slides 30-33: School uniforms (1) to (4)

Activities

Small-group activity

- Get the students to work on *School uniforms (1)* in groups.
- *School uniforms (1)* brings out the distinction between part-part and part-whole relations. Its numerical values have been chosen to focus attention on the basic definition of each representation (and on comparison between these definitions) without issues of simplification arising.





Whole-class activity

- As previously, do not collect the students' strategies for solving this immediately but instead complete *School uniforms (2)* through dialogic whole-class teaching. Remember to record answers by writing or typing on the slide.
- You may want to discuss how to check if answers to *School uniforms (2)* are correct.
- The numerical values in *School uniforms (2)* have been chosen to prompt attention to issues of arithmetical equivalence (including simplification), and the second instruction box is intended to raise these explicitly.

Small-group activity

- Then, get the students to return to *School uniforms (1)* and let them revisit it in groups (as some groups may need to revise their previous strategies).

Whole-class activity

- In whole-class, get the students to work on *School uniforms (3)* and *School uniforms (4)* (which can be found on the next page).
- As always, encourage the students to explain their reasoning as well as provide answers. Take time to establish if differing strategies were used, even when they lead to the same answer. If strategies are recorded on a board or flipchart, the students can be asked to compare them and, under your guidance, identify the most effective approach.

School uniforms (3)

In one school, $\frac{3}{10}$ of the students preferred blue-yellow.

What percentage was this?

30%

In another school, 65% of the students preferred blue-yellow.

What decimal was this?

0.65

In another school, 0.7 of the students preferred blue-yellow.

What fraction was this?

7/10

School uniforms (4)

In one school, 25% preferred blue-yellow, and 300 pupils liked red-white.

How many pupils took part in the survey in that school?

400

In another school, $\frac{1}{5}$ preferred blue-yellow, and 200 pupils liked red-white.

How many pupils took part in the survey in that school?

250

Note

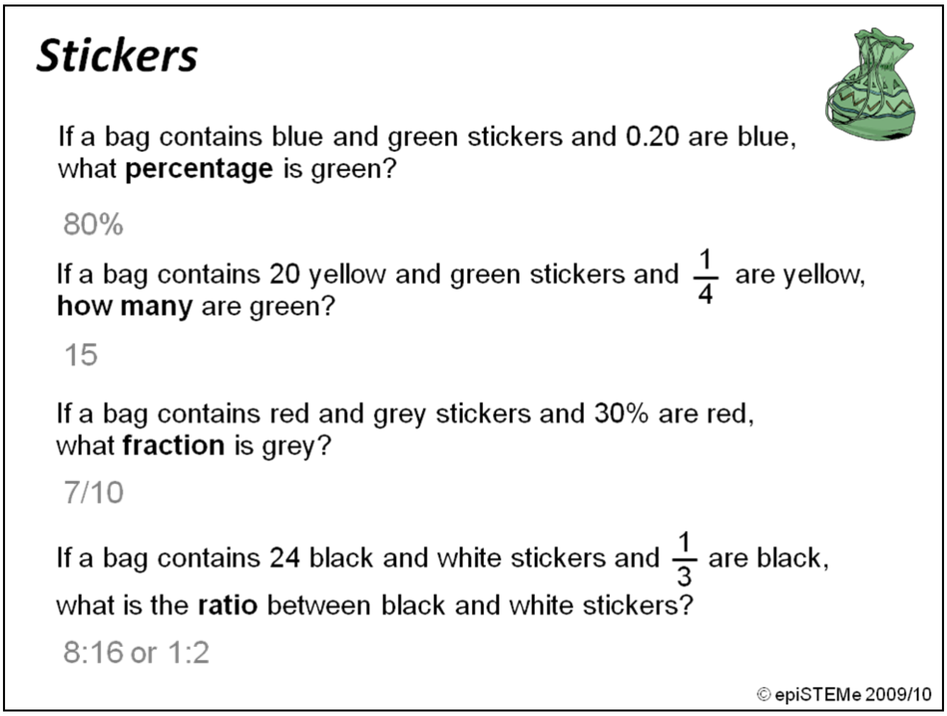
- It is important that the students are encouraged to use and give proportional reasons (not just calculation procedures) throughout these tasks.
- Introduce the language of "is equivalent to" in rephrasing, summarising and extending student contributions. *16 to 4 is exactly the same ratio as 160 to 40; we say that 16 to 4 is "equivalent" to 160 to 40 ... Can anyone suggest another ratio that is equivalent to both of these? 40 students out of 200 is the same proportion as 4 out of 20. And both of these are the same proportion as 1 out of 5. So all these fractions describe exactly the same proportion; they are equivalent.*

LESSON 4: PART 3

Objective
To provide practice in conversion between proportional representations and from proportional representations to other integers

Time
20 minutes

Resources
- Slide 34: Stickers
- Slide 35: Science (1)
- Slide 36: Science (2)



Activities
Individual work
- Let the students work individually on the *Stickers* task to practise converting between proportional representations, and from proportional representations to integers.

Whole-class activity
- Debrief using whole-class dialogic mode.

LESSON 4: PART 3 CONTINUED

Science (1)

70% of the Earth's surface is covered by water and the rest by land. What is the **ratio** between water and land on the Earth's surface?

7:3

0.20 of the human water intake comes from food. What **fraction** of the water intake comes from other sources, including drinking water and beverages?

8/10 or 4/5

40% of the human body is made up of muscle. What **fraction** comes from other material?

