NEW PERSPECTIVES ON RESEARCH IN MATHEMATICS EDUCATION

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DEVELOPING RESEARCH IN MATHEMATICS EDUCATION

Twenty Years of Communication, Cooperation and Collaboration in Europe

Edited by
Tommy Dreyfus
Michele Artigue
Despina Potari
Susanne Prediger
Kenneth Ruthven

ROUTLEDGE

DEVELOPING RESEARCH IN MATHEMATICS EDUCATION

Developing Research in Mathematics Education is the first book in the series *New Perspectives on Research in Mathematics Education*, to be produced in association with the European Society for Research in Mathematics Education (ERME). This inaugural volume sets out broad advances in research in mathematics education which have accumulated over the last 20 years through the sustained exchange of ideas and collaboration between researchers in the field.

An impressive range of contributors provide specifically European and complementary global perspectives on major areas of research in the field on topics that include:

- the content domains of arithmetic, geometry, algebra, statistics and probability;
- the mathematical processes of proving and modelling;
- teaching and learning at specific age levels from early years to university;
- teacher education, teaching and classroom practices;
- special aspects of teaching and learning mathematics such as creativity, affect, diversity, technology and history;
- theoretical perspectives and comparative approaches in mathematics education research.

This book is a fascinating compendium of state-of-the-art knowledge for all mathematics education researchers, graduate students, teacher educators and curriculum developers worldwide.

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New Perspectives on Research in Mathematics Education – ERME series

ERME, the European Society for Research in Mathematics Education, is a growing society of about 900 researchers from all over Europe and beyond.

The ERME series documents the growing body of substantial research on mathematics education within the context of ERME. Volumes in the ERME series can be monographs or collections growing out of the collaboration of European researchers in mathematics education.

The volumes are written by and for European researchers, but also by and for researchers from all over the world. An international advisory board guarantees that ERME stays globally connected. A rigorous and constructive review procedure guarantees a high quality of the series.

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SERIES FOREWORD

New perspectives on research in mathematics education – ERME series

ERME, the European Society for Research in Mathematics Education, is a growing society of about 900 researchers from all over Europe and beyond. Between 1998 and 2018, ten biannual Congresses of ERME (CERME), nine summer schools (YESS) and six ERME Topic Conferences (ETC) have taken place.

The ERME series documents the growing body of substantial research on mathematics education within the context of ERME. Volumes in the ERME series can be monographs or collections growing out of the collaboration of European researchers in mathematics education; for example, post-conference publications of selected contributions to ETCs or research in EU-funded projects.

The volumes are written by and for European researchers, but also by and for researchers from all over the world. An international advisory board guarantees that ERME stays well connected to the rest of the world and includes results of non-European research. A rigorous and constructive review procedure guarantees the high quality of the series.

The inaugural volume of the ERME Series is titled *Developing Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration in Europe*. We thank all involved editors, authors and reviewers for the joint work. It is a highly interesting start of the series which will – hopefully – fuel the international communication, cooperation and collaboration.

The series editors: Viviane Durand-Guerrier,
Konrad Krainer, Susanne Prediger, Naďa Vondrová

PREFACE

The aim of this book is to present, on the occasion of the 20th anniversary of the European Society for Research in Mathematics Education (ERME), the most important directions and trends of European research in mathematics education. The book reports on the main lines of development in ERME over the course of these 20 years, showing the ERME spirit in the process: a spirit of communication between different sub-areas and different countries, a spirit of cooperation between different theoretical approaches and research paradigms, a spirit of collaboration between established and developing researchers. Prior to the creation of ERME, several research traditions had already developed within Europe, each with its own identity reflecting cultural and educational particularities of its birthplace. With this book, we hope to establish shared understandings in which to ground future European research in mathematics education as well as to showcase the specific character of European research traditions for audiences inside and outside Europe.

At ERME conferences (CERMEs) participants spend most of the time in thematic working groups (TWGs). New working groups are added at each conference, typically two or three, following an open call for proposals and then selection by the International Program Committee for the conference. Equally, working groups are discontinued when they no longer attract enough interest.

Given how central the TWGs are to the scientific interaction that takes place within ERME, it was natural that the core of this book should reflect the work and achievements of these working groups. All groups that have been operational over sufficiently many CERMEs to warrant a substantial chapter have been included. The authors of each chapter have been selected from among those who have recently been active in leading the work of the relevant groups.

The chapters in the book have been ordered so as to begin with those focusing on specific content domains, age levels, and mathematical processes (such as proving and modelling). The following chapters then examine aspects of teaching and learning that range across contents and processes (including learning environments, affect and research about teachers). The final chapters deal with linguistic, cultural and social aspects of learning and teaching mathematics, as well as with the role of theories in mathematics education research. These 18 core chapters of the book are preceded by an introduction that provides an account of the evolution of ERME as a society, written by its former presidents.

The book concludes with two commentary chapters contributed by eminent researchers from outside Europe. Together these commentary chapters situate the research done in ERME and presented in the book in the wider context of mathematics education research worldwide. At the same time, they show how ERME might, in the future, inspire and be inspired by developments outside Europe.

Nowadays it is increasingly hard for researchers even to read all the relevant work that has been published on their topic, let alone to take comprehensive account of it in conducting and reporting their own research. There is a pressing need for greater coordination of research efforts; in particular, in the form of synthesis of frameworks and findings. Thus, this book seeks to identify cumulative achievements in research and to highlight starting points for further development within and beyond ERME. The book does not, therefore, aim to summarise 20 years of highly diverse research by hundreds of researchers but, rather, aims to display the patterns and threads that have the potential to unify these research efforts into a cumulative and coherent body of knowledge providing a view from the past into the future.

In this sense, the authors of the core chapters were asked to focus on the sustained development of ideas in the working groups over several years and ERME conferences. Also stressed were ideas that arose within a group and have become relevant for other groups, ideas that arose within ERME and initiated or influenced developments in mathematics education research beyond ERME, and ideas that originated in one geographical area, were brought into ERME and, as a consequence, were taken up more widely.

The book was initiated by the board of ERME in early 2015. When the board invited us to edit the book, we designed a process according to which initial chapter drafts would be available prior to CERME 10 (February 2017), enabling them to be discussed in the TWGs at the conference. Thus, the CERME spirit of communication and cooperation came to bear also in the process of writing the book. Chapters were then finalised, taking into account the feedback from discussions at CERME 10 as well as new developments reported at that conference. Chapter lengths were tailored to the size of the group (or, in a few cases, of groups that had recently split) and the number of years that the group had been working. We are looking forward to launching the book at CERME 11 (February 2019).

Given the nature of the book, it contains many references to CERME proceedings. In order to save space, a shortened format has been used for references to these proceedings in the chapters. The full references of all proceedings are listed below this preface.

In conclusion, some general patterns seem to appear across the different core chapters. In many groups, work in the earlier years of CERME was characterised by members' difficulties in understanding each other, due to the very diverse research traditions and theoretical frameworks in use within Europe. Through comparing and contrasting these, mutual understandings of the different lines of research have been reached. Even if most researchers kept on working within their respective traditions, they succeeded in finding commonalities, and understanding differences. Moreover, efforts of networking theories have been described, not only for purposes of comparing and contrasting but for coordinating or combining theoretical approaches. Indeed, in quite a number of cases where researchers have started to combine theoretical frameworks, this has led to a widening of perspectives from rather narrow research topics to more comprehensive foci that require more complex approaches.

It will, of course, be for the reader of this book to form an opinion as to what extent ERME has succeeded in showing the existence of a dynamic community which, over 20 years, has been able to create an identity for European research in mathematics education, structured around a common spirit, while remaining respectful of diversity and open to outside influences; to create original ways of collaborative research and to support the acculturation and integration of young researchers; and to contribute substantially to the advance of international research on a wide range of issues, while playing a pioneering role in some areas.

The editors: Tommy Dreyfus, Michèle Artigue,
Despina Potari, Susanne Prediger, Kenneth Ruthven

References of CERME proceedings

Note: The complete references of the CERME proceedings are listed here. All CERME proceedings are accessible via http://www.mathematik.uni-dortmund.de/~erme/index.php?slab=proceedings

CERME 1: Schwank, I. (Ed.) (1999). *European Research in Mathematics Education I: Proceedings of the First Conference of the European Society for Research in Mathematics Education* (CERME 1, August 1998). Osnabrück, Germany: Forschungsinstitut für Mathematikdidaktik and ERME.

CERME 2: Novotná, J. (Ed.) (2002). *European Research in Mathematics Education II: Proceedings of the Second Conference of the European Society for Research in Mathematics Education* (CERME 2, February 24–27, 2001). Mariánské Lázně, Czech Republic: Charles University, Faculty of Education and ERME.

CERME 3: Mariotti, M. A. (Ed.) (2004). *European Research in Mathematics Education III: Proceedings of the Third Conference of the European Society for Research in Mathematics Education* (CERME 3, February 28–March 3, 2003). Bellaria, Italy: University of Pisa and ERME.

CERME 4: Bosch, M. (Ed.) (2006). *European Research in Mathematics Education IV: Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (CERME 4, February 17–21, 2005). Sant Feliu de Guíxols, Spain: FUNDEMI IQS – Universitat Ramon Llull and ERME.

CERME 5: Pitta-Pantazi, D., & Philippou, C. (Eds.) (2007). *European Research in Mathematics Education V: Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (CERME 5, February 22–26, 2007). Larnaca, Cyprus: University of Cyprus and ERME.

CERME 6: Durand-Guerrier, V., Soury-Lavergne, S., & Arzarello, F. (Eds.) (2010). *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (CERME 6, January 28–February 1, 2009). Lyon, France: Institut National de Recherche Pédagogique and ERME.

CERME 7: Pytlak, M., Rowland, T., & Swoboda, E. (Eds.) (2011). *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (CERME 7, February 9–13, 2011). Rzeszów, Poland: University of Rzeszów and ERME.

CERME 8: Ubuz, B., Haser, C., & Mariotti, M. A. (Eds.) (2013). *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (CERME 8, February 6–10, 2013). Ankara, Turkey: Middle East Technical University and ERME.

CERME 9: Krainer, K., & Vondrová, N. (Eds.) (2015). *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (CERME 9, February 4–8, 2015). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

CERME 10: Dooley, T., & Gueudet, G. (Eds.) (2017). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME 10, February 1–5, 2017). Dublin, Ireland: DCU Institute of Education and ERME.

THE EUROPEAN SOCIETY FOR RESEARCH IN MATHEMATICS EDUCATION

Introduction by its former presidents

Ferdinando Arzarello, Paolo Boero, Viviane Durand-Guerrier and Barbara Jaworski

1 Introduction

During the weekend of 2–4 May 1997, 16 representatives from European countries met in Osnabrück, Germany, to establish a new society, the European Society for Research in Mathematics Education, in short ERME, to promote Communication, Cooperation and Collaboration (the three Cs) in mathematics education research in Europe. The three Cs came, over the years, to characterize the "ERME spirit".

The full foundation of this society took place at CERME 1, the first conference of the society, in August 1998, also in Osnabrück. The first president of ERME was Jean-Philippe Drouhard (France, 1997–2001). Jean-Philippe had accepted to join us in writing this chapter, but he passed away suddenly in November 2015. We miss him. Paolo Boero (Italy), who was an active member of the group of scholars who initiated the process of establishing the new society and was a member of the first Board, became the second president of ERME (2001–2005), and Barbara Jaworski (United Kingdom), also a founding member, was the third (2005–2009). They were followed by Ferdinando Arzarello (Italy, 2009–2013) and Viviane Durand-Guerrier (France, 2013–2017). From February 2017, the ERME president is Susanne Prediger (Germany). The society has been governed from the beginning by an elected board with representation criteria from the regions of Europe.

In what follows, we first present the motivations for *establishing the new society* 20 years ago and the steps through which the original aims were achieved. In Section 3, we present a brief summary of a significant reflection that took place on *quality and inclusion* in the CERME conferences. In the last section, we report on further developments aimed at the *ongoing strengthening of the society*.

2 Establishing the new society

After CERME 1, three main challenges had to be met by the ERME Board elected in Osnabrück: (1) to take initiatives to prepare a new generation of European researchers in mathematics education according to the ERME spirit; (2) to involve a sufficiently large number of European researchers in mathematics education, in order for CERME to become the representative forum of European research in the field; and (3) to ensure stability to the new-born society through a legal status anchored in the laws of one of the European countries. At the same time, the society and its initiatives had to be opened to countries beyond Europe, in order to promote worldwide scientific exchanges in the field.

From the very beginning, the society has been linked to the organisation of the CERME conferences with a wide spectrum of themes and orientations to profit from the rich diversity in European research, as will be demonstrated in the various chapters of this book. The conferences have been structured around a number of Thematic Working Groups (TWGs), each with a designated research focus. Participants have particularly appreciated the concrete possibility, offered by the TWGs, to develop their personal research through systematic, collaborative work with other researchers engaged in the same area, and to get constructive feedback from them on their papers (before, during and after the conference). Since CERME 10 there has been an open call for new TWGs: the current call can be found on the ERME website (www.mathematik.uni-dortmund.de/~erme/). The International Programme Committee (IPC) of each CERME is elected by the ERME Board, as is the IPC chair. The choice of TWGs and their leaders is discussed in the IPC and approved by the ERME Board. The decision to include a new group is taken according to the nature, focus and relevance of the research, its potential to attract participants, and its distinction from existing groups. The choice of group leaders and co-leaders is an important lever for ensuring the quality of work in the group, for planning activities to include all those who wish to participate and for opening the society to the diversity of research in Europe.

From CERME 1, the tasks of drawing up a constitution and establishing a legal status were undertaken by the ERME Board mainly thanks to the extraordinary work performed by Board member Graham Littler (UK). On behalf of ERME, he approached the UK Charity Commission to request charitable status in the UK, and dealt with the very complicated paperwork involved. In CERME 3 a draft was presented and voted on, as a basis for the formal establishment of the society. This was proposed in the General Meeting at CERME 4 and ratified in CERME 5.

An important issue of ERME policy consists of supporting and educating new/young researchers in mathematical education. The Board elected at CERME 2 in 2001 gave the task of designing a Summer School to Paolo Boero, Barbara Jaworski and Konrad Krainer. It was to be held every two years in alternate years from the CERMEs. The main target population consisted of PhD students in mathematics education. By analogy with CERME, the Summer School was conceived as a working place for students. TWGs (led by "expert" researchers – about 60% of the whole time) offered students a unique opportunity to present the current status of their research (be it initial, or near to the conclusion), to receive constructive feedback from the "expert" and from the other participants and to establish links with other young researchers interested in the same subject.

The first Summer School, held in Klagenfurt in 2002, showed that the design of the school was suitable to meet the students' expectations. Gradually the number of applicants rose from 40 in Klagenfurt to more than 100 (resulting in 72 participants) for the last four schools, including students from other continents. In parallel with the design and the implementation of the school, the Board helped to set up an informal branch of the society, YERME (Young European Researchers in Mathematics Education), to be involved in the preparation of the school and in other initiatives of interest to young researchers. The decision was formally taken in the General Meeting in CERME 8 when the first two representatives of young researchers in the ERME Board were elected. From the institution of YERME, the summer schools took on the abbreviation of YESS – YERME Summer School. YESS 9 is being prepared as this book goes to press.

Another important issue, from the beginning of ERME, CERME and YESS, was the encouragement of mathematics education researchers from Eastern Europe to join in ERME activities. For this purpose, ERME designated funds to contribute to travel and accommodation where financial hardship was demonstrated. When, in 2009, very tragically, Graham Littler died, the ERME Board decided to name this fund the Graham Littler Fund. Since then, this fund has been topped up regularly and used to provide financial support for participants to CERME and YERME where a need has been identified.

During the years 2001–2005, when it was important to ensure the representativeness of the new society, the Board worked hard to establish relationships with several research groups and existing national societies in the field of mathematics education; also researchers from other continents were invited to join ERME initiatives. As a result, the number of participants in CERME doubled at each conference until CERMEs 5, 6 and 7, when it stabilized at about 450–500 participants. It increased anew up to about 700 participants in CERMEs 9 and 10.

3 Scientific quality and inclusion in CERME conferences

From the very beginning, the issues of quality and inclusion in CERME conferences were main concerns of the society. Quality refers to scientific standards relating to papers presented and published, and to activity in the TWGs. Inclusion refers to ways in which participants are included in activities in the groups, and in presentations and published papers. CERME's policy of encouraging presentation (after two rounds of revision) of as many papers as possible was sometimes seen to act against high scientific standards. Seeking a balance between quality and inclusion was seriously problematic.

In CERME 6, held in Lyon, France, members of the ERME Board collected data in several ways from delegates at the conference concerning issues related to quality and inclusion in CERME conferences. In response, one group leader wrote: "Being all-inclusive and academically qualitative are *a priori* incompatible." While this is an individual view, it nevertheless flags a tension between inclusivity and scientific quality. Participants acknowledged that newcomers are drawn quickly into the activity of the group. There were almost no comments that suggested that group work was not friendly and welcoming, that participants were not (overtly) encouraged to take part and join in the discussions. In scientific terms, we can see inclusion to be facilitated through the review process in which a critical review can be helpful and supportive and enable the improvement of a paper. However, to quote Gates and Jorgensen (2009, p. 164), we were aware that "the field . . . in which the participants engage recognizes and conveys power to those whose habitus is represented and privileged in the field". Thus, and according to Atweh, Boero, Jurdak, Nebres and Valero (2008, p. 445), "collaboration between educators with varying backgrounds, interests and resources may lead to domination of the voice of the more able and marginalization of the less powerful".

Such considerations led Jaworski, da Ponte and Mariotti (2011) to start characterising inclusion and quality and to relate the characterisation to the specific aims of ERME in terms of Communication, Cooperation and Collaboration, in contributing to the ERME spirit. Inclusion is characterised in affective and scientific terms. Quality is characterised scientifically through "key ideas" and their development. The key ideas need to be there for scientific quality to exist at all; they need to be engaged with for scientific quality to be recognisable in the activity of a group. We present below how it works in CERME.

Starting communication: Participants have read the accepted papers; they come together with friendliness and the sincere desire to work inclusively. There are key ideas in the accepted papers to which the review process draws attention. Activity and discussion begin to encourage communication related to the ideas where the objective is to know each other's ideas and relate them to each other.

Developing cooperation in engaging with debate: Group organisation enables a focus on the key ideas. Friendly and considerate interaction, with attention to language enables participants to start to engage with the ideas. The emphasis is on including all people attending a TWG, including those who do not present accepted papers, in order to get reactions, questions and comments. This group activity may contribute, on the one hand, to improving the accepted papers (in terms of their revision for publication in the final proceedings), and on the other hand, to improving the quality of all participants' research work.

Developing cooperation in recognising ideas: Group leaders create activity to encourage a focus on getting participants engaged with the key ideas which are recognised. The emphasis is on reaching a quality of interaction relating to scientific ideas.

Enabling collaboration: Here we see an enabling of critical inquiry into the essences of the ideas; deep engagement of a scientific quality with deep probing of ideas and corresponding critical debate. From here, collaboration can begin.

It seems clear that, for the last point to be possible, both the second and third have to be achieved. This means dealing with the organisational challenges raised in the CERME 6 survey, which are far from trivial. However, it could be that a theoretical perspective of this sort, of what is involved in achieving inclusion and quality in group work in CERME, can act as a basis for thinking about dealing with the challenges and conceptualising in practical terms what we are aiming for in CERME.

4 Carrying on strengthening the society

From 2009, the main activities of ERME, in particular CERME and YESS, continued and expanded further. We stress below some of the main ERME activities contributing to the development and visibility of the society.

During this period, the number of participants from developing countries was increasing, due to the growing effort of ERME to support people from those countries through the Graham Littler Fund. Two decisions, taken during the General Meeting in CERME 7, contributed to expanding ERME beyond the strict boundaries of Europe. The first concerned countries from North Africa. While these countries were living the exciting period of the so-called "Arab Spring", some researchers were attending CERME 7 and claimed that the possibility for them to take part in events like CERME was important also for the progress of democracy in their own countries. Thus it was decided that researchers from these countries could apply for financial support from the Graham Littler Fund. The second was the decision of the ERME Board to organize the first CERME outside the European Union, namely CERME 8 in Turkey.

Since 2011 ERME has become an affiliated society of ICMI (International Commission on Mathematical Instruction). As such, ERME has the opportunity to present a quadrennial report to the General Assembly of ICMI, so that ERME activities can be known by the whole world community working in mathematics education. More generally, an important concern was to encourage the European research community to develop relationships with mathematicians and to take part in the ICMI activities. ICME-13 in Europe in July 2016 was a wonderful opportunity to improve the international visibility of ERME. As an ICMI affiliate organisation, ERME organised two sessions, one a general presentation of ERME with a focus on teacher education research, the other on YERME.

A further action of ERME in recent years concerned the issue of rating of publication outlets. As is well known, it is very difficult to escape the influence of the ranking and grading of scientific journals for the development of researchers' careers. However, the current systems are often based on crude bibliometric analyses that have little to do with scientific quality. This represents a disadvantage for researchers in the field of mathematics education. For this reason, ERME, together with the Education Committee of the European Mathematical Society (EMS), and supported by the ICMI, decided in 2011 to appoint a joint commission, with the aim to propose a grading of research journals in mathematics education based on expert judgment. The result of their careful and joint analysis, based on consultation with 91 experts from 42 countries, was a document rating the 17 most important journals in mathematics education on a scale with four grades (Törner & Arzarello, 2012). We understand that this document has been used by some official boards charged with assessing the work and the consequent promotion of mathematics educators.

Another very important issue is the publication of the proceedings for each CERME conference. All the proceedings are available online on the ERME website and since CERME 9, thanks to Nad'a Vondrovà and Konrad Krainer, CERME Proceedings are also available on the open Archiv HAL, managed by the French CNRS (Centre national de la recherche scientifique). A detailed list appears at the end of the Preface of this book.

A further decision concerned the ERME Topic Conferences (ETC), which are conferences organised on a specific research theme or themes related to the work of ERME as presented in associated working groups at CERME conferences. Their aim is to extend the work of the group or groups in specific directions with clear value to the mathematics education research community. These initiatives take place during the year in which CERME does not take place. After careful preparatory work, in the General Meeting in Prague (2015), the rules for ETCs were approved. Three ETCs were held in 2016, and three were approved in 2017 for appearance in 2018.

5 Conclusion

A main objective of this Introduction is to present the evolution of the spirit and the activities of ERME to the reader of this book as background for the core chapters of the book (Chapters 1–18), which are based on the work of the TWGs at and between the conferences of the society. Another objective is that present and future members of the society, who have not taken part in its evolution and its activities, should know how choices have been made, and then re-thought and deepened (according to accumulated experience). In doing so, we wish to stress that the future of the society is in the hands of the members and not in the paragraphs of the Constitution.

In addition, we wish to illustrate how ideas about the society and its activities have progressively matured and how emerging needs have been taken into account. We hope that ERME's "historical" evolution will give a concrete idea of the openness of the society and of the deep reasons why CERME, YESS and the other ERME initiatives are organized in the present way.

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References

Atweh, B., Boero, P., Jurdak, M., Nebres, B., & Valero, P. (2008). International cooperation in mathematics education: promises and challenges. In M. Niss & E. Emborg (Eds.), *Proceedings of the 10th International Congress on Mathematical Education (ICME 10), July 4–11, 2004, Copenhagen, Denmark* (pp. 443–447). Roskilde, Denmark: Roskilde University.

Gates, P., & Jorgensen, R. (2009). Foregrounding social justice in mathematics teacher education. *Journal of Mathematics Teacher Education*, *12*, 161–170.

Jaworski, B., da Ponte, J. P., & Mariotti, M. A. (2011). The CERME spirit: issues of quality and inclusion in an innovative conference style. In B. Atweh, M. Graven, W. Secada & P. Valero (Eds.), *Mapping Equity and Quality in Mathematics Education* (pp. 457–478). London: Springer.

Törner, G., & Arzarello, F. (December 2012). Grading mathematics education research journals. *EMS Newsletter*, *86*, 52–54.

References

The European Society for Research in Mathematics Education: introduction by its former presidents

Atweh, B., Boero, P., Jurdak, M., Nebres, B., & Valero, P. (2008). International cooperation in mathematics education: promises and challenges. In M. Niss & E. Emborg (Eds.), Proceedings of the 10th International Congress on Mathematical Education (ICME 10), July 4–11, 2004, Copenhagen, Denmark (pp. 443–447). Roskilde, Denmark: Roskilde University.

Gates, P., & Jorgensen, R. (2009). Foregrounding social justice in mathematics teacher education. Journal of Mathematics Teacher Education, 12, 161–170.

Jaworski, B., da Ponte, J. P., & Mariotti, M. A. (2011). The CERME spirit: issues of quality and inclusion in an innovative conference style. In B. Atweh, M. Graven, W. Secada & P. Valero (Eds.), Mapping Equity and Quality in Mathematics Education (pp. 457–478). London: Springer.

Törner, G., & Arzarello, F. (December 2012). Grading mathematics education research journals. EMS Newsletter, 86, 52–54.

1 Chapter 1: From geometrical thinking to geometrical working competencies

Space and shape

Transformations

Methodology

of research

betters models Bridging practical + theoretical approaches Need of theory, paradigm and model

FIGURE 1.1 Main themes or issues of the group

allow the connection between theoretical and empirical aspects. Beside refer

ences to general psychological (Piaget's or Vygotski's works) or didactical theories

(Brousseau's or Chevallard's theories), specific theoretical and methodological

developments were provided during the meetings. The van Hiele levels, Fischbein's

figural concept and Duval's registers were among the most relevant cognitive and

semiotic approaches that were adopted. Regarding the epistemological and didacti

cal approaches, a decisive importance was granted to the geometrical paradigms

and the Geometrical Working Spaces, and more recently to a lesser extent, on the

notion of geometrical competencies. A great number of researches focused on differ

ent aspects of geometrical understanding, such as figural apprehension, visualisation

and the effort of conceptualisation in geometry using different methodologies and

frameworks. Among the numerous papers presented in the

Working Group on

Geometry, we can distinguish some recurrent topics that we present below in paral

lel with the history of the group.

The first working group specifically dedicated to geometry was created in

CERME 3. Since this meeting various names were given to this group including

some nuances, from thinking of researching through teaching and learning, ending at

the last edition (CERME 10) with the sole name Geometry. On Figure 1.1, which

is drawn from the synthesis of CERME 7, themes and issues generated by the con

tributions are organised in a conceptual tree.

On Figure 1.2, some unifying characteristics related to geometric competencies

that have been developed during CERME 8 are presented.

Further on, we will illustrate the evolution of the group with a short overview

of the papers presented during these conferences, intending to show the richness

of the themes developed throughout these meetings.

2.1 Before the official creation of the group

In the first two sessions, even if the group did not officially exist, some communica

tions were already centred on geometry with a diversity of viewpoints. Some reflexive

and the reform of mathematics examination at a time when social and professional

mobility was a major issue. Other contributions insisted on the need of metaphors and images (Parzysz, 2002) or focused on epistemological and cultural approaches

about geometrical knowledge (e.g. Arzarello, Dorier, Hefendehl-Hebeke, & Turnau,

1999; Burton, 1999). Mostly descriptive, the studies addressed the geometric content and often men

tioned some general mathematical skills or cognitive and instrumental dimensions

in geometric activity. They also indicated some “frictions” with other mathemati

cal domains like algebra, and some typical interactions in the use of technologies

with digital geometric software (Dreyfus, Hillel & Sierpinska, 1999). In addition, the natural relationship of geometry to proof and proving was regu

larly examined, both in theoretical and philosophical essays and in reports of studies

involving empirical research. They link proving, arguing, modelling and discover

ing processes. They also deal with the students’ way for proving and their use of

a logical discourse, figural signs and technology when they are faced with proof

exercises in geometry. Attention was also paid to the links between the discovery

process, visualisation and instrumentation processes with software (Jones, Lagrange

& Lemut, 2002). There were also studies concerned with the teacher education and professional

development in geometry and the organisation of courses (Houdement & Kuzniak,

2002). Despite the fact that several works deal with mathematical proofs, visualisation and dynamic geometry, surprisingly, other natural links with geometry are virtually

missing during the first congresses. In fact, few researches focus on modelling of

physical phenomena using geometrical tools. Visual • observation • exploration Operational • instrumentation • instrumentalization Reasoning • argumentation • deductive Figural • to model • conjecture • define

FIGURE 1.2 Geometrical competencies

2.2 Thematic from CERME 3 to CERME 5

During these conferences, the new geometry group starts the study of geometrical

thinking using and developing the key concepts of paradigms, developmental stages

and generalisation in space (Houdement & Kuzniak, 2004; Gueudet-Chartier,

2004; Houdement, 2007). At the core of many researches, visualisation is often

directly connected to Duval's registers of representation (Perrin-Glorian, 2004;

Kurina, 2004; Pittalis, Mousoulides & Christou, 2007). In an original study,

the visualisation issue is raised for sighted and blind students (Kohanová, 2007).

Other papers focus on the notion of instrumentation, both with classical drawing

tools (Vighi, 2006; Bulf, 2007; Kospentaris & Spyrou, 2007) or digital tools (Rolet,

2004). Education for future teachers (Kuzniak & Rauscher, 2006) and reasoning

(Ding, Fujita & Jones, 2006; Markopoulos & Potari, 2006) appear as topics of

continuity, while concepts and conceptions remain key themes in many studies (e.g. Modestou, Elia, Gagatsis & Spanoudes, 2007; Marchini & Rinaldi, 2006).

2.3 Thematic from CERME 6 to CERME 7

The theoretical and methodological dimensions of research in geometry remain

prominent topics, especially in further development of the notions of geometrical

work and Geometrical Working Spaces. Many points were considered as a com

mon background, as they were developed during former sessions. These points

were related to the use of geometrical figures and diagrams (Deliyianni et al., 2011)

and to the understanding and use of concepts and proof in geometry (Gagatsis,

Michael, Deliyianni, Monoyiou & Kuzniak, 2011; Fujita, Jones, Kunimune,

Kumakura & Matsumoto, 2011). For an epistemological and didactical approach,

researchers used the geometrical paradigms and geometrical work spaces. Attention

is also paid to 3D geometry forms of representation through the possible use of

digital tools (Mithalal, 2010; Hattermann, 2010; Steinwandel & Ludwig, 2011).

In addition to the usual geometrical topics, special attention is paid to general

or cross-cutting aspects, such as educational goals and curriculum in geometry

(Girnat, 2011; Kuzniak, 2011), communication and language (Bulf, Mathé &

Mithalal, 2011), the teaching, the thinking and the learning processes in geom etry. Moreover, Mackrell (2011) questions the interrelations between numbers, algebra and geometry, especially in digital environments.

2.4 Restructuring in CERME 8 to CERME 10

More recently, the working group sought to revisit and extend the issue of geometrical thinking and geometrical work by reformulating it in terms of four geometrical competencies (reasoning, figural, operational and visual) organised in a tetrahedron (see Figure 1.2). Each geometric competence constitutes a pole of geometrical thought and it is the study of the link between these competences that makes it possible to better understand the global functioning of geometrical thought (Maschietto, Mithalal, Richard & Swoboda, 2013). Further on, at CERME 9, there were more specific contributions about the way geometry is, or should be, taught and the four competencies were used as a general way of describing the geometrical activity and for creating links between different points of view (theoretical and empirical). In this group the discussions were related to geometry teaching and learning (Douaire & Emprin, 2015; Kuzniak & Nechache, 2015) and issues such as teaching practices and task design (Mithalal, 2015; Pytlak, 2015). Furthermore, cultural and educational contexts modifying the geometry curricula were also discussed, introducing a new issue about the role of language

and social interactions in the teaching and learning of geometry. In CERME 10,

the four competencies were used to describe geometrical thinking: reasoning,

figural, operational and figural. The group took these dimensions as a background

that was very helpful to understand each other and to compare our approaches to

the issue of what is at stake in the teaching and learning of geometry. Three main issues were addressed during the working group: The role of

material activity in the construction of mathematical concepts, including using

instruments, manipulation, investigation, modelling . . .; Visualisation and spa

tial skills; Language, proof and argumentation. In comparison to the previous

CERME, this time psychological points of view, among others, were represented.

This raised new questions, often with very different theoretical and methodologi

cal backgrounds sometimes far from mathematics.

3 Studying the teaching and learning of geometry

through a common lens

Since the creation of the group, the development of shared theoretical frameworks

was central to ground collaboration between participants. This point is particularly

evident in CERME 3, where the main theoretical concerns of the group were

summarised by Dorier, Gutiérrez and Strässer (2004). Their classification is used to highlight some theoretical approaches which have been reinforced by their presentation and discussion during CERME conferences.

3.1 Geometrical paradigms

The history of geometry shows two contradictory trends. First, geometry is used as a tool to deal with situations in real life but, on the other hand, geometry for more than two thousand years was considered the prototype of logical, mathematical thinking and writing after the publication of Euclid’s “Elements”. These contradictory perspectives are taken into account in geometry education by Houdement and Kuzniak (2004) with geometrical paradigms. Three paradigms are distinguished. “Geometry I: natural geometry” is intimately related to reality; experiments and deduction grounded on material objects. “Geometry II: natural/axiomatic geometry” is based on hypothetical deductive laws as the source of validation with a set of axioms as close as possible to intuition and may be incomplete. “Geometry III: formal axiomatic geometry” is formal, with axioms that are no more based on the sensory reality.

These various paradigms are not organised in a hierarchy making one preferable to another, but their work horizons are different and the choice of a path toward the solution is determined by the purpose of the problem and the researcher’s viewpoint.

Useful to provide a method to classify geometrical thinking, geometrical paradigms have also been helpful to interpret tasks eventually given to students and future teachers and can be used to classify the students' productions, offering an orientation for the teacher of geometry. In contrast to van Hiele's theory, this approach is not dependent on the general thinking and reasoning development, but relates more to an epistemological viewpoint.

3.2 Development stages

The so-called "van Hiele levels" of geometrical reasoning are among the most used theoretical frameworks for organising the teaching and learning of geometry according to development stages. The description of the "van Hiele levels" already gives hints to fundamental links between these levels and the model suggested by Houdement and Kuzniak (2004). There was some discussion whether geometrical knowledge progresses through sequences of stages. Some papers presented (Braconne-Michoux, 2011) appear as contradictory to the traditional "van Hiele levels". This is especially true if the levels are linked to clearly identified and fixed ages or if individual persons are thought to necessarily follow the order of the respective levels. Positioning itself on ideas of stages of development and/or learning is obviously important for constructing and discussing geometric tasks or

research projects. The stage levels can help to find and further develop appropriate

tasks, and they are obviously helpful for exploratory activities.

3.3 Registers of representation

Semiotic consideration has always been important in geometry and the distinction

of figure, in the sense of the most general object of geometry (either 2D or 3D) ver

sus its material representation, is classic in the field. Three main groups of semiotic

representation can be distinguished: material representation (in paper, cardboard,

wood, plaster, etc.), a drawing (made either with paper and pencil, or on a com

puter screen), and a discursive representation (a description with words using a

mixture of natural and formal languages). Each register bears its own internal func

tioning, with rules more or less explicit. Moreover, students have to move from

one register to another, sometimes explicitly, sometimes implicitly, sometimes

back and forth. Questions about registers of semiotic representation and cogni

tive processes have been studied in depth by Duval (1993). He defines “semiotic

representations as productions made by use of signs belonging to a system of rep

resentation which has its own constraints of meaning and functioning”. Semiotic representations are necessary to mathematical activity, because its objects cannot be directly perceived and must, therefore, be represented.

3.4 Instrumentation with artefacts and digital tools

There is a broad consensus in the community of mathematics teachers and educators that learning geometry is much more effective if concepts, properties, relationships, etc. are presented to students materialised by means of instruments modelling their characteristics and properties. Furthermore, the use of didactic instruments is very convenient, if not necessary, in primary and lower secondary grades. There is a huge pile of literature reporting the continuous efforts devoted by mathematics educators since long ago to explore the teaching and learning of geometry with the help of manipulative, computers, and other tools. Unfortunately, many researchers interested in this topic have generally preferred to participate in the CERME group on technology and thus have deprived, in some part, the group of discussions on this subject.

3.5 On Geometrical Working Spaces

In relation to semiotic, instrumental and discursive proof, the idea of global thinking on geometric work was first introduced in CERME 4 (Kuzniak & Rauscher, 2006) and was based on the model of Geometrical Working Spaces (GWS) pre sented during a plenary lecture at CERME 8 (Kuzniak, 2013). The basic idea

behind this approach is that some real geometrical work appears only when a

student's activity is both coherent and complex to develop a reasoning using intui

tion, experimentation and deduction. The GWS is conceived as a dynamic abstract

place that is organised to foster the work of people solving geometry problems.

The model articulates two main concerns, the one of epistemological nature, in a

close relationship with mathematical content of the studied area, and the other of

cognitive nature, related to the thinking of the person solving mathematical tasks.

This complex organisation is generally summarised in the diagrams in Figure 1.3

(for details, see Kuzniak, Tanguay & Elia, 2016). Visualisation Representamen Referential Epistemological plane Cognitive plane i n s t r u m e n t a l g e n e s i s Construction Proving [Sem-Ins] [Sem-Dis] [Ins-Dis] Artefacts R V P T A C s e m i o t i c g e n e s i s d i s c u r s i v e g e n e s i s

FIGURE 1.3 The Geometrical Working Space diagrams

For describing the work in its epistemological dimension, there are three com

ponents in interaction, organised according to purely mathematical criteria: a set

of concrete and tangible objects, the term sign or representamen is used to sum

marise this component; a set of artefacts such as drawing instruments or software;

and a theoretical system of reference based on definitions, properties and theorems.

In close relation to the components of the epistemological level, three cogni

tive components are introduced as follows: visualisation related to deciphering and

interpreting signs; construction depending on the used artefacts and the associated

techniques; and proving conveyed through processes producing validations, and

based on the theoretical frame of reference.

The process of bridging the epistemological plane and the cognitive plane can

be identified through the lens of GWSs as genesis related to a specific dimension

in the model: semiotic, instrumental and discursive geneses. In order to understand

this complex process of interrelationships, the three vertical planes of the diagram

are useful and can be identified by the genesis which they implement [Sem-Ins],

[Ins-Dis] and [Sem-Dis] (Figure 1.3).

To the coordination of the geneses in the GWS model, there are three inter

nal fibrations that focus on the role of tools, controls and representations in the

mathematical work which allow us to describe the complexity of managing

the relationships between all the components involved in geometrical activity.

Richard, Oller and Meavilla (2016) have introduced three internal fibrations that

focus on the role of tools, controls and representations in the mathematical work.

This model was briefly discussed in CERME 10.

3.6 Within and beyond the model of GWS

The GWS model results largely from the work done in the group of geometry

where its use led to understanding that the model can easily be adapted to relate

with other theoretical approaches. This means that, depending on the issues and

problems dealt with, it can be complementarily associated with other theories:

(a) to increase its contextual perspective, such as the Mathematics Teachers'

Specialised Knowledge (MTSK), the Action, Process, Object and Schema (APOS)

theory, the Activity Theory (AT) or the Theory of Didactical Situations in

Mathematics (TDSM); (b) to refine it according to the needs of the studies, such as

Peirce's semiotics (1931), Rabardel's cognitive ergonomics (1995) or Richard and

Sierpinska's (2004) functional-structural approach.

Using the GWS model makes it possible to question in a didactic and scientific

way – mostly non-ideological – the teaching and learning of geometry, straddling

the mathematical education and the didactics of the field. For example, we know

that curricula change frequently or that the meanings of the words or concepts of

reference vary from one region to another, which is a problem from a research

point of view.

4 Building and developing a spirit of communication and collaboration

As previously underlined, a strong point of the group is participants' emphasis on relating theoretical and empirical aspects of research in geometry education. The continuity of this trend was evident through the meetings of the group so far, as the participants were mainly discussing results of empirical or developmental research studies and theoretical reports about the teaching and learning of geometry. The need for a common framework related to geometry education appeared necessary in the working group in order to stimulate the discussions among members and to allow the capitalisation of knowledge in the domain. Due to collaborations initiated during CERME meetings with colleagues from France, Cyprus, Spain, Canada, Mexico and Chile, it has been made possible to develop a joint theoretical framework. The framework should be dedicated to study the teaching and learning of geometry, space and shape on the whole educational system and should be neutral in the sense that it can be used to facilitate exchanges in different countries and institutions. According to this need, the framework of geometrical paradigms, as explained above, was introduced in the CERME 3 conference. In the pursuit of results at the CERME 3, the geometry working group of

CERME 4 and 5 continued by looking into geometrical paradigms. In particular, in CERME 4, Kuzniak and Rauscher (2006) analysed pre-service

schoolteachers’ geometrical approaches, based on the notion of geometrical para

digms and levels of argumentation. They found that students’ levels of understanding

and memorisation of the bases of the elementary geometry differ greatly and that

they keep the practical use of geometry. Although their study was conducted with a

particular population, their results can be useful for evaluating the long-term effects

of education in geometry. Moving forward, in CERME 5, during discussions about the possible uses of

geometrical paradigms, new participants of the group initiated a discussion about

the real benefit of this approach. Perspectives in these directions were given by

Houdement’s (2007) and Bulf’s (2007) papers. In fact, Houdement highlighted the

uses of this approach for comparing curricula in different countries and for reflect

ing on the necessity to teach Geometry II and the proper way to introduce it. In

Bulf’s effort to examine the link between geometrical knowledge and the reality in

relation to the concept of symmetry, this approach was useful for tracing a double

play between the Geometry I and Geometry II on one side and Reality/Theory on

the other side. Furthermore, Kospentaris and Spyrou (2007) used the approach of

paradigms in teachers’ education, as done in CERME 4 by

(2006). Their results were in line with previous results presented at CERMEs 3 and 4 about pre-service teachers' geometrical thinking. They actually found that visual strategies or measurement using tools are used by students at the end of secondary school, interpreting it not by a developmental approach, but based on the geometrical paradigms. The discussion of the aforementioned papers gave a future perspective for the group in order to make precise the sufficiency of the so far existing theoretical tools for determining the nature and the construction of the GWS used by students and teachers. In fact, no paper was traced in the following CERMEs that challenged these new theoretical tools.

Following up the spirit of the previous years, the participants in CERME 6, who came from both Europe and America, have extended and enriched the results obtained so far. Until then a common background was built and known by expe rienced participants, thus the participants worked within the continuity of the former sessions of CERME and their discussions were effectively facilitated by this common culture. In fact, the participants came to two main conclusions regarding the use of theory in research: (1) theory can serve as a starting point for initiating a research study and (2) theory can act as a lens to look into the data. An example

of such research is that of Kuzniak and Vivier (2010) who examined the Greek

Geometrical Work at secondary level from the French viewpoint, using a theo

retical frame based on paradigms and GWSs. Also, Panaoura and Gagatsis (2010)

compared the geometrical reasoning of primary and secondary school students,

based on the way students confronted and solved specific geometrical tasks, find

ing difficulties and phenomena related to the transition from Natural Geometry

to Natural Axiomatic Geometry. Therefore, a perspective for future research on

geometry theories and their articulation for the group was the use of geometrical

paradigms as a tool for analysing existing curricula and students’ behaviour.

The creation of a common spirit of communication has also built ideas of collab

oration between participants through the discussions of the group. This was evident

regarding the focus of research in specific educational levels. Actually, at the first

discussions of the working group the attention was given to primary education, as

many of the papers were about young students’ geometrical concepts (Marchetti,

Medici, Vighi & Zaccomer, 2006; Marchini & Rinaldi, 2006) and the role of spe

cific tools for the teaching and learning of geometry at that age level (Vighi, 2006).

However, in subsequent meetings of the group (in CERME 5), collaborations

were envisaged about the transition from a lower to a higher educational level and

also the adaptation of a common framework to work out such kinds of studies as

paradigms, GWS, spatial abilities and conceptions about the figure.

This was succeeded in the next meeting of the group in CERME 6 as, among

the research presented in the group, the dimension of the students’ transition from

primary to secondary school was also taken into account. For example, Deliyianni,

Elia, Gagatsis, Monoyiou, and Panaoura (2010) investigated the role of various

aspects of figural apprehension in geometrical reasoning in relation to the students’

transition from primary to secondary education, revealing differences between the

two groups of students’ performance and strategies in solving geometrical tasks.

In a similar sense, Panaoura and Gagatsis (2010) compared primary and secondary

school students’ solutions of geometrical tasks and stressed the need for helping

students progressively move from the geometry of observation to the geometry of

deduction as they transit to a higher educational level.

Finally, in an effort to build a spirit of communication and collaboration

in the group, collaborations between experienced and new researchers were accomplished (e.g. the work of Deliyianni et al. (2011) and Gagatsis et al. (2011)). These common works facilitated not only the communication between old and new research, but also the collaboration between researchers from different countries (e.g. France and Cyprus).

5 Toward a common research agenda?

Even if exchange within the group is still very rich and exciting, the geometry group seems to have gradually forsaken some of its initial ambitions, because of the existence of various groups specifically interested in technology use, proof, teacher education, semiotic aspects, etc. It has been partially disembodied and deprived of what has always been the strength of geometry: its transdisciplinary contribution to human thinking. Another challenge faced by the group comes from its difficulty to capitalise its results and findings because of two kinds of volatility. The first is natural and comes from the renewal of participants who are younger and younger, and sometimes beginners in the field. Experienced researchers were attracted by other groups, developing topics closer to their own researches. On the other hand, this movement is also positive in the sense that new participants might give new ideas and perspectives to these groups. Another reason for the difficulty to capitalise in the field is the constant curriculum changes in geometry. This can be illustrated by the erratic presence of geometric transformations. Moreover, and fundamentally, geometrical activity seems more and more oriented to other mathematical fields through modelling activity based on geometrical support, such as physics, geography, etc. To overcome some of these problems, we suggest that the viewpoint on geometrical work could help to shape a common research agenda aiming at understanding better the competencies involved in geometrical work through and beyond the whole education. It requires coordination between cognitive, epistemological and sensible approaches, structured around three complementary dimensions which relate to visual, experimental and reasoning competencies.

5.1 On semiotic work and visual competency

Geometry is traditionally viewed as a work on geometrical configurations which are both tangible signs and abstract mathematical objects. This difference is clearly identified under the classical opposition drawing versus figure, which focuses on strong interactions and differences between semiotic and discursive dimensions.

The semiotic dimension, especially worked through the visualisation process, is at the centre of Duval's research, which developed very powerful tools, such as registers of semiotic representation, to explore the question. In his view, a real understanding of mathematical objects requires interplay between different reg

isters which are the sole tangible and visible representations of the mathematical

objects. Within a multi-modal approach on semiotic resources, it is possible to

study the entanglement of mathematical objects and their various semiotic repre

sentations. One of the main issues will be to see how these links are formed and

reformed and can hide the very nature of geometrical objects to students.

5.2 On instrumental work and experimental competency

Geometry could not exist without instrumented activities or drawing tools, and the

study of their different uses allows identification of two types of geometry which

are well described by Geometry I and Geometry II paradigms. Some construc

tions are possible or impossible depending on tools or milieus. Thus, the trisection

of the angle is impossible with ruler and compass, but quite possible with origami

or mechanical instruments. The same is true for duplicating the cube. In general,

it is possible to understand how, and justify why, constructions with drawing tools

are effective, by questioning the structure of the mathematical objects, especially

the nature of numbers, involved in these constructions. From precise draw

ings based on mathematically wrong constructions (like the Dürer pentagon) to

mathe matically correct but imprecise, in the sense of their measure, constructions

(like Euclid’s pentagon), it is possible to see all the epistemic conflicts opposing

constructions based on an approximation to constructions based on purely deduc

tive arguments. The tension between precise and correct constructions has been

renewed with the appearance of dynamic geometry software (DGS). As Strässer

(2002) suggested, we need to think more about the nature of geometry embedded

in tools, and reconsider the traditional opposition between practical and theoretical

aspects of geometry.

5.3 On the work around proof and reasoning competency

Since antiquity, demonstration work in its demonstrative form has always been

emphasised in geometry, considered as a kind of ideal of rational thought grasped

in its most intuitive and visible form. But in education, this idealised form of

advanced mathematical work is not so obvious to discover and to implement,

because it is hidden behind the play between practical and formal geometric para

digms. Moreover, thanks to its graphical precision, the software finally convinces

users of the validity of the results. Proofs are no longer only formal and the modes

of argumentation are enriched by experiments that give new meaning to the clas

sical epistemological distinctions between the iconic and non-iconic reasoning and relate to discursive-graphic reasoning (Richard, 2004). Increasingly, human reasoning is supported on representation and processing delegated to a machine (Richard et al., 2016). It remains to understand how to animate the figures (instrumental aspect) by coordinating better the semiotic aspects associated with figures and the discourse of proof in natural language.

5.4 On modelling competency in geometrical work

Being as old as the first forms represented on walls of prehistoric caves, the very constitution of the geometric model is certainly an incarnation of what is the modelling of space and forms. Unfortunately, modelling is not widely practised in compulsory education, and problem solving in geometry classes too often relies on classical and not open tasks. In several European curricula, emphasis is now placed on the notion of measurement in order to understand certain geometric properties. The passage from synthetic geometry to arithmetic and algebra is thus favoured. However, the geometric model remains necessary to develop the visual discovery of new properties, but also of proofs without recourse to measurement and algebra. Geometry is always one of the ingredients of the discovery of the beauty of mathematics and is also deeply linked to geography, physics or the arts and thus to the understanding

of the world.

Arzarello, F., Dorier, J.-L., Hefendehl-Hebeker, L., & Turnau, S. (1999). Mathematics as a cultural product. CERME1 (pp. 70–77).

Braconne-Michoux, A. (2011). Relations between geometrical paradigms and van Hiele levels. CERME7 (pp. 618–627).

Bulf, C. (2007). The use of everyday objects and situations in mathematics teaching: The symmetry case in French geometry teaching. CERME5 (pp. 962–971).

Bulf, C., Mathé, A.-C., & Mithalal, J. (2011). Language in the geometry classroom. CERME7 (pp. 649–659).

Burton, L. (1999). Mathematics and their epistemologies: And the learning of mathematics. CERME1 (pp. 87–102).

Deliyianni, E., Elia, I., Gagatsis, A., Monoyiou, A., & Panaoura, A. (2010). A theoretical model of students' geometrical figure understanding. CERME6 (pp. 696705).

Deliyianni, E., Gagatsis, A., Monoyiou, A., Michael, P., Kalogirou, P., & Kuzniak, A. (2011). Towards a comprehensive theoretical model of students' geometrical figure understanding and its relation with proof. CERME7 (pp. 598–607).

Ding, L., Fujita, T., & Jones, K. (2006). Developing geometrical reasoning in the classroom: Learning from highly experienced teachers from China and Japan. CERME4 (pp. 727–737).

Dorier, J. L., Gutiérrez, A., & Strässer, R. (2004). Introduction to the thematic working group on geometry. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG7/TG7_list.php.

Douaire, J., & Emprin, F. (2015). Teaching geometry to students (from five to eight years old). CERME9 (pp. 529–535).

Dreyfus, T., Hillel, J., & Sierpinska, A. (1999). Cabri based linear algebra: Transformations. CERME1 (pp. 209–221).

Duval, R. (1993). Registres de représentation sémiotique et fonctionnement de la pensée. Annales de didactique et de sciences cognitive, 5, 37–65.

Fujita, T., Jones, K., Kunimune, S., Kumakura, H., & Matsumoto, S. (2011). Proofs and refutations in lower secondary school geometry. CERME7 (pp. 660-669).

Gagatsis, A., Michael, P., Deliyianni, E., Monoyiou, A., & Kuzniak, A. (2011). Secondary students' behaviour in proof tasks: Understanding and the influence of the geometrical figure. CERME7 (pp. 608–617).

Girnat, B. (2011). Geometry as propaedeutic to model building: A reflection on secondary school teachers' beliefs. CERME7 (pp. 628–637).

Gueudet-Chartier, G. (2003). Geometric thinking in a n-space. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG7/TG7_list.php.

Hattermann, M. (2010). The drag-mode in three dimensional dynamic geometry environments: Two studies. CERME6 (pp. 786–795).

Houdement, C. (2007). Geometrical working space: A tool for comparison. CERME5 (pp. 954–972).

Houdement, C., & Kuzniak, A. (2002). Pretty (good) didactical provocation as a tool for teacher's training in geometry. CERME2 (pp. 292–303).

Houdement, C., & Kuzniak, A. (2004). Elementary geometry split into different geometrical paradigms. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG7/TG7_list.php.

Jones, K., Lagrange, J.-B., & Lemut, E. (2002). Introduction to WG2: Tools and technologies in mathematical didactics. CERME2 (pp. 125–127).

Kilpatrick, J. (1999). Ich bin europäisch. CERME1 (pp. 49–68).

Kohanová, I. (2007). Comparison of observation of new space and its objects by sighted and

non-sighted pupils. CERME5 (pp. 982–991).

Kospentaris, G., & Spyrou, P. (2007). Assessing the attainment of analytic-descriptive geo

metrical thinking with new tools. CERME5 (pp. 992–1011).

Kurina, F. (2004). Geometry: The resource of opportunities. CERME3: www.mathematik.

uni-dortmund.de/~erme/CERME3/Groups/TG7/TG7_list.php.

Kuzniak, A. (2011). Geometric work at the end of compulsory education. CERME7

(pp. 638–648).

Kuzniak, A. (2013). Teaching and learning geometry and beyond: Plenary conference.

CERME8 (pp. 33–49).

Kuzniak, A., & Nechache, A. (2015). Using the geometric working spaces to plan a coher

ent teaching of geometry. CERME9 (pp. 543–549).

Kuzniak, A., & Rauscher, J. C. (2006). On the geometrical thinking of pre-service school

teachers. CERME4 (pp. 738–747).

Kuzniak, A., & Vivier, L. (2010). A French look on the Greek Geometrical Working Space

at secondary school level. CERME6 (pp. 686–695).

Kuzniak, A., Tanguay, D., & Elia, I. (2016). Mathematical Working Spaces in schooling: an

introduction. ZDM-Mathematics Education, 48(6), 721–737.

Mackrell, K. (2011). Integrating number, algebra, and geometry with interactive geometry

software. CERME7 (pp. 691–700).

Marchetti, P., Medici, D., Vighi, P., & Zaccomer, E. (2006). Comparing perimeters and

area: Children's pre-conceptions and spontaneous procedures. CERME4 (pp. 766–776).

Marchini, C., & Rinaldi, M. G. (2006). Geometrical pre-conceptions of 8 year old pupils.

CERME4 (pp. 748–755).

Markopoulos, C., & Potari, D. (2006). Using dynamic transformations of solids to promote

children's geometrical reasoning. CERME4 (pp. 756–765).

Maschietto, M., Mithalal, J., Richard, P., & Swoboda, E. (2013). Introduction to the papers

and posters of WG4: Geometrical thinking. CERME8 (pp. 578–584).

Mithalal, J. (2010). 3D geometry and learning of mathematical reasoning. CERME6

(pp. 796–805).

Mithalal, J. (2015). Combining epistemological and cognitive approaches of geometry with cKΦ. CERME9 (pp. 557–563).

Modestou, M., Elia, I., Gagatsis, A., & Spanoudes, G. (2007). Problem solving in geometry: The case of the illusion of proportionality. CERME5 (pp. 1052–1061).

Panaoura, G., & Gagatsis, A. (2010). The geometrical reasoning of primary and secondary school students. CERME6 (pp. 746–755).

Parzysz, B. (2002). Introduction to WG7: Working together on metaphors and images. CERME2 (pp. 531–532).

Peirce, C. S. (1931). Collected Papers, vols. 1–6. Cambridge, MA: Harvard University Press.

Perrin-Glorian, M.-J. (2004). Studying geometric figures at primary schools: From surfaces to points. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG7/TG7_list.php.

Pittalis, M., Mousoulides, N., & Christou, C. (2007). Spatial ability as a predictor of students' performance in geometry. CERME5 (pp. 1072–1081).

Pytlak, M. (2015). Learning geometry through paper-based experiences. CERME9 (pp. 571–577).

Rabardel, P. (1995). Les hommes et les technologies: Une approche cognitive des instruments contemporains. Paris: Armand Colin.

Richard, P. R. (2004). Raisonnement et stratégies de preuve dans l'enseignement des mathématiques. Berne: Peter Lang.

Richard, P. R., & Sierpinska, A. (2004). Étude fonctionnelle-structurelle de deux extraits de manuels anciens de géométrie. Revue des Sciences de l'Education, 30(2), 379–409.

Richard, P. R., Oller, A. M., & Meavilla, V. (2016). The concept of proof in the light of mathematical work. ZDM-Mathematics Education, 48(6), 843–859.

Rolet, C. (2004). Teaching and learning plane geometry in primary school: Acquisition of a first geometrical thinking. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG7/TG7_list.php.

Steinwandel, J., & Ludwig, M. (2011). Identifying the structure of regular and semiregular solids: A comparative study between different forms of representation. CERME7 (pp. 670–680).

Strässer, R. (2002). Cabri-géomètre: Does Dynamic Geometry Software (DGS) change geometry and its teaching and learning? International Journal of Computers for Mathematical Learning, 6(3), 319–333.

Vighi, P. (2006). Measurement on the squared paper. CERME4 (pp. 777–785).

2 Chapter 2: Number sense in teaching and learning arithmetic

Aunio, P., Aubrey, C., Godfrey, R., Liu, Y., & Yuejuan, P. (2007). Young children's number sense in China, England and Finland. CERME5 (pp. 1577–1586).

Carvalho, R., & da Ponte, J. P. (2013). Student's mental computation strategies with fractions. CERME8 (pp. 283–292).

Chrysostomou, M., Tsingi, C., Cleanthous, E., & Pitta-Pantazi, D. (2011). Cognitive styles and their relation to number sense and algebraic reasoning. CERME7 (pp. 287–296).

Dehaene, S. (1997). The number sense: How the mind creates mathematics. London: Penguin Press.

Ejersbo, L. R. (2016). Number sense as a bridge to number understanding. In L. Lindenskov (Ed.), Special needs in mathematics education (pp. 189–202). Aarhus: Aarhus University.

Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. Child Development Perspectives, 7(2), 74–79. doi: 10.1111/cdep.12019.

Ferreira, E., & Serrazina, L. (2011). Strategies and procedures: What relationship with the development of number sense of students? CERME7 (pp. 307–315).

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. Nature, 447, 589–592. doi: 10.1038/nature05850.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "Number Sense": The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. Developmental Psychology, 44(5), 1457–1465. doi: 10.1037/a0012682.

Halberda, J., Mazzocco, M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievements. Nature, 455, 665–669.

Hedrén, R. (1999). The teaching of traditional standard algorithms for the four arithmetic operations versus the use of pupils' own methods. CERME1 (pp. 233–244).

Lübke, S. (2015). Investigating fourth graders' conceptual understanding of computational estimation using indirect estimation questions. CERME9 (pp. 302-308).

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining

basic number sense. For the Learning of Mathematics, 12(3), 2–8.

Meissner, H. (2011). Teaching arithmetic for the needs of the society. CERME7 (pp. 346–356).

Morais, C., & Serrazina, L. (2013). Mental computation strategies in subtraction problem

solving. CERME8 (pp. 333–343).

Rechtsteiner-Merz, C., & Rathgeb-Schnierer, E. (2015). Flexible mental calculation and

“Zahlenblickschulung”. CERME9 (pp. 354-361).

Rezat, S. (2011). Mental calculation strategies for addition and subtraction in the set of

rational numbers. CERME7 (pp. 396–406).

Sayers, J., & Andrews, P. (2015). Foundational number sense: Summarising the development

of an analytical framework. CERME9 (pp. 361–368).

Schütte, S. (2004). Rechenwegnotation und Zahlenblick als Vehikel des Aufbaus flexibler

Rechenkompetenzen. Journal für Mathematik-Didaktik, 25(2), 130–148.

Turvill, R. (2015). Number sense as sorting machine in primary mathematics education.

CERME9 (pp. 1658–1664).

Weinert, F. E. (2001). Concept of competence: A conceptual clarification. In D. S. Rychen & L. H. Salganik (Eds.), Defining and selecting key competencies (pp. 45–65). Seattle, WA: Hogrefe & Huber.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. Cognition, 74, B1-B11.

Yang, D. C. (2003). Teaching and learning number sense: An intervention study of fifth grade students in Taiwan. International Journal of Science and Mathematics Education, 1(1), 115-134.

3 Chapter 3: Algebraic thinking

Ainley, J., Bills, L., & Wilson, K. (2004). Designing tasks for purposeful algebra. CERME3:

www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG6/TG6_ainley_

cerme3.pdf.

Alexandrou-Leonidou, V., & Philippou, G. N. (2011). Can they 'see' the equality?

CERME7 (pp. 410-419).

Ayalon, M., & Even, R. (2010). Offering proof ideas in an algebra lesson in different classes

and by different teachers. CERME6 (pp. 430-439).

Bagni, G. (2006). Inequalities and equations: History and didactics. CERME4 (pp. 652-662).

Bartolini Bussi, M. G., & Mariotti, M.-A. (2008). Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective. In L. English, M. G. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), Handbook of international research in mathematics education (2nd ed., pp. 720-749). Mahwah, NJ: Lawrence Erlbaum Associates.

Bazzini, L. (1999). On the construction and interpretation of symbolic expressions. CERME1 (pp. 112-122).

Bergsten, C. (1999). From sense to symbol sense. CERME1 (pp. 123-134).

Block, J. (2015). Flexible algebraic action on quadratic equations. CERME9 (pp. 391-397).

Bolea, P., Bosch, M., & Gascón, J. (2004). Why is modelling not included in the teaching of algebra at secondary school? CERME3 : www.mathematik.uni-dortmund.de/~erme/

CERME3/Groups/TG6/TG6_bolea_cerme3.pdf.

Brousseau, G. (1997). The theory of didactical situations in mathematics: Didactique des mathématiques, 1970-1990 (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. & Trans.). Dordrecht: Kluwer.

Cañadas, M. C., Castro, E., & Castro, E. (2011). Graphical representation and generalization in sequences problems. CERME7 (pp. 460–469).

Caspi, S., & Sfard, A. (2011). The entrance to algebraic discourse: Informal meta-arithmetic as the first step toward formal school algebra. CERME7 (pp. 470–478).

Chiappini, G. (2011). The role of technology in developing principles of symbolical algebra. CERME7 (pp. 429–439).

Chua, B. L., & Hoyles, C. (2011). Secondary school students’ perception of best help generalising strategies. CERME7 (pp. 440–449).

Didi ş, M. G., Baş, A. S., & Erbaş, K. (2011). Students’ reasoning in quadratic equations with one unknown. CERME7 (pp. 479–489).

Dooley, T. (2011). Using epistemic actions to trace the development of algebraic reasoning in a primary classroom. CERME7 (pp. 450–459).

Dörfler, W. (2007). Matrices as Peircean diagrams: A hypothetical learning trajectory. CERME5 (pp. 852–861).

Douady, R. (1999). Relation function/al algebra: An example in high school (age 15–16). CERME1 (pp. 113–124).

Drouhard, J.-P., Panizza, M., Puig, L., & Radford, L. (2006). Working Group 6: Algebraic thinking. CERME4 (pp. 631–642).

Gerhard, S. (2011). Investigating the influence of student’s previous knowledge on their concept of variables: An analysis tool considering teaching reality. CERME7 (pp. 490–499).

Gerhard, S. (2013). How arithmetic education influences the learning of symbolic algebra. CERME8 (pp. 430–439).

Godino, J., Neto, T., Wilhelmi, M., Aké, L., Etchegaray, S., & Lasa, A. (2015). Algebraic reasoning levels in primary and secondary education. CERME9 (pp. 426–432).

Hadjidemetriou, C., Pampaka, M., Petridou, A., Williams, J., & Wo, L. (2007). Developmental assessment of algebraic performance. CERME5 (pp. 893–902).

Henz, D., Oldenburg, R., & Schöllhorn, W. (2015). Does bodily movement enhance mathematical problem solving? Behavioural and neurophysiological evidence. CERME9 (pp. 412–418).

Hewitt, D. (2011). What is algebraic activity? Consideration of 9–10 year olds learning to solve linear equations. CERME7 (pp. 500–510).

Hodgen, J., Küchemann, D. E., Brown, M., & Coe, R. (2010). Children's understandings of algebra 30 years on: What has changed? CERME6 (pp. 539–548).

Isler, I., Strachota, S., Stephens, A., Fonger, N., Blanton, M., & Gardiner, A. (2017). Grade 6 students' abilities to represent functional relationships. CERME10 (pp. 432–439).

Izsák, A., Remillard, J. T., & Templin, J. L. (Eds.). (2016). Psychometric methods in mathematics education: Opportunities, challenges, and interdisciplinary collaborations. Reston, VA: National Council of Teachers of Mathematics.

Jones, I. (2010). Presenting equality statements as diagrams. CERME6 (pp. 549–558).

Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), Algebra in the early grades (pp. 5–17). New York: Lawrence Erlbaum.

Kieran, C. (2004). The core of algebra: Reflections on its main activities. In K. Stacey, H. Chick, & M. Kendal (Eds.), The future of the teaching and learning of algebra: The 12th ICMI Study (pp. 21–33). Dordrecht: Kluwer.

Kilhamn, C. (2013). Hidden differences in teachers' approach to algebra: A comparative case study of two lessons. CERME8 (pp. 440–449).

Küchemann, D. (1981). Algebra. In K. M. Hart (Ed.), Children's understanding of mathematics:

11–16 (pp. 102–119). London, UK: John Murray.

Lagrange, J. B. (2013). Covariation, embodied cognition, symbolism and software design in

teaching/learning about functions: The case of CASYOPÉE. CERME8 (pp. 460–469).

Maffei, L., & Mariotti, M. A. (2011). The role of discursive artefacts in making the structure

of an algebraic expression emerge. CERME7 (pp. 511–520).

Malara, N. A., & Iaderosa, R. (1999). The interweaving of arithmetic and algebra: Some

questions about syntactic and structural aspects and their teaching and learning. CERME1

(pp. 159–171).

Mason, J. (2007). Research and practices in algebra: Interwoven influences. CERME5 (pp. 913–923).

Mestre, L., & Oliveira, H. (2013). Generalising through quasi-variable thinking: A study with grade 4 students. CERME8 (pp. 490–499).

Novotna, J., & Sarrazy, B. (2006). Model of a professor's didactical action in mathematics education: Professor's variability and students' algorithmic flexibility in solving arithmetical problems. CERME4 (pp. 696–705).

Oldenburg, R., Hodgen, J., & Küchemann, D. (2013). Syntactic and semantic items in algebra tests: A conceptual and empirical view. CERME8 (pp. 500–509).

Peacock, G. (1940). Treatise on algebra. New York: Dover Publications. (Original work published 1830).

Pittalis, M., Pitta-Pantazi, D., & Christou, C. (2015). The development of student's early number sense. CERME9 (pp. 446–452).

Postelnicu, V. (2013). Students' difficulties with the Cartesian connection. CERME8 (pp. 520–529).

Proulx, J. (2013). Mental mathematics and algebraic equation solving. CERME8 (pp. 530–539).

Radford, L. (2010). Signs, gestures, meanings: Algebraic thinking from a cultural semiotic perspective. CERME6 (pp. XXXIII–LIII).

Rinvold, R. A., & Lorange, A. (2010). Allegories in the teaching and learning of mathematics. CERME6 (pp. 609–618).

Schwartz, B., Herschkowitz, R., & Dreyfus, T. (2002). Emerging knowledge structures in and with algebra. CERME2 (pp. 81-91).

Strømskag, H. (2015). A pattern-based approach to elementary algebra. CERME9 (pp. 474-480).

Weber, C. (2017). Graphing logarithmic functions: Multiple interpretations of logarithms as a basis for understanding. CERME10 (pp. 537-544).

Zwetzschler, L., & Prediger, S. (2013). Conceptual challenges for understanding the equivalence of expressions: A case study. CERME8 (pp. 558-567).

4 Chapter 4: Research on probability and statistics education: trends and directions

Abrahamson, D., & Wilensky, U. (2004). The quest of the bell curve: A constructionist

designer's advocacy of learning through designing. CERME3: www.mathematik.uni

dortmund.de/~erme/CERME3/Groups/TG5/TG5_abrahamson_cerme3.pdf.

Abrahamson, D., & Wilensky, U. (2006). Problab goes to school: Design, teaching, and learning of probability with multi-agent interactive computer models. CERME4 (pp. 570–579).

Ainley, J., Jarvis, J., & McKeon, F. (2011). Designing pedagogic opportunities for statistical thinking within inquiry based science. CERME7 (pp. 705–714).

Alldredge, J. R., & Brown, G. (2004). Association of course performance with student beliefs: an analysis by gender and instructional software environment. CERME3: www.mathematik.

Batanero, C., Arteaga, P., & Ruiz, B. (2010). Statistical graphs produced by prospective teachers in comparing two distributions. CERME6 (pp. 368–377).

Batanero, C., Estrada, A., Díaz, C., & Fortuny, J. M. (2006). A structural study of future teachers' attitudes towards statistics. CERME4 (pp. 508–517).

Becta (2000). Secondary mathematics with ICT: A pupil's entitlement to ICT in secondary mathematics, downloaded on December 6, 2016 from: www.stem.org.uk/elibrary/resource/29209.

Ben-Zvi, D., Makar, K., Bakker, A., & Aridor, K. (2011). Children's emergent inferential reasoning about samples in an inquiry-based environment. CERME7 (pp. 745–754).

Biehler, R., Ben-Zvi, D., Bakker, A., & Makar, K. (2013). Technology for enhancing statistical reasoning at the school level. In M. A. (Ken) Clements et al. (Eds.), Third international handbook of mathematics education (pp. 643–689). New York: Springer Science and Business Media. doi: 10.1007/978-1-4614-4684-2_21.

Borovcnik, M. (2006). Probabilistic and statistical thinking. CERME4 (pp. 484-506).

Contreras, J. M., Batanero, C., Díaz, C., & Fernandes, J. A. (2011). Prospective teachers' common and specialized knowledge in a probability task. CERME7 (pp. 776-775).

Diaz-Rojas, D., & Soto-Andrade, J. (2015). Enactive metaphoric approaches to randomness. CERME9 (pp. 629-635).

Doerr, H. M., & Jacob, B. (2011). Investigating secondary teachers' statistical understandings. CERME7 (pp. 776-786).

Donati, M. A., Primi, C., Chiesi, F., & Morsanyi, K. (2015). Interest in statistics: Examining the effects of individual and situational characteristics. CERME9 (pp. 740-745).

Eckert, A., & Nilsson, P. (2013). Contextualizing sampling: Teaching challenges and possibilities. CERME8 (pp. 766-776).

Eichler, A. (2007). The impact of a typical classroom practice on students' statistical knowledge. CERME5 (pp. 722-731).

Eichler, A. (2010). The role of context in statistics education. CERME6 (pp. 378-387).

Eichler, A., & Vogel, M. (2015). Aspects of students' changing mental models when acting within statistical situations. CERME9 (pp. 636-642).

Frischemeier, D., & Biehler, R. (2013). Design and exploratory evaluation of a learning trajectory leading to do randomization tests facilitated by Tinkerplots. CERME8 (pp. 798-808).

Frischemeier, D., & Biehler, R. (2015). Preservice teachers' statistical reasoning when comparing groups facilitated by software. CERME9 (pp. 643-650).

Gonzales, O. (2013). Conceptualizing and assessing secondary mathematics teachers' professional competencies for effective teaching of variability-related ideas. CERME8 (pp. 809-818).

González Astudillo, M. T., & Pinto Sosa, J. E. (2011). Instructional representations in the teaching of statistical graphs. CERME7 (pp. 797-806).

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. Journal for Research in Mathematics Education, 38(5), 427–437.

Groth, R. E. (2015). Working at the boundaries of mathematics education and statistics education communities of practice. Journal for Research in Mathematics Education, 46(1), 4–16.

Hacking, I. (1975). The emergence of probability: A philosophical study of early ideas about probability, induction and statistical inference. Cambridge: Cambridge University Press.

Hauge, K. I. (2013). Bridging policy debates on risk assessment and mathematical literacy. CERME8 (pp. 819–828).

Helmerich, M. (2015). Rolling the dice: Exploring different approaches to probability with primary school students. CERME9 (pp. 678–684).

Henriques, A., & Oliveira, H. (2015). Student’s informal inference when exploring a statistical investigation. CERME9 (pp. 685–691).

Henry, M., & Parzysz, B. (2011). Carrying out, modelling and simulating random experiments in the classroom. CERME7 (pp. 864–874).

Jacob, B., & Doerr, H. M. (2013). Students’ informal inferential reasoning when working with the sampling distribution. CERME8 (pp. 829–839).

Jacob, B., Lee, H., Tran, D., & Doerr, H. (2015). Improving teachers’ reasoning about sampling variability: A cross institutional effort. CERME9 (pp. 692–699).

Kazak, S. (2015). A Bayesian inspired approach to reasoning about uncertainty: ‘How confident are you?’ CERME9 (pp. 700–706).

Koleza, E., & Kontogianni, A. (2013). Assessing statistical literacy: What do freshmen know? CERME8 (pp. 840–849).

Lee, H., Tran, D., & Nickel, J. (2015). Simulation approaches for informal inference: Models to develop understanding. CERME9 (pp. 707–714).

Martins, M., Monteiro, C., & Carvalho, C. (2015). How teachers understand sampling when using Tinkerplots. CERME9 (pp. 715–721).

Maxara, C., & Biehler, R. (2007). Constructing stochastic simulations with a computer tool: Students' competencies and difficulties. CERME5 (pp. 762–771).

Monteiro, C., & Ainley, J. (2004). Developing critical sense in graphing. CERME3: www.

Nilsson, P., Schindler, M., & Bakker, A. (2018). The nature and use of theory in statistics education. In D. Ben-Zvi, K. Makar, & J. Garfield (Eds.), International handbook of research in statistics education (pp. 359–386). Cham: Springer.

Paparistodemou, E. (2006). Young children's expressions for the law of large numbers. CERME4 (pp. 611–618).

Paparistodemou, E., Potari, D., & Pitta, D. (2007). Looking for randomness in tasks of prospective teachers. CERME5 (pp. 791–800).

Paparistodemou, E., & Noss, R. (2004). Fairness in a spatial computer environment. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG5/TG5_paparistodemu_cerme3.pdf.

Plicht, C., Vogel, M., & Randler, C. (2015). An interview study on reading statistical representations in biology education. CERME9 (pp. 734–739).

Pratt, D. (2004). The emergence of probabilistic knowledge. CERME3: www.mathematik.

Pratt, D., & Prodromou, T. (2006). Towards the design of tools for the organization of the stochastic. CERME4 (pp. 619–626).

Pratt, D., Levinson, R., Kent, P., & Yogui, C. (2011). Risk-based decision-making by mathematics and science teachers. CERME7 (pp. 875–884).

Prodromou, T. (2007). Making connections between the two perspectives on distribution.

CERME5 (pp. 801–810).

Prodromou, T., & Pratt, D. (2009). Students' causal explanations for distribution. CERME6 (pp. 394-403).

Quintas, S., Oliveira, H., & Tomás Ferreir, R. (2013). The didactical knowledge of one secondary mathematics teacher on statistical variation. CERME8 (pp. 860–869).

Santos, R., & De Ponte, J. P. (2013). Prospective elementary school teachers' interpretation of central tendency measures during a statistical investigation. CERME8 (pp. 870–879).

Schnell, S. (2013). Coping with patterns and variability: Reconstruction of learning pathways towards chance. CERME8 (pp. 880–889).

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15(2), 4–14.

Soto-Andrade, J. (2013). Metaphorical random walks: A royal road to stochastic thinking? CERME8 (pp. 890–900).

Stigler, S. M. (1986). The history of statistics: The measurement of uncertainty before 1900. Cambridge, MA: Harvard University Press.

Sturm, A., & Eichler, A. (2015). Changing beliefs about the benefit of statistical knowledge. CERME9 (pp. 761–767).

5 Chapter 5: Research on university mathematics education

Adam, S. (2002). Towards a common framework for Mathematics degrees in Europe.

Newsletter of the European Mathematical Society 4526–4528.

Artigue, M. (1994). Didactical engineering as a framework for the conception of teaching

products. In R. Biehler, R. W. Scholz, R. Strässer, & B. Winkelmann (Eds.), Didactics of

mathematics as a scientific discipline (pp. 27–39). Dordrecht: Kluwer.

Artigue, M. (2007). Digital technologies: A window on theoretical issues in mathematics

education. CERME5 (pp 68–82).

Artigue, M. (2015). Perspectives on design research: The case of didactical engineering. In:

A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), Approaches to qualitative research

in mathematics education (pp. 467–496). Dordrecht: Springer.

Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K. (1997). The development of students' graphical understanding of the derivative. Journal of Mathematical Behavior, 16(4), 399–431.

Barquero, B., Bosch, M., & Gascȍn, J. (2011). 'Applicationism' as the dominant epistemology at university. CERME7 (pp. 1937–1948).

Barquero, B., Serrano, L., & Serrano, V. (2013). Creating the necessary conditions for mathematical modelling at university. CERME8 (pp. 950–959).

Bergsten, C. (2011). Why do students go to lectures? CERME7 (pp. 1960–1970).

Bergsten, C., & Jablonka, E. (2013). Mathematics as 'meta-technology' and 'mindpower': Views of engineering students. CERME8 (pp. 2284–2293).

Biehler, R., Fischer, P., Hochmuth, R., & Wassong, T. (2011). Designing and evaluating blended learning bridging courses in mathematics. CERME7 (pp. 1971–1980).

Biehler, R., Kortemeyer, J., & Schaper, N. (2015). Conceptualizing and studying students' processes of solving typical problems in introductory engineering courses requiring mathematical competences. CERME9 (pp. 2060–2066).

Biza, I., Souyoul, A., & Zachariades, T. (2006). Conceptual change in advanced mathematical thinking. CERME4 (pp. 1727–1736).

Breen, S., Larson, N., O'Shea, A., & Pettersson, K. (2015). Students' concept images of inverse functions. CERME9 (pp. 2228–2234).

Burton, L. (1999). Mathematics and their epistemologies – and the learning of mathematics. CERME1 (pp. 87–102).

Burton, L. (2004). Mathematicians as enquirers: Learning about learning mathematics. Netherlands: Springer.

Cazes, C., Gueudet, G., Hersant, M., & Vandebrouck, F. (2006). Problem solving and web resources at tertiary level. CERME4 (pp. 1737–1747).

Chevallard, Y. (2002). Organiser l'étude, 3: Écologie et régulation. In J.-L. Dorier et al., XIe école d'été de didactique des mathématiques (pp. 41–56). Grenoble: La Pensée Sauvage.

De Vleeschouwer, M., & Gueudet, G. (2011). Secondary-tertiary transition and evolutions of didactic contract: the example of duality in linear algebra. CERME7 (pp. 2113–2122).

Dorier, J.-L., Robert, A., Robinet, J., & Rogalski, M. (1999). Teaching and learning linear algebra in first year of science university in France. CERME1 (pp. 103–112).

Dreyfus, T., Hillel, J., & Sierpinska, A. (1999). Cabri-based linear algebra: Transformations. CERME1 (pp. 209–221).

Farah, L. (2015). Students' personal work in mathematics in French business school preparatory classes. CERME9 (pp. 2096–2102).

Fischbein, E. (1994). The interaction between the formal, the algorithmic and the intuitive components in a mathematical activity. In R. Biehler, R. W. Scholz, R. Strässer, & B. Winkelmann (Eds.), Didactics of mathematics as a scientific discipline (pp. 232–245). Dordrecht: Kluwer Academic Publishers.

González-Martín, A. (2013). Students' personal relationship with series of real numbers as a consequence of teaching practices. CERME8 (pp. 2326–2335).

González-Martín, A., Biza, I., Cooper, J., Ghedamsi, I., Hausberger, T., Pinto, A., et al. (2017). Introduction to the papers of TWG14: University mathematics education. CERME10 (pp. 2073–2080).

Grønbæk, N., & Winsløw, C. (2015). Media and milieus for complex numbers: An experiment with Maple based text. CERME9 (pp. 2131–2137).

Gueudet, G. (2013). Digital resources and mathematics teachers' professional development at university. CERME8 (pp. 2336–2345).

Hähkiöniemi, M. (2006). Is there a limit in the derivative? Exploring students' understanding of the limit of the difference quotient. CERME4 (pp. 1758–1767).

Halliday, M. (1978). Language as social semiotics. The social interpretation of language and meaning. London, UK: Edward Arnold.

Hausberger, T. (2015). Abstract algebra, mathematical structuralism and semiotics. CERME9 (pp. 2145–2151).

Hiebert, J. (Ed.) (1986). Conceptual and procedural knowledge: The case of mathematics. Hillsdale, NJ: Erlbaum.

Holton, D. (Ed.) (2001). The teaching and learning of mathematics at university level. New ICMI Study Series, Vol. 7. New York: Kluwer.

Inglis, M., & Simpson, A. (2006). Characterising mathematical reasoning: Studies with the Wason selection task. CERME4 (pp. 1768–1777).

Jaworski, B., & Matthews, J. (2011). How we teach mathematics: Discourses on/in university teaching. CERME7 (pp. 2022–2032).

Klein, F. (1908). Elementharmathematik vom höherem Standpunkte aus. Leipzig: B. G. Teubner.

Liebendörfer, M., & Hochmuth, R. (2015). Perceived autonomy in the first semester of mathematics studies. CERME9 (pp. 2180–2186).

Meehan, M. (2007). Student generated examples and the transition to advanced mathematical thinking. CERME5 (pp. 2349–2358).

Mesa, V., & Cawley, A. (2015). Faculty knowledge for teaching inquiry based mathematics. CERME9 (pp. 2194–2200).

Misfeldt, M., & Sanne, A. (2007). Flexibility and cooperation: Virtual learning environments in online undergraduate mathematics. CERME5 (pp. 1470–1479).

Morgan, C., & Tang, S. (2012). Studying changes in school mathematics over time through the lens of examinations: The case of student position. In T. Y. Tso (Ed.), Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education (Vol. 3, pp. 241–248). Tapei, Taiwan: PME.

Nardi, E. (2013). Shifts in language, culture and paradigm: The supervision and teaching of graduate students in mathematics education. CERME7 (pp. 2396–2405).

Nardi, E., & Iannone, P. (2006). To appear and to be: Mathematicians on their students' attempts at acquiring the 'genre speech' of university mathematics. CERME4 (pp. 1737–1747).

Nardi, E., Biza, I., González-Martín, A., Gueudet, G., & Winsløw, C. (Eds.) (2014). Institutional, sociocultural and discursive approaches to research in university mathematics education. Research in Mathematics Education, 16(2) [special issue].

Pettersson, K., Stadler, E., & Tambour, T. (2013). Transformation of the students' discourse on the threshold concept of function. CERME8 (pp. 2406–2415).

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. Educational Studies in Mathematics, 22, 1–36.

Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses, and mathematizing. New York: Cambridge University Press.

Sikko, S., & Pepin, B. (2013). Students' perceptions of how they learn best in higher education mathematics courses. CERME8 (pp. 2446–2455).

Smith, G., Wood, L., Coupland, M., Stephenson, B., Crawford, K., & Ball, G. (1996). Constructing mathematical examinations to assess a range of knowledge and skills. International Journal of Mathematical Education in Science and Technology, 27(1), 65–77.

Stadler, E. (2011). The secondary-tertiary transition: A clash between two mathematical discourses. CERME7 (pp. 2083–2092).

Stadler, E., Bengmark, S., Thunberg, H., & Winberg, M. (2013). Approaches to learning mathematics: Differences between beginning and experienced university students. CERME8 (pp. 2435–2445).

Tabach, M., Rasmussen, C., Hershkowitz, R., & Dreyfus, T. (2015). First steps in re-inventing Euler's method: A case for coordinating methodologies. CERME9 (pp. 2249–2255).

Tall, D. (1991). Advanced mathematical thinking. Dordrecht: Kluwer.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with special reference to limits and continuity. Educational Studies in Mathematics, 12, 151–169.

Thoma, A., & Iannone, P. (2015). Analysing university closed book examinations using two frameworks. CERME9 (pp. 2256–2262).

Trigueros, M., Oktaç, A., & Manzanero, L. (2007). Understanding of systems of equations in linear algebra. CERME5 (pp. 2359–2368).

Vandebrouck, F. (2011). Students' conceptions of functions at the transition between secondary school and university. CERME7 (pp. 2093–2102).

Winsløw, C. (2006). Research and development of university level teaching: The interaction of didactical and mathematical organisations. CERME4 (pp. 1821-1830).

Xhonneux, S., & Henry, V. (2011). A didactic survey of the main characteristics of Lagrange's theorem in mathematics and in economics. CERME7 (pp. 2123-2133).

6 Chapter 6: Argumentation and proof

Antonini, S., & Mariotti, M. A. (2007). Indirect proof: An interpreting model. CERME5 (pp. 541–550).

Arzarello, F., & Sabena, C. (2011). Meta-cognitive Unity in indirect proof. CERME7 (pp. 99–109).

Azrou, N. (2013). Proof in Algebra at the university level: Analysis of students' difficulties. CERME8 (pp. 76–85).

Balacheff, N. (2008). The role of the researcher's epistemology in mathematics education: An essay on the case of proof. ZDM. The International Journal on Mathematics Education, 40, 501–512.

Balacheff, N., & Mariotti, M. A. (2008). Introduction to the special issue on didactical and epistemological perspectives on mathematical proof. ZDM. The International Journal on Mathematics Education, 40, 341–344.

Barrier, T., Mathe, A. C., & Durand-Guerrier, V. (2010). Argumentation and proof: A discussion about Toulmin's and Duval's models. CERME6 (pp. 191–200).

Bartolini-Bussi, M. G. (2010). Experimental mathematics and the teaching and learning of proof. CERME6 (pp. 221–230).

Bergwall, A. (2015). On a generality framework for proving tasks. CERME9 (pp. 86–92).

Boero, P. (2011). Argumentation and proof: Discussing a 'successful' classroom discussion. CERME7 (pp. 120–130).

Boero, P. (2015). Analysing the transition to epsilon-delta Calculus: A case study. CERME9 (pp. 93–99).

Boero, P., Garuti, R., & Lemut, E. (2007). Approaching theorems in grade VIII. In P. Boero (Ed.), Theorems in schools: From history, epistemology and cognition to classroom practice (pp. 249–264). Rotterdam: Sense Publishers.

Boero, P., Garuti, R., & Mariotti, M. A. (1996). Some dynamic mental processes underlying producing and proving conjectures. Proceedings of the 20th PME Conference, Valencia, Spain, 2, 121–128.

Chellougui, F., & Kouki, R. (2013). Use of formalism in mathematical activity – case study: The concept of continuity in higher education. CERME8 (pp. 96-105).

Conejo, L., Arce, M., & Ortega, T. (2015). A case study: How textbooks of a Spanish publisher justify results related to limits from the 70s until today. CERME9 (pp. 107-113).

Cramer, J. (2011). Everyday argumentation and knowledge construction in mathematical tasks. CERME7 (pp. 141-150).

Cramer, J. C. (2013). Possible language barriers in processes of mathematical reasoning. CERME8 (pp. 116-125).

Deloustal-Jorrand, V. (2007). Relationship between beginner teachers in mathematics and the mathematical concept of implication. CERME5 (pp. 601-610).

Douek, N. (2010). Approaching proof in school: From guided conjecturing and proving to a story of proof construction. CERME6 (pp. 332-342).

Durand-Guerrier, V. (2004). Logic and mathematical reasoning from a didactical point of view: A model-theoretic approach. CERME3: www.erme.tu-dortmund.de/~erme/CERME3/Groups/TG4/TG4_Guerrier_cerme3.pdf.

Durand-Guerrier, V. (2006). Natural deduction in Predicate Calculus: A tool for analysing proof in a didactic perspective. CERME4 (pp. 409-419).

Durand-Guerrier, V., Boero, P., Douek, N., Epp, S., & Tanguay, D. (2012). Examining the role of logic in teaching proof. In G. Hanna & M. de Villiers (Eds.), ICMI Study 19 Book: Proof and proving in mathematics education (pp. 369-389). New York: Springer.

Fischbein, E., & Kedem, I. (1982). Proof and certitude in the development of mathematical thinking. Proceedings of the sixth international conference for the psychology of mathematics education (pp. 128-131). Antwerp, Belgium: Universitaire Instelling Antwerpen.

Habermas, J. (2003). Truth and justification. Cambridge, MA: MIT Press.

Hanna, G. (1989). More than formal proof. For the Learning of Mathematics, 9(1), 20-25.

Hanna, G., Jahnke, H. N., & Pulte, H. (Eds.) (2010).

Explanation and proof in mathematics: Philosophical and educational perspectives. New York: Springer.

Jahnke, H. N. (2006). A genetic approach to proof. CERME4 (pp. 428–437).

Jahnke, H. N. (2007). Proofs and hypotheses. ZDM–The International Journal on Mathematics Education, 39(1-2), 79–86.

Jahnke, H. N., & Wambach, R. (2013). Understanding what a proof is: A classroom-based approach. ZDM – The International Journal on Mathematics Education, 45, 469–482.

Johnson-Laird, P. N. (1983). Mental models: Towards a cognitive science of language, inference, and consciousness. Cambridge, MA: Harvard University Press.

Mariotti, M. A. (2006). Proof and proving in mathematics education. In A. Gutiérrez & P. Boero (Eds.), Handbook of research on the psychology of mathematics education (pp. 173–204). Rotterdam: Sense Publishers.

Mariotti, M. A. (2011). Proof and proving as an educational task. CERME7 (pp. 61–89).

Mesnil, Z. (2013). New objectives for the notions of logic teaching in high school in France: Complex request for teachers. CERME8 (pp. 166–175).

Morselli, F. (2013). Approaching algebraic proof at lower secondary school level: Developing and testing an analytical toolkit. CERME8 (pp. 176–185).

Morselli, F., & Boero, P. (2010). Proving as a rational behaviour: Habermas' construct of rationality as a comprehensive frame for research on the teaching and learning of proof. CERME6 (pp. 211–220).

Müller-Hill, E. (2013). The epistemic status of formalizable proof and formalizability as a meta-discursive rule. CERME8 (pp. 186–195).

Pedemonte, B. (2002). Relation between argumentation and proof in mathematics: Cognitive unity or break. CERME2 (Vol. 2, pp. 70–80).

Pedemonte, B. (2004). What kind of proof can be constructed following an abductive argumentation? CERME3:

www.erme.tu-dortmund.de/~erme/CERME3/Groups/TG4/TG4_Pedemonte_cerme3.pdf.

Pedemonte, B. (2007a). Structural relationships between argumentation and proof in solving open problems in algebra. CERME5 (pp. 643–652).

Pedemonte, B. (2007b). How can the relationship between argumentation and proof be

analysed? Educational Studies in Mathematics, 66(1), 23–42.

Perry, P., Molina, O., Camargo, L., & Samper, C. (2011). Analyzing the proving activity of

a group of three students. CERME7 (pp. 151–160).

Reid, D. (2005). The meaning of proof in mathematics education. CERME4 (pp. 458–467).

Reid, D., & Vargas, E. V. (2017). Proof-based teaching as a basis for understanding why.

CERME10 (pp. 235–242).

Silverman, B., & Even, R. (2015). Textbook explanations: Modes of reasoning in 7th grade

Israeli mathematics textbooks. CERME9 (pp. 205–212).

Stenning, K., & van Lambalgen, M. (2008). Human reasoning and cognitive science. Cambridge,

UK: Bradfors Books.

Stylianides, A. J. (2007). Proof and proving in school mathematics. Journal for Research in Mathematics Education, 38, 289–321.

Stylianides, A. J., & Stylianides, G. J. (2007). The mental models theory of deductive reasoning: Implications for proof instruction. CERME5 (pp. 665–674).

Stylianides, G. J., & Stylianides, A. J. (2009). Facilitating the transition from empirical arguments to proof. Journal for Research in Mathematics Education, 40, 314–352.

Tarski, A. (1936). Introduction to logic and to the methodology of deductive sciences (4th ed., 1994). New

York: Oxford University Press.

Thompson, D. R., Senk, S. L., & Johnson, G. J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. Journal for Research in Mathematics Education, 43, 253–295.

Thurston, W. P. (1994). On proof and progress in mathematics. Bulletin of The American Mathematical Society, 30(2), 161–177.

Toulmin, S. E. (1969). The uses of arguments. Cambridge, UK: Cambridge University Press.

Vinner, S. (1983). The notion of proof: Some aspects of students' views at the senior high level. Proceedings of the Seventh International Conference for the Psychology of Mathematics Education (pp. 289–294).

7 Chapter 7: Theory-practice relations in research on applications and modelling

Ärlebäck, J. B. (2010). Towards understanding teachers' beliefs and affect about mathematical modelling. CERME6 (pp. 2096–2105).

Ärlebäck, J. B., Doerr, H. M., & O'Neil, A. H. (2013). Students' emerging models of average rate of change in context. CERME8 (pp. 940–949).

Barbosa, J. C. (2007). Mathematical modelling in parallel discussion. CERME5 (pp. 2101–2109).

Barquero, B., Serrano, L., & Serrano, V. (2015). Creating necessary conditions for mathematical modelling at university level. CERME9 (pp. 950–959).

Bautista, A., Wilkerson-Jerde, M. H. Tobin, R., & Brizuela, B. M. (2013). Diversity in middle school mathematics teachers' ideas about mathematical models: The role of educational background. CERME8 (pp. 960–969).

Blomhøj, M., & Højgaard Jensen, T. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. Teaching Mathematics and its Applications, 22(3), 123–139.

Blomhøj, M., & Kjeldsen, T. H. (2007). Learning the integral concept through mathematical modelling. CERME5 (pp. 2070–2079).

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In S. J. Cho (Ed.), The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and Attitudinal Challenges (pp. 73–96). New York: Springer.

Blum, W., Galbraith, P. L., Henn, H.-W., & Niss, M. (Eds.) (2007). Modelling and applications in mathematics education. The 14th ICMI study. New York: Springer.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. Zentralblatt für Didaktik der Mathematik, 38(2), 86–95.

Borromeo Ferri, R., & Blum, W. (2013). Barriers and motivations of primary teachers for implementing modelling in mathematics lessons. CERME8 (pp. 1000–1009).

Borromeo Ferri, R., & Mousolides, N. (2017). Mathematical modelling as a prototype for interdisciplinary mathematics education? CERME10 (pp. 900-907).

Burkhart, H. (2006). Modelling in mathematics classrooms: Reflections on past developments and the future. Zentralblatt für Didaktik der Mathematik, 38(2), 178-195.

Cabassut, R. (2010). The double transposition in mathematisation at primary. CERME6 (pp. 2156-2165).

Carriera, S., & Baioa, A. M. (2015). Assessing the best staircase: Students' modelling based on experimentation with real objects. CERME9 (pp. 834-840).

Doerr, H. M., & O'Neil, A. H. (2011). A modelling approach to developing an understanding of average rate of change. CERME7 (pp. 937-946).

Doerr, H. M., Ärlebäck, J. B., & O'Neil, A. H. (2013). Teaching practices and modelling changing phenomena. CERME8 (pp. 1041-1050).

Frejd, P. (2013). An investigation of mathematical modelling in the Swedish national course tests in mathematics. CERME8 (pp. 947-956).

García, F., Gascón, J., Higueras, L., & Bosch, M. (2006). Mathematical modelling as a tool for the connection of school mathematics. Zentralblatt für Didaktik der Mathematik, 38(3), 226-246.

Grigoras, R. (2010). Modelling in environments without numbers: A case study. CERME6 (pp. 2206-2015).

Henning, H., & Keune, M. (2006). Levels of modelling competencies. CERME4 (pp. 1666-1674).

Kaiser, G., Blomhøj, M., & Sriraman, B. (2006). Towards a didactical theory for mathematical modelling. Editorial. Zentralblatt für Didaktik der Mathematik, 38(2), 82-85.

Kaiser, G., Sriraman, B., Blomhøj, M., & Garcia, F. J. (2007). Report from the working group modelling and applications: Differentiating perspectives and delineating commonalties. CERME5 (pp. 2035-2041).

Lesh, R. A., & Doerr, H. M. (Eds.). (2003). Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching.

Mahwah, NJ: Erlbaum.

Malle, G. (1986). Variable: Basisartikel mit Überlegungen zur elementaren Algebra. Mathematik Lehren, 15, 2–8.

Ramirez, P. (2017). Teachers' beliefs about mathematical modelling. CERME10 (pp. 972–979).

Rocard, M., Csermely, P., Jorde, D., Lenzen, D., Walberg-Henriksson, H., & Hemmo V. (2007). L'enseignement scientifique aujourd'hui: une pédagogie renouvelée pour l'avenir de l'Europe. Brussels: Commission Européenne, Direction générale de la recherche, Science, économie et société.

Roorda, G., Vos, P., & Goedhart, M. (2007). Derivatives in applications: How to describe students' understanding. CERME5 (pp. 2160–2169).

Ruiz, N., Bosch, M., & Gascón, J. (2007). The functional algebraic modelling at Secondary level. CERME5 (pp. 2170–2179).

Schmidt, B. (2010). Modelling in the classroom: Motives and obstacles from the teachers' perspective. CERME6 (pp. 2066–2075).

Schweiger, F. (2006). Fundamental ideas: A bridge between mathematics and mathematics education. In J. Maazs & W. Schloeglmann (Eds.), New mathematics education research and practice (pp. 63–73). Rotterdam: Sense Publishers.

Siller, H.-S., Kuntze, S., Lerman, S., & Vogl, C. (2011). Modelling as a big idea in mathematics with significance for classroom instruction: How do pre-service teachers see it? CERME7 (pp. 990–999).

Skovsmose, O. (1994). Towards a philosophy of critical mathematical education. Dordrecht: Kluwer.

Sriraman, B. (2006). Conceptualizing the model-eliciting perspective of mathematical problem solving. CERME4 (pp. 1686–1695).

Sriraman, B., & Lesh, R. (2006). Modeling conception revisited. Zentralblatt für Didaktik der Mathematik, 38(3), 247–254.

Treffers, A. (1987). Three dimensions: A model of goal and theory descriptions in mathematics instruction – the

Wiskobas Project. Dordrecht, NL: Kluwer.

Vos, P. (2006). Assessment of mathematics in a laboratory-like environment: The importance of replications. CERME4 (pp. 1696–1705).

Wake, G. (2011). Modelling in an integrated mathematics and science curriculum: Bridging the divide. CERME7 (pp. 1000–1009).

Zell, S., & Beckmann, A. (2010). Modelling activities while doing experiments to discover the concept of variable. CERME6 (pp. 2116–2225).

8 Chapter 8: Early years mathematics

Alpaslan, Z., & Erden, F. (2015). The status of early childhood mathematics education

research in the last decade. CERME9 (pp. 1933–1939).

Bartolini Bussi, M. G. (2013). Bambini Che Contano: A long term program for preschool teachers’ development. CERME8 (pp. 2088–2097).

Bartolini Bussi, M. G., & Baccaglini-Frank, A. (2015). Using pivot signs to reach an inclusive definition of rectangles and squares. CERