

EXPANDING CURRENT PRACTICE IN USING DYNAMIC GEOMETRY TO TEACH ABOUT ANGLE PROPERTIES

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Introductory comments

An earlier article (Ruthven, Hennessy & Deaney, 200#) reported on current practice in using dynamic geometry to teach about angle properties. The way of using dynamic geometry systems (DGS) found to be most widespread involved establishing angle properties through dragging a geometric figure and observing its angle measures. Typical topics were angle properties of the circle ('the circle theorems'); and the angle sums of the triangle and other polygons. There was an emphasis on mediating geometric properties through numeric measures and data pattern generalisation. This article will make some speculative suggestions as to how these approaches might be adapted and extended so as to support a broader range of mathematical thinking, expanding –in particular– its visuo-spatial and logico-deductive aspects.

The two topics mentioned above will be examined in turn. First, a summary will be given of how the topic was found to be framed in current practice. Then, two further ways of framing the topic will be suggested. The first will retain numeric measures, but create a situation which brings a system of geometric properties into play rather than one or two isolated ones. The second will suppress numeric measures, and propose a situation which focuses on direct geometrical analysis. In line with the preference expressed by teachers in the course of the earlier study, these situations will be carefully structured to bring out the relevant properties. This depends, of course, on attention to how the figure changes under dragging, and –more important still– to what stays the same –as an indicator of what is generic and paradigmatic within the figure.

Angle properties of the circle

The 'circle theorems' appear to be a classic case of curricular drift. Originally, the primary rationale for including them was their epistemic value in providing an exemplary system of local deduction:

This is the ideal group; the results are interesting and are also perfect examples of the deductive rather than the experimental method. (Mathematical Association, 1959: p. 62).

Within this rationale, the 'angle-chasing' exercises which now typify the textbook and examination treatment of this topic were regarded as secondary:

Calculations of angles by the use of these properties are needed to drive home the facts, and are generally appreciated by the duller boy as easier than most riders. (Mathematical Association, 1959: p. 63)

We must, of course, recall that these extracts were written (as their language indicates) in an earlier age, with reference to the curriculum of academically selective schools. Nevertheless, as long as we continue to teach this topic, it is important to be clear about what distinctive educational function it serves; and that function is clearly in exemplifying geometrical analysis of a system of properties.

As observed, current practice involved taking the simplest figure possible (see Figure 1; in some cases, the angle-at-the-centre was only added later; in others, both angles were shown from the start). First, the size of the angle-at-the circumference was explored by dragging its vertex, P ; then its relationship to the size of the angle-at-the-centre, by dragging the common points, A and B , to generate several pairs of angle measures; and last, the special case where A and B are dragged to form a straight angle-at-the-centre. Teachers reported that they found this an efficient and effective way of covering these topics.

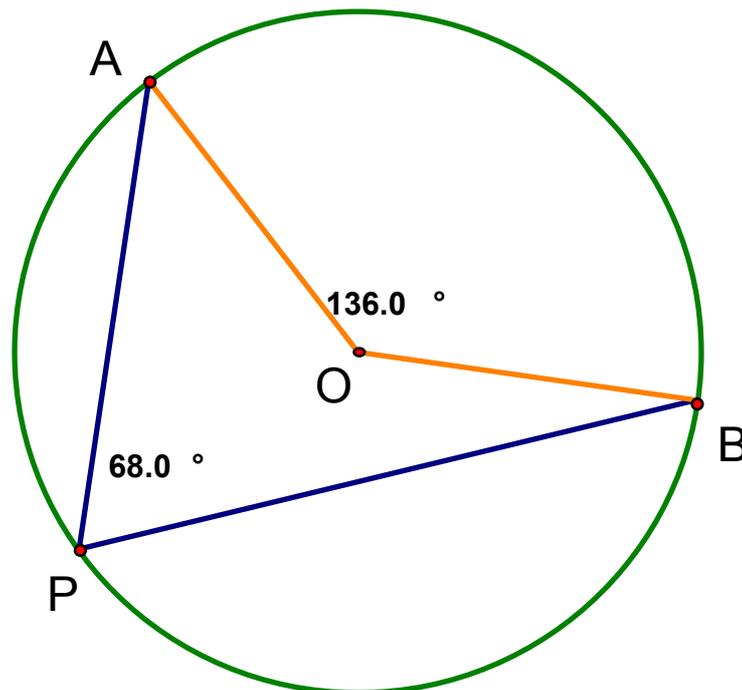
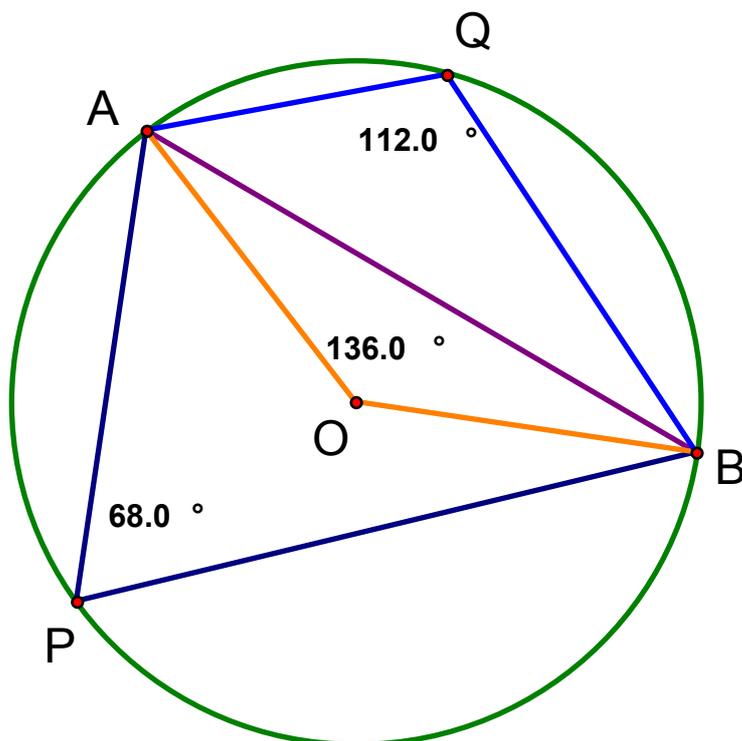


Figure 1

The first reframing of this topic is designed to create a mathematically richer situation for empirical exploration (see Figure 2). It involves two additions to the figure. First, the chord AB is marked, bringing out the division of the circle into two segments: this chord can now be dragged in its own right, so establishing it (rather than separate points A and B) as a single entity governing the situation; as it is dragged towards or away from the vertex of a particular angle, the size of that angle

increases or decreases; as it is dragged over the centre of the circle, the two segments come into balance, forming semicircles as the chord becomes a diameter.



The chord **AB** divides the circle into two segments.
P and **Q** are points on the circumference of the circle.

AB can be dragged to change the dividing position.
P and **Q** can be dragged to change their position within
a segment, or to move them into the other segment.

*How does dragging affect the size of the marked angles?
Are the sizes of the marked angles connected in any way?*

Figure 2

Second, a further point *Q*, analogous to *P*, is added on the circumference: when *P* and *Q* are positioned in the same segment, they create the standard static figure for the angles-in-the-same-segment property (see Figure 3; variants of this idea were seen in lessons observed in the earlier study); when they are positioned in opposite segments, they create the standard static figure for the angles-in-opposite-segments, or opposite-angles-of-cyclic-quadrilateral, property (see Figure 2). Recognising the case where a point-on-the-circumference moves from one segment to the other, or two points-on-the-circumference lie in opposite segments (cases avoided in the lessons observed) not only provides a more complete analysis of the situation, but creates a fuller and

more coherent system of angle properties (compatible with the current national curriculum).

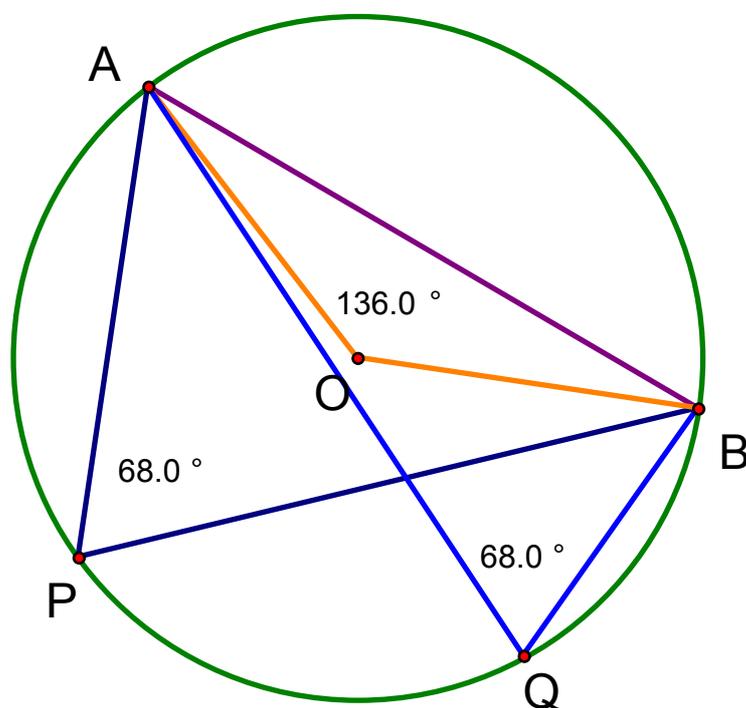
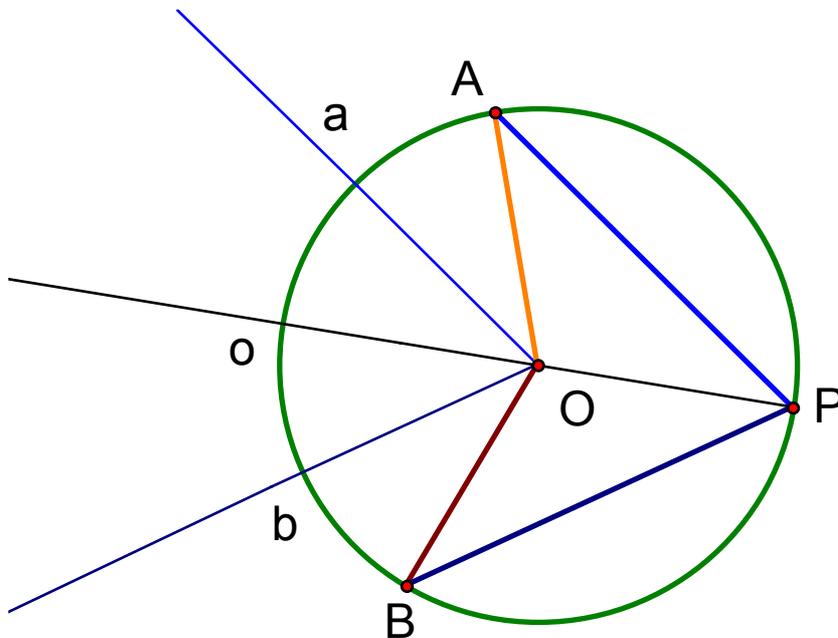


Figure 3

The second reframing of this topic is designed to create a mathematically richer situation for analysing the geometric mechanisms underpinning the central property (see Figure 4). The rays which have been added to the normal figure provide visual guides when points are dragged; the dragging of the dynamic figure helps to bring out which angles are equivalent in size (particularly for students not yet familiar with the proof analysis). When A is dragged, a remains parallel to its defining arm PA , creating a system of alternate and corresponding angles; it is also seen to bisect the angle formed by the radius OA and the ray o . As this is happening, the point B , and its associated arm, radius and ray remain static. This dynamic-static situation is reversed when B is dragged rather than A . Under both these forms of dragging, the ray o remains static, separating the ‘A’ and ‘B’ components of the geometric mechanism. Finally, when P is dragged, o does moves, dividing both the angle APB and its counterpart angle aOb in identical ways. Through these dragging operations, then, the geometric structure of the figure emerges, notably the angle relationships which underpin a proof analysis.



This figure helps to analyse the geometric structure creating a connection between angles **APB** and **AOB**. The ray **a** has been constructed parallel to **PA**, the ray **b** parallel to **PB**, and the ray **o** extends **PO**.

*Which angles in the figure change when **A** is dragged, when **B** is dragged, and when **P** is dragged?*

Which sets of angles in the figure keep the same size as one another when any of these points is dragged?

Why do these sets of angles keep the same size?

*Why does angle **AOB** stay double angle **APB** in size?*

Figure 4

Angle sums of polygons

This topic provides an example of a different type of curricular shift, in which what was a relatively minor geometric topic has taken on a new significance as an example of the algebraic formulation of sequence patterns. In the course of this shift, emphasis has switched from shape-and-space towards number-and-algebra. The new framings of the topic offered here seek to re-emphasise shape-and-space aspects, while retaining a concern with inductive sequence. The intention is to move from empirical induction towards mathematical induction, through decomposing a dynamic polygon into two components: a stable component in the form of a polygon of smaller degree, and a variable component in the form of a triangle; a more accessible form of mathematical induction than that normally taught.

As observed, current practice involved taking a simple polygon figure with the measures of all angles marked (see Figure 5), dragging points to generate several examples, and summing the angles for each example. The development followed an inductive sequence, starting with the familiar cases of the triangle and quadrilateral, using these to introduce the dragging approach; then proceeding to pentagon and later polygons, so establishing a table of angle sums from which a data pattern could be formulated. Later, additional lines were drawn onto figures to show how they could be broken down into triangles (although, in the lessons that we observed, it could have been made more explicit that the purpose was to decompose the angles of the polygon into triangular sets).

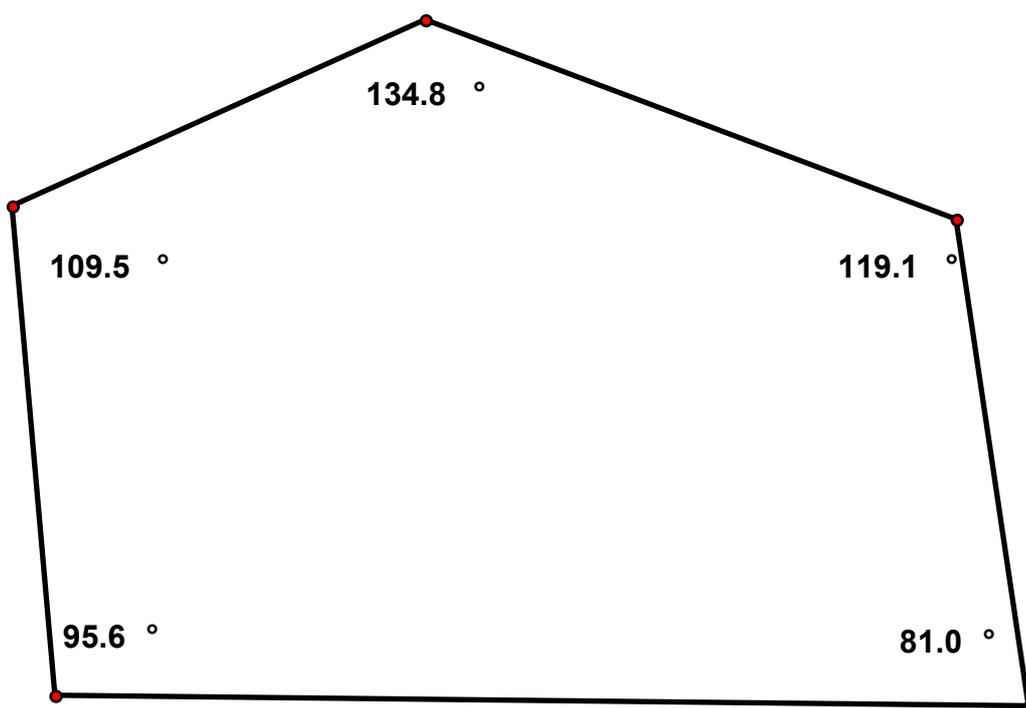
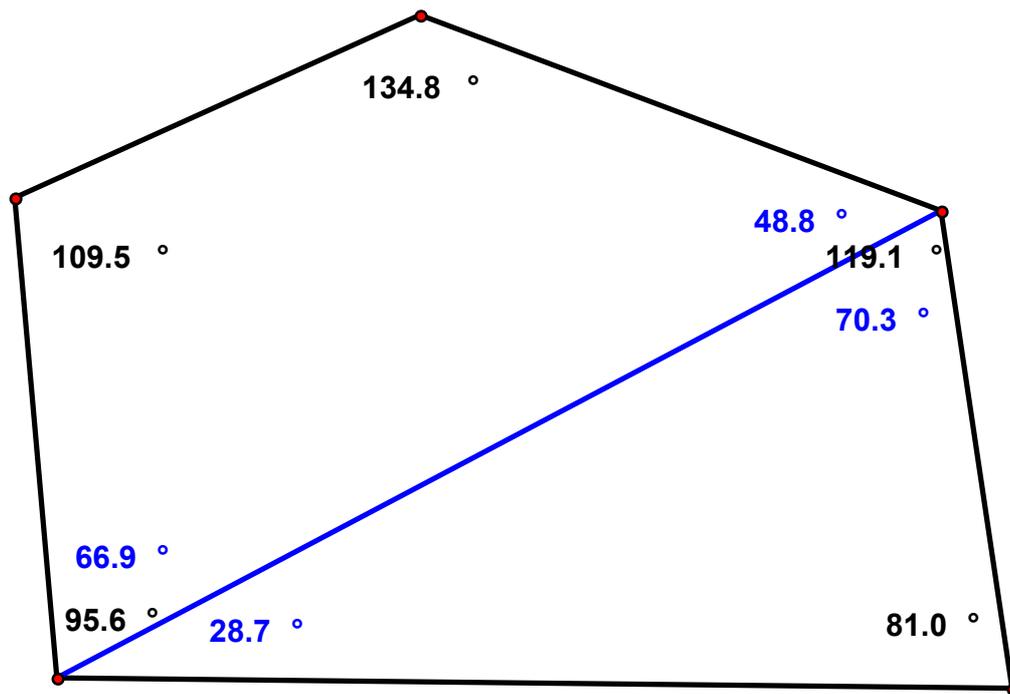


Figure 5

The first reframing of this topic is designed to create a mathematically richer situation for empirical exploration (see Figure 6). Rather than using separate figures for triangle, quadrilateral, and pentagon, all three are included in a single figure. Moreover, this figure incorporates the relationship that any pentagon can be decomposed into a triangle and a quadrilateral in a way which also partitions the angles of the pentagon. In the situation proposed, this decomposition provides a way of calculating the required angle sums more efficiently.



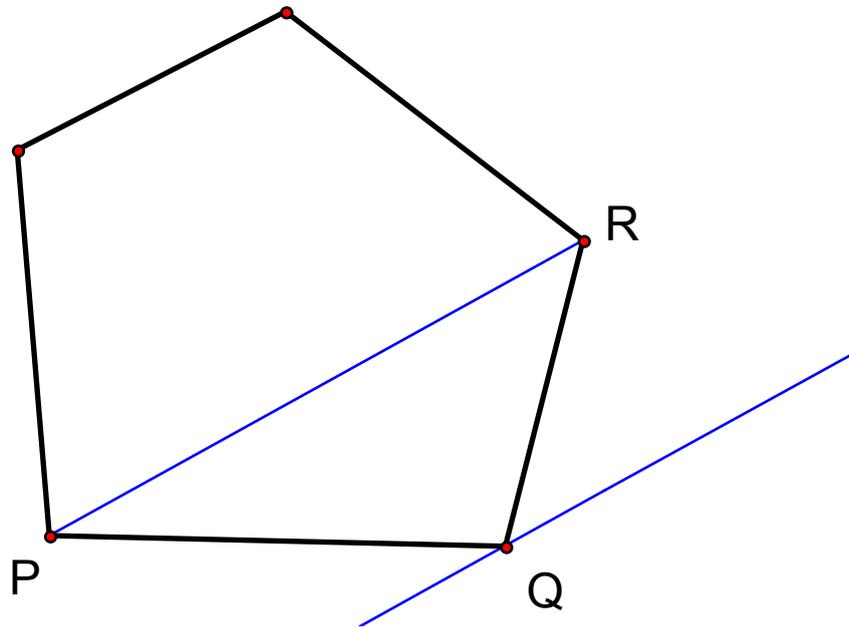
This figure contains a triangle, a quadrilateral, and a pentagon. It shows the sizes of all the angles in each of these polygons. *Find an efficient way of calculating the angle sums of all three polygons.*

Dragging points in the figure changes the shape of the polygons. *How does dragging affect the angle sum of each polygon? Do the angle sums of the three polygons form a pattern?*

Figure 6

The second reframing of this topic is designed to create a mathematically richer situation for analysis of the geometric mechanisms underpinning the inductive step (see Figure 7). The starting point is the observation that when a vertex is dragged, only its adjacent edges move with it; all other elements of the figure remain static. Dragging the vertex Q also moves the edges QP and QR, but no other vertices or edges of the pentagon. Introducing some supplementary guide-lines assists analysis of this situation: the segment PR has been marked to help emphasise how dragging Q affects only the triangular form PQR, leaving the remainder of the pentagon (which takes a quadrilateral form) unchanged. In particular, the angles (and part-angles) in this stable component of the pentagon are unaffected by dragging (as long as Q does not cross PR), and sum to the total for a quadrilateral. The angles (and part-angles) of the pentagon which do change under this form of dragging are those within triangle PQR. Although students are likely to be familiar with the fact that the angles of a

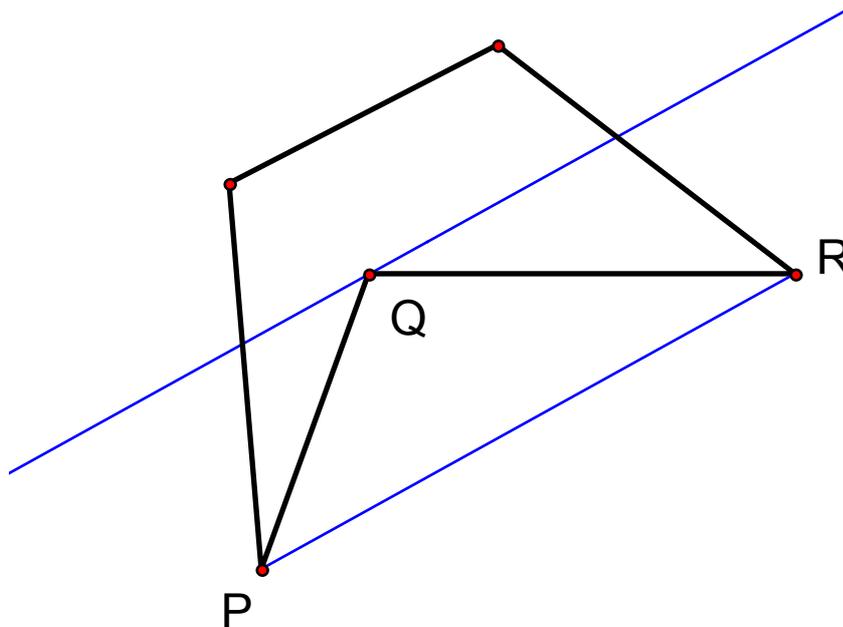
triangle sum to a straight angle, they may not have met –or may be unable to recollect– a geometrical explanation for this. The line marked through Q, parallel to PR provides a framework for providing such an explanation, through observing the



effects of dragging and analysing them in terms of alternate angles.

Figure 7

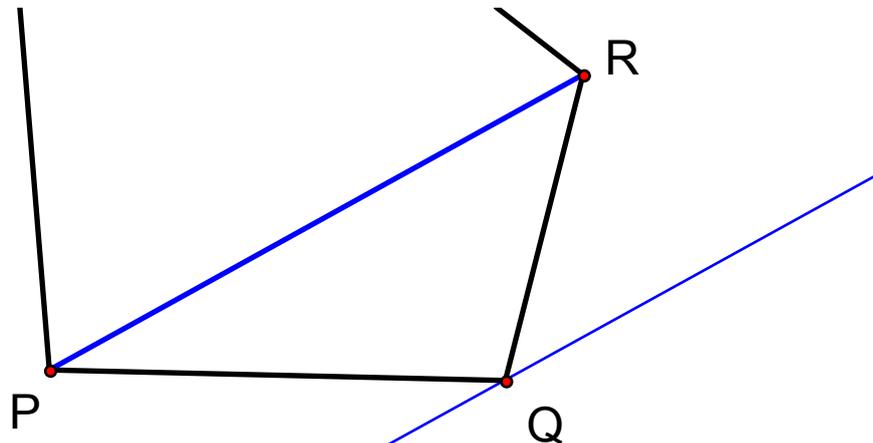
Moving beyond this simplest case, Q can be dragged so that it lies on PR (creating a degenerate pentagon), or even ‘beyond’ PR so that the angle PQR within the pentagon becomes reflex (creating a concave pentagon). Again, in each of these



cases, the additional line through Q assists analysis of the situation (see Figure 8).

Figure 8

The final figure (see Figure 9) was created simply by dragging the pentagon figure slightly out of the DGS window (but who can imagine what monstrous polygon might be lurking there in its place), and is intended to help reframe the pentagon as a generic polygon (for which we do not know the specific number of sides).



This diagram shows a polygon which is being varied by dragging its vertex **Q**.

The part of the polygon which is changing forms the triangular shape **PQR**.

The other part of the polygon is not changing, and not all of it can be seen.

What kind of shape does this other part form?

Compared to this other shape, how many sides does the whole polygon have? and what size is the angle sum of the whole polygon?

If this other shape were a triangle, how many sides and what angle sum would the polygon have?

If this other shape were a quadrilateral, how many sides and what angle sum would the polygon have?

If this other shape were a pentagon, how many sides and what angle sum would the polygon have?

Figure 9

Of course, this whole approach depends on a tacit mathematics of dragging; in particular, on the idea that more complex dragging of a polygon can be decomposed into a sequence of vertex dragging operations of the type examined here.

Concluding comments

These suggestions are speculative starting points for adapting and extending current approaches to using dynamic geometry to teach about angle properties. They are intended to promote and support a broader range of mathematical thinking; in particular, visuo-spatial and logico-deductive aspects. Against the background of current practice, and in the professional context that shapes it, these ideas may seem far-fetched and unrealistic. If not that, they certainly need to be operationalised and tested, and adapted to different classroom situations. I would be pleased to hear from anyone attempting that, or thinking of doing so. In the future, I hope to find some support for sustained collaborative work to develop the classroom use of dynamic geometry to support visuo-spatial and logico-deductive aspects of mathematical thinking. Again, I would be pleased to hear from any departments or teachers interested in working on this.

Acknowledgements

Thanks to the Economic and Social Research Council for funding the *Eliciting Situated Expertise in ICT-integrated Mathematics and Science Teaching* project (2002/04), award ref. R000239823, and to my colleagues Rosemary Deaney, Sara Hennessy and Tim Rowland for their various contributions.

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