6/10 or 3/5

Science (2)

There are an estimated 350,000 species of plants on Earth, of which $\frac{1}{7}$ have not been identified yet.

How many species of plant have been identified so far?

300,000

0.90 of flowering plants depend on animal assistance for pollination. What **percentage** does not depend on animal assistance for pollination?

10%

Nitrogen makes up 3/4 of the air. What **percentage** is made up of other gases?

25%

Activities

Individual work

- In order to make a link with Science, let the students work individually on *Science (1)* and *Science (2)*.

Whole-class activity

- Debrief using whole-class dialogic mode.

LESSON 4 HOMEWORK (OPTIONAL)

School food

In 2003, the BBC Food Magazine published the results of a survey that asked adults which school food they hated most.

600 pupils at Seaside Middle School were asked which school food they disliked most.

The following table shows the results of that survey.

Work out the missing numbers to describe for each school lunch the proportion of pupils who disliked this particular food.

600 pupils in total	Beetroot	Cabbage	Stew	Semolina
Number of pupils	300	150	120	30
Fraction	1/2	$\frac{1}{4}$	1/5	1/20
Decimal	0.5	0.25	0.2	0.05
Percentage	50%	25%	20%	5%

Note

If the students attempt this exercise, it may be useful to establish whether they converted the representations directly (e.g. 1/4 = 0.25 = 25%) or whether they calculated the whole numbers first (e.g. 1/4 of 600 is 150) and then worked out the representations independently.

END OF LESSON 4

LESSON 5: EQUIVALENCE BETWEEN REPRESENTATIONS

Overview

Lesson 5 is detailed on Pages 31 to 35. The lesson starts by teaching students how to simplify and scale up ratios. It continues with simplifying and scaling up fractions, and ends with a task that combines equivalence within ratios as well as equivalence within fractions. [Please note that if simplification was covered during Lessons 3 and 4 (as may have been appropriate in some classes – see earlier), some tasks in Lesson 5 will not require extended coverage.]

Aims

Lesson 5 aims to teach students:

- Equivalences within proportional representations
- How ratios and fractions can be simplified

Structure

The lesson is in two parts and uses a mixture of whole-class and small-group teaching.

- *Part 1* (whole-class and small-groups): Covers simplification and scaling up of ratios
- *Part 2* (whole-class and small-groups): Covers simplification and scaling up of fractions

Resources

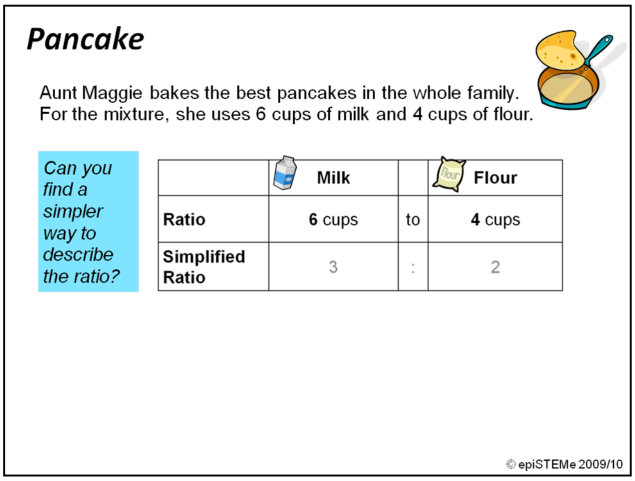
- PowerPoint Slides 38-44 (Possible homework is on Slide 45)
- Fraction domino task - a master copy is in a plastic folder

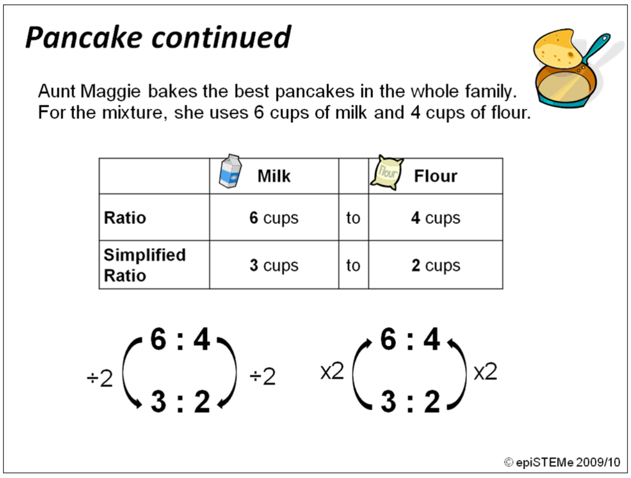
LESSON 5: PART 1

Objective
To cover simplification and scaling up of ratios

Time
20 minutes

Resources
- Slide 38: Pancake
- Slide 39: Pancake continued
- Slide 40: Pancake equivalence





Activities
Whole-class activity

- Refer back to previous tasks that allowed for simplification, e.g. by using the *Pancake* example.
- Ask the students how they would simplify the ratios in the table. In the *Pancake* example, the ratio of 6:4 can be simplified to 3:2. As you discuss the example, make sure that terms like “is in proportion with” or “is equivalent to” are used in rephrasing, summarising and extending student contributions. For instance *6 to 4 is the same proportion as 3 to 2. 6 to 4 is equivalent to 3 to 2.*
- Then use the *Pancake continued* example to show the students how to simplify ratios and how to scale them up.

LESSON 5: PART 1 CONTINUED

Activities

Small-group activity

- Let the students work in groups on *Pancake equivalence*.

Whole-class activity

- Ask the students to explain how they have derived the numbers.
- The first three rows should be relatively easy. The last two rows may be more difficult, and the students may need guidance. For instance, since 9 is 1.5 times 6, one needs to add 6 cups of flour because 4 times 1.5 is 6. Since 1 is a quarter of 4, one needs to add 1.5 cups of milk because 6 divided by 4 is 1.5. As before, using a board or flipchart to record and support comparison of student strategies may be helpful in leading them to the most effective approach.

Pancake equivalence

Aunt Maggie bakes the best pancakes in the whole family.
For the mixture, she uses 6 cups of milk and 4 cups of flour.

Fill the missing numbers into the table so that they show equivalent ratios.

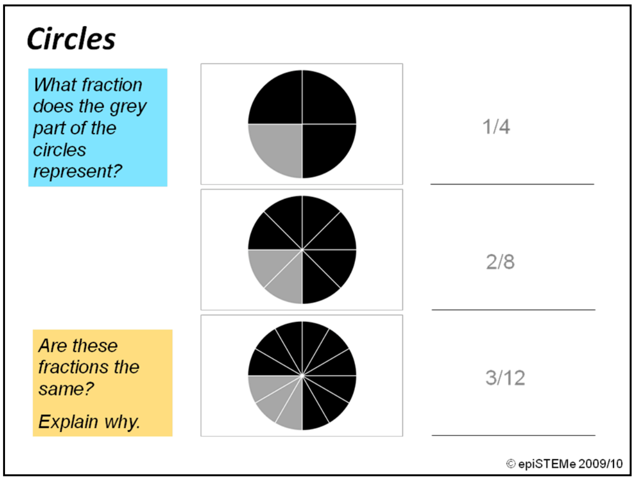
Milk		Flour
6 cups	:	4 cups
12	:	8
18	:	12
30	:	20
9	:	6
1.5	:	1

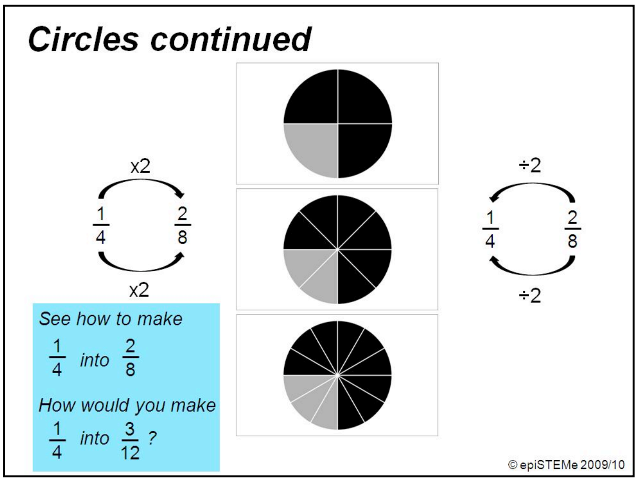
LESSON 5: PART 2

Objective
To cover simplification and scaling up of fractions

Time
30 minutes

Resources
- Slide 41: Circles
- Slide 42: Circles continued
- Slide 43: Vinaigrette equivalence
- Slide 44: Fraction domino slide
- Fraction domino task - a master copy is in the plastic folder

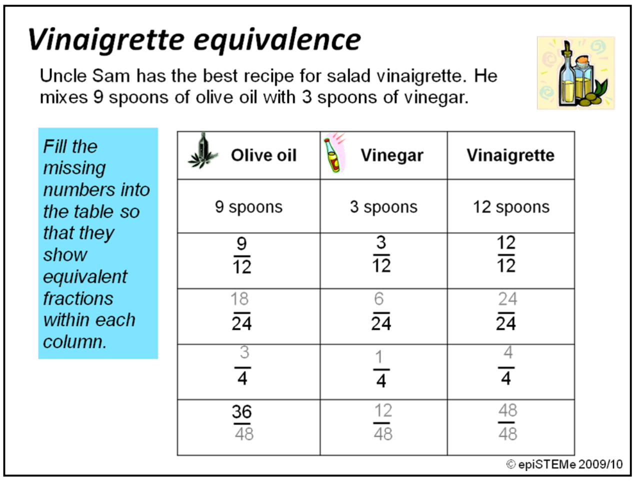




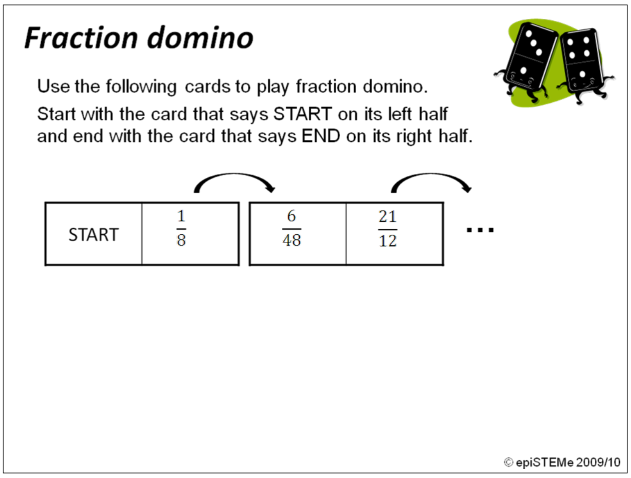
Activities
Whole-class activity
- With the whole-class, use the *Circles* task to show equivalences within fractions. The three circles visualise the fraction of 1/4 in the first circle and its equivalent fractions of 2/8 and 3/12 in the second and third circle.
- Ask the students what fraction the grey part in the circles represents. Ask them also whether these fractions are the same. The students should try to explain their answer.
- Use the *Circles continued* example to show the students HOW to simplify fractions and how to scale them up.
- Ask the students what calculation they need to undertake in order to make 1/4 into 3/12.

Note
If the class is already familiar with simplification, they may start by suggesting 1/4 for all three circles. If so, encourage them to find 2/8 and 3/12 as alternatives (plus many others if you wish), and discuss the relation between the various representations.

LESSON 5: PART 2 CONTINUED



Olive oil	Vinegar	Vinaigrette
9 spoons	3 spoons	12 spoons
$\frac{9}{12}$	$\frac{3}{12}$	$\frac{12}{12}$
$\frac{18}{24}$	$\frac{6}{24}$	$\frac{24}{24}$
$\frac{3}{4}$	$\frac{1}{4}$	$\frac{4}{4}$
$\frac{36}{48}$	$\frac{12}{48}$	$\frac{48}{48}$



Activities

Small-group activity

- Let the students work in groups on *Vinaigrette equivalence* to practise equivalences within fractions.

Whole-class activity

- Ask the students to explain how they derived the numbers.
- Encourage them to use the language of proportional reasoning, i.e. “is the same proportion as” or “is equivalent to”.

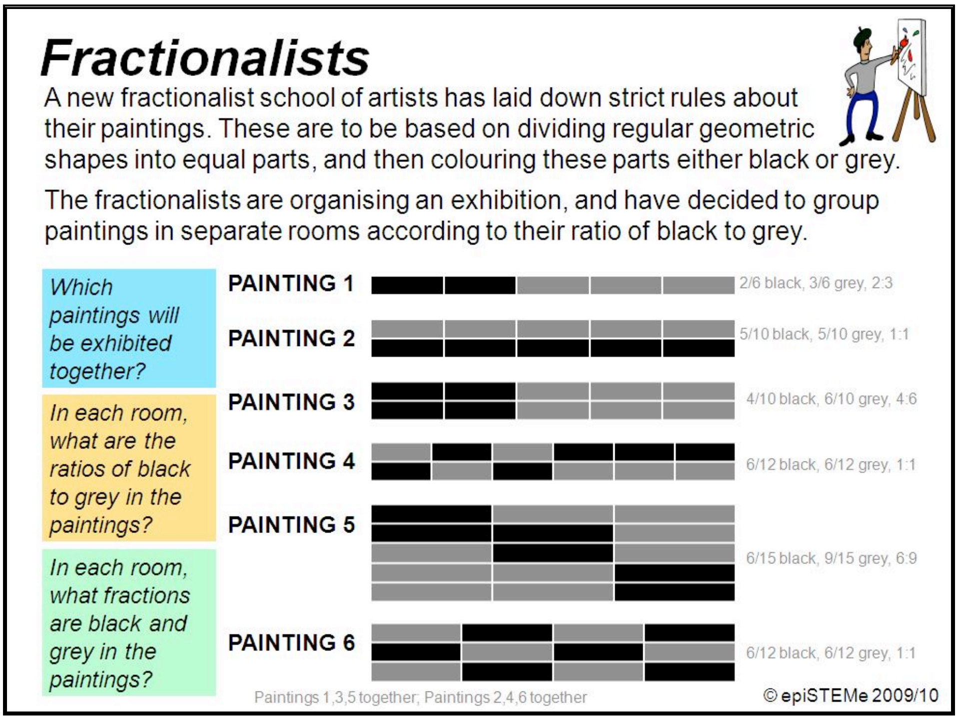
Small-group activity

- Let the students work in groups on *Fraction domino* to practise equivalences within fractions.
- Explain the domino rules to the students if they are not familiar with the game.

Note

The pairs in the *Fraction domino* game are chosen in a way that one part of the pair is an integer multiple of the other part, e.g. 8/2 and 16/4 (multiplied by 2) instead of 8/2 and 28/7 (multiplied by 3.5), with the easy exception of 5/5 and 213/213. This should make it possible to play the game without the need for a calculator.

LESSON 5 HOMEWORK (OPTIONAL)



Note

As well as highlighting equivalences *within* ratios and fractions (and therefore that some forms are simplified versions of other forms), this task gently revisits equivalences *across* ratios and fractions.

END OF LESSON 5

LESSONS 6 AND 7 ARE HARDER THAN LESSONS 1 TO 5, AND THEREFORE OPTIONAL

LESSON 6: CALCULATOR WORK (Optional lesson)

Overview

Optional Lesson 6 is detailed on Pages 37 to 40. The lesson starts by teaching students how to derive percentages using calculators. It continues with problems that invite students to identify ‘best offers’, as often occur in real-life situations. The lesson ends with a whole-class task, in which students show their understanding of the relative magnitudes of different proportional representations.

Aim

Lesson 6 aims to teach students:

- How to use calculators in order to work out proportional relationships with larger or more complex numbers

Structure

The lesson is in three parts and uses a mixture of whole-class, small-group and individual teaching.

- *Part 1* (whole-class and small-groups): Introduces the use of calculators in proportional representation
- *Part 2* (individual work, whole-class and small-groups): Presents further calculator work in the everyday context of spotting bargains
- *Part 3* (whole-class): Requires students to place fractions, decimals and percentages in order of magnitude

Resources

- PowerPoint slides 46-52 (Possible homework is on Slides 53-55)

LESSON 6: PART 1

Objective

To introduce the use of calculators in proportional representation

Time

15 minutes

Resources

- Slides 46-48: Trips to school (1) – (3)

Trips to school (1)

In 2006, the Office for National Statistics published survey results of how 11-16 year old children get to school.

23,485 children participated in this survey.

How would you work out the percentages in the table?

	Walk	Bus	Car	Bike	Other ?
Number of children	9629	7280	4697	705	1174
Percentage	41%	31%	20%	3%	5%
Decimal	0.41	0.31	0.2	0.03	0.05
Fraction	41/100	31/100	1/5	3/100	1/20

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Activities

Whole-class activity

- Use the first column of the *Trips to school (1)* task and ask the students how they would calculate the percentage of students walking to school.
- If the students don't know how to derive percentages, use the *Trips to school (2)* example to explain the procedures. The key thing is that in order to get percentages, the students need to work out the decimals first using their calculators.

Small-group activity

- Return to the *Trips to school (1)* task and get the students to work in small-groups on the remaining columns.
- Make sure that the students' calculators are set to show decimals instead of fractions.

Whole-class activity

- Debrief in whole-class using *Trips to school (3)*.
- You can ask the students how they could check if their calculations are right (the percentages add up to 100% and the decimals add up to 1.0).
- If time permits, ask the students to calculate the ratios for two example combinations of travel modes.

Trips to school (2)

In 2006, the Office for National Statistics published survey results of how 11-16 year old children get to school.

23,485 children participated in this survey.

How would you work out the percentages in the table?

	Walk
Number of children	9629
Percentage	
Decimal	
Fraction	

9629 ÷ 23,485 = 0.41 (decimal)

= 41% (percentage)

= 41/100 (fraction)

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Trips to school (3)

	Walk	Bus	Car	Bike	Other ?
Number of children	9629	7280	4697	705	1174
Percentage	41%	31%	20%	3%	5%
Decimal	.41	.31	0.2	0.03	.05
Fraction	$\frac{41}{100}$	$\frac{31}{100}$	$\frac{1}{5}$	$\frac{3}{100}$	$\frac{1}{20}$

What is the ratio of "children being driven to school" to "children using other travel modes"?

4697:1174 or 4:1

What is the ratio of "carbon-neutral travel modes (walk + bike)" to "carbon-using travel modes (car + bus)"?

10334:11977 or 44:51

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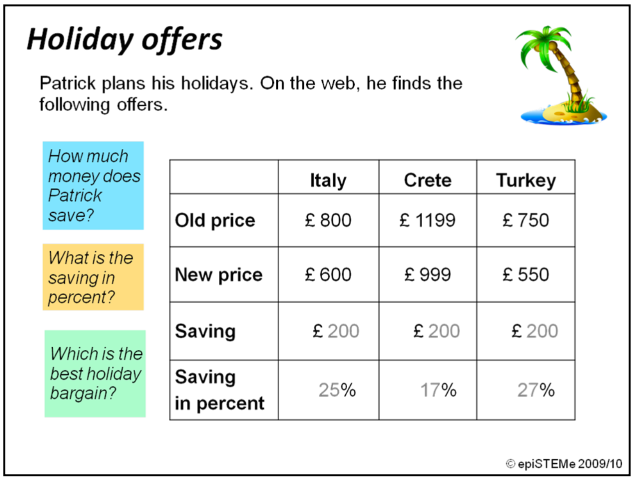
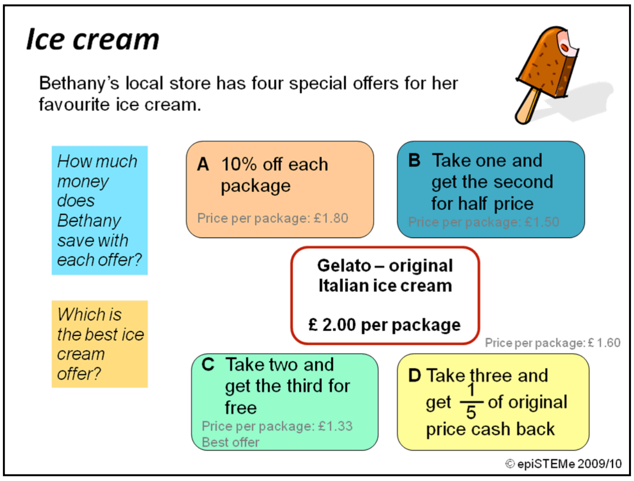
LESSON 6: PART 2

Objective
To practise calculation of percentages in the everyday context of spotting bargains

Time
20 minutes

Resources
- Slide 49: Holiday offers
- Slide 50: Ice cream





Activities
Individual work
- Get the students to work individually on *Holiday offers*.

Whole-class activity
- Debrief using whole-class dialogic mode.
- The students need to derive the saving in £ first and then calculate what this saving is compared to the original price, e.g. Italy, £200 saving; £200/£800 = 0.25 = 25%.

Small-group activity
- Get the students to work in small-groups on *Ice cream*.
- As before, circulate to gain information about the thinking and respond only to requests for clarification (see Introductory Module). Remind the students to agree about which offer is best and give reasons.

Whole-class activity
- Ask the students which offer is best, and let them explain their reasoning and calculations.

LESSON 6: PART 3

Objective

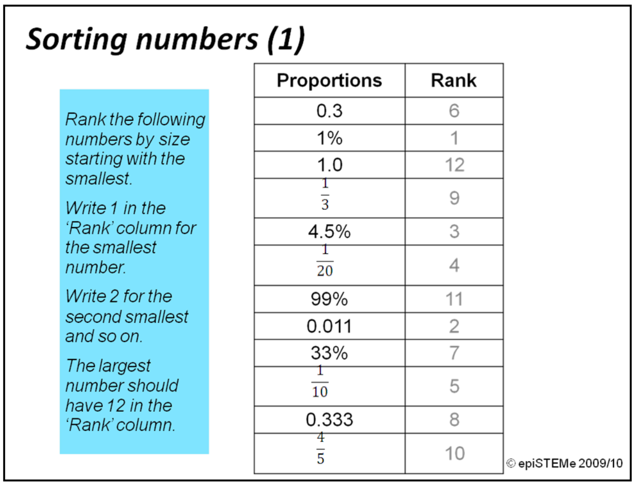
To help students recognise the relative magnitude of different proportional representations

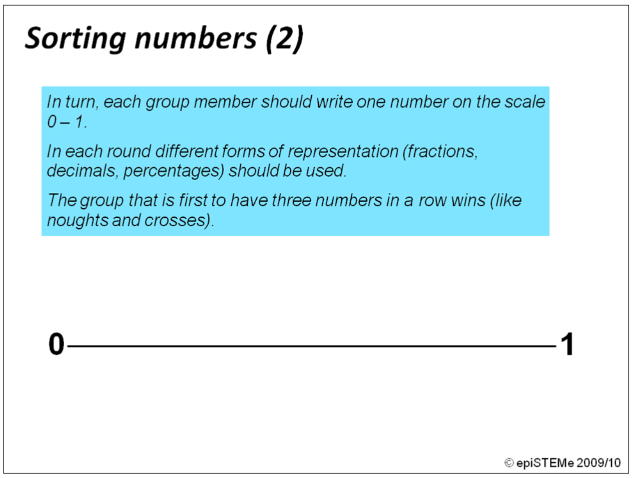
Time

15 minutes

Resources

- Slides 51 and 52: Sorting numbers (1) and (2)





Activities

Whole-class activity

- In whole-class, use *Sorting numbers (1)* and *Sorting numbers (2)* to let the students order different proportional representations by size.
- You could use *Sorting numbers (2)* as a whole-class game with competition between two groups, e.g. boys vs. girls or half of the tables vs. the other half.

LESSON 6 HOMEWORK (OPTIONAL)

National statistics (1)

Fill in the missing numbers in the text.

The number of people living in the UK increased by 8% (per cent) from 55.9 million in 1971 to 60.6 million in 2006.

In the UK in 2004, males could expect to live 76.6 years and females 81.0 years. The life expectancy of men is 95% (per cent) of that of women.

In 1901 males born in the UK could expect to live around 45 years and females around 49 years. This is an increase in life expectancy of 70% (per cent) for males and 65% (per cent) for females from 1901 to 2004.

In 2006, the death rates for circulatory diseases (which include heart disease and stroke) were 2,462 per million for males and 1,559 per million for females. The ratio of males to females dying of circulatory diseases is 2,462 to 1,559.

National statistics (2)

Fill in the missing numbers in the text.

Ten of the warmest years globally have all occurred between 1995 and 2006. This means that 5/6 or 10/12 (fraction) of this period has seen the warmest years globally.

In 2006, 18,133 gigawatt hours (GWh) of electricity were produced from renewable energy. Currently biofuels dominate the amount of electricity generated (9,295 GWh in 2006), which is 51% (per cent) of electricity generated by renewable energy. This is followed by hydro (4,605 GWh), which is around 1/4 (fraction) of electricity generated. The energy use per household between 1971 and 2005 increased by 1.5 per cent, from 1.87 to 1.90 (decimal) tonnes of oil equivalent per household (22,100 kilowatt hours).

The total amount of municipal waste produced in England rose from around 24.6 million tonnes in 1996/97 to peak at nearly 29.6 million tonnes in 2004/05. This is an increase of around 1/5 (fraction). In 2006/07, municipal waste amounted to 29.1 (decimal) million tonnes, of which 89 per cent (25.9 million tonnes) was generated by households.

National statistics (3)

Fill in the missing numbers in the text.

In 2006/07, there were nearly 33,900 schools in the UK, attended by 9.8 million pupils, providing a school-to-pupil ratio of approximately 1 to 289 (integers).

In this year, the number of female teachers in mainstream schools in the UK was at its highest level in the last 24 years at 310,000, while the number of male teachers fell to its lowest level over the same period, to 132,000, giving a ratio of female to male teachers of 310 to 132 (integers).

The number of support staff, such as teaching assistants and technicians, in maintained schools in England, increased by 7 per cent from 225,000 in 2006 to 240,750 (integers) in 2007.

Note

The statements and numbers are adapted from the ONS:
http://www.statistics.gov.uk/downloads/theme_social/Social_Trends38/Social_Trends_38.pdf

END OF LESSON 6

LESSON 7: PROPORTIONAL REASONING AND PROPORTIONAL REPRESENTATIONS (Optional lesson)

Overview

Optional Lesson 7 is detailed on Pages 42 to 45. Using the context of the Harry Potter story, the lesson revisits proportional reasoning (as introduced in Lessons 1 and 2), but uses more complicated numbers that normally require calculators. So Lesson 7 concludes the module by bringing several themes together.

Aims

Lesson 7 aims to:

- Consolidate students' use of multiplicative strategies in proportional reasoning
- Use calculators in the context of proportional reasoning

Structure

Lesson 7 is in three parts and uses a mixture of whole-class, small-group and individual work. All three parts cover scaling-up and scaling-down with complex numbers.

- *Part 1* (whole-class)
- *Part 2* (small-groups and whole-class)
- *Part 3* (individual work and whole-class)

Resources

- PowerPoint slides 56-61 (Possible homework is on Slides 62 and 63)

LESSON 7: PART 1

Objective

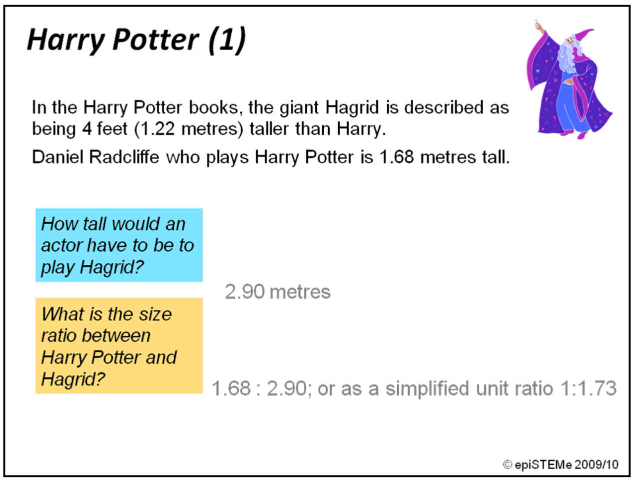
To learn how to address scaling-up problems with complex numbers

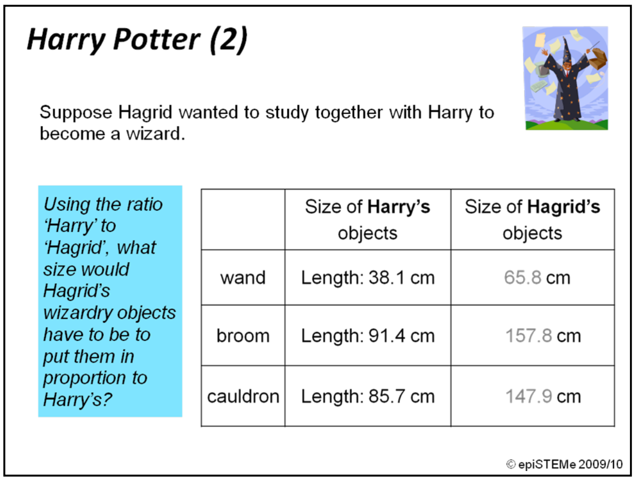
Time

15 minutes

Resources

- Slide 56: Harry Potter (1)
- Slide 57: Harry Potter (2)





Activities

Whole-class activity

- Ask the students how to find the length of Hagrid in *Harry Potter (1)*.
- Then ask the students to work out the ratio between Harry Potter and Hagrid. Point them towards simplifying the ratio to a 'unit ratio', which will help to answer the problems that follow. (Use your discretion as to whether to use the term 'unit ratio'.)
- Get the students to work in whole-class on *Harry Potter (2)*.
- Ask them to explain how they derived their answers, encouraging them to use the language of proportionality.

Note

In *Harry Potter (1)*, an actor would have to be 2.90m to play Hagrid. Therefore, the ratio between Harry Potter and Hagrid is 1.68:2.90, or as a simplified unit ratio 1:1.73 (i.e. 1.68 divided by 2.90 = 1 divided by ?)

LESSON 7: PART 2

Objective

To extend principles to scaling-within problems and scaling-down problems

Time

15 minutes

Resources

- Slide 58: Harry Potter (3)
- Slide 59: Harry Potter (4)

Harry Potter (3)

Hagrid would love to try on Harry's invisibility cloak. Although Hagrid is not supposed to use magic he tries three different charms to change the cloak's size in proportion to Harry's.

Which of the three charms was the correct one?

Length of Harry's invisibility cloak	Length of invisibility cloak after charm A	Length of invisibility cloak after charm B	Length of invisibility cloak after charm C
2.1 metres ×	1.2 metres ×	3.8 metres ×	3.6 metres ✓

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Harry Potter (4)

Hagrid lives in a hut that is made for him being a giant. Suppose Harry and Hagrid wanted to switch accommodation for one night.

What size would Harry's belongings in the hut have to be to put them in proportion to Hagrid's giant-sized belongings?

	Size of **Hagrid's** belongings	Size of **Harry's** belongings
toothbrush	Length: 30.5 cm	17.7 cm
bed	Length: 3.5 metres	2.0 m
pillow	Length: 1.1 metres	0.6 m

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Activities

Small-group activity

- Get the students to work in small-groups on *Harry Potter (3)*, a 'scale-within problem'.

Whole-class activity

- Debrief using whole-class dialogic mode.

Small-group activity

- Get the students to work in small-groups on *Harry Potter (4),* a 'scale-down problem'.

Whole-class activity

- Debrief using whole-class dialogic mode.

LESSON 7: PART 3

Objective

To practise solving scaling-up, scaling-within, and scaling-down problems

Time

20 minutes

Resources

- Slide 60: Harry Potter (5)
- Slide 61: Harry Potter (6)

Harry Potter (5)

There are no actors who are 2.90 m tall. Robbie Coltrane, the actor who played Hagrid, is actually 1.85 m tall. Set designers fool people into believing that Hagrid is a giant by altering the size of objects in such a scene.

In one scene, Hagrid frees Harry from a cabin on a remote island. Hagrid lifts the cabin's door from its hinges and crouches to enter the room.

What size must the set designer make the items in that scene so that Robbie Coltrane (1.85 metres) looks as tall as the giant Hagrid (2.90 metres)?

	standard size	set size
door	1.90 metres high	1.21 m
table	90 cm high	57 cm
window	85 cm wide	54 cm

Harry Potter (6)

The set designer's assistant has designed more film objects for a scene, in which Robbie Coltrane (1.85 metres) should look as tall as the giant Hagrid (2.90 metres).

What object(s) has the assistant scaled incorrectly?

	standard size	set size
chair	60 cm high	94.1 cm × 38.2 cm
wardrobe	2.3 metres high	1.47 metres ✓
book	25 cm long	39.2 cm × 15.9 cm

When Harry (Daniel Radcliffe being 1.68 metres) is filmed in Hagrid's (2.90 metres) hut, the audience should get the impression that Harry sits at a giant's table and drinks from a giant's cup.

What size must the set designer make the items in that scene so that Harry looks like being in a giant's home?

	standard size	set size
tea cup	8 cm high	13.8 cm
window	85 cm wide	146.7 cm
watering can	20 cm long	34.5 cm

Activities

Individual work

- The students should work individually on *Harry Potter (5)* and *Harry Potter (6)*, which incorporate all three scaling problems from the previous Harry Potter tasks.

Whole-class activity

- Debrief using whole-class dialogic mode.

LESSON 7 HOMEWORK (OPTIONAL)

Fuel efficiency

Susan thinks of buying an environmentally friendly car.

On a test run where she drove 85 km, she used 4.3 litres of fuel.

Assuming that the consumption of fuel was stable for this car, how much fuel would Susan have used for the following distances?

Please use calculators.

Distance in km	Litres
35	1.8
60	3.0
85	4.3
100	5.1
145	7.3

Sheep shearer

Each summer approximately 500 shearers from Australia and New Zealand come to the UK to help local farmers to remove the wool from the 14.5 million of sheep in the UK.

It takes these 500 shearers 10 hours in total to shear 200,000 sheep.

How many hours would it take the following number of shearers to shear these 200,000 sheep?

Shearers	Hours
50	100
100	50
250	20
500	10
1000	5
2000	2.5

Note

Sheep Shearer is an inverse proportionality problem. If you use this homework task, you may want to tell the students to calculate the man hours first.

END OF LESSON 7

APPENDIX: epiSTEMe

The epiSTEMe project [Effecting Principled Improvement in STEM Education] is part of a national programme of research that aims to strengthen understanding of ways to increase young people's achievement in physical science and mathematics, and their participation in courses in these areas. Drawing on relevant theory and earlier research, the epiSTEMe project has developed a principled model of curriculum and pedagogy designed to enhance engagement and learning during a particularly influential phase in young people's development: the first year of secondary education. The module on 'Fractions, Ratios and Proportions' is one of four topic-specific modules that have been developed to operationalise that model and support its classroom implementation.

The teaching model

The epiSTEMe teaching model builds on current thinking in the field and on promising exemplars that have been extensively researched. These suggest that students' learning and engagement can be enhanced through classroom activity organised around carefully crafted problem situations designed to develop key disciplinary ideas. These situations are posed in ways that appeal to students' wider life experience, and draw them more deeply into mathematical and scientific thinking. Such an approach is intended not just to help students master challenging new ways of thinking, but also to help them develop a more positive identity in relation to mathematics and science.

An important feature of the teaching model is the way in which it makes explicit links between mathematics and science. Within mathematics modules, the primary rationale for this is that science represents a major area where an unusually wide range of mathematics is applied, often for a variety of purposes. Within science modules, the primary rationale is that understanding of scientific ideas is deepened by moving from expressing them in qualitative terms to representing them mathematically. Thus, the present 'Fractions, Ratios and Proportions' module has a partner module, which addresses proportionality in the context of Key Stage 3 science teaching about forces. Although the modules are self-contained, the ideal scenario is that this module is studied concurrently with or slightly before its science equivalent.

The teaching model also emphasises the contribution of dialogic processes in which students are encouraged to consider and debate different ways of reasoning about situations. These dialogic processes are designed to take place in the course of joint activity and collective reflection at two levels of classroom activity: student-led (and teacher-supported) collaborative activity within small-groups, and teacher-led (and student-interactive) whole-class activity. Because of the importance of developing dialogic processes that support effective learning, these processes are the focus of a separate Introductory Module, which is additional to the four topic-specific modules.

The design of the topic-specific modules

The function of all four topic-specific modules is to provide examples of concrete teaching sequences that incorporate classroom tasks that reflect the teaching model. The tasks will, in particular, support dialogic processes and will be supported by these processes.

First, each module has been designed to cover those aspects of the topic that are suitable for the start of the Key Stage 3 curriculum, and to do so in a way that is appropriate for students across a wide range of achievement levels. Taking account of available theory and research on the development of students' thinking around the topic, the module 'fills out' the official prescriptions in ways intended to build strong conceptual foundations for the topic. This includes providing means of deconstructing common misconceptions related to the topic.

In this way, the modules take account of students' informal knowledge and thinking related to a topic. They also make connections with widely shared student experiences relevant to a topic. Equally, with a view to helping students understand how mathematics and science play a part in their wider and future lives, the modules try to bring out the human interest, social relevance, and scientific application of topics.

Finally, while the modules place a strong emphasis on exploratory dialogic talk, they also make provision for later codification and consolidation of key ideas, and build in individual checks on student understanding that can be used to provide developmental feedback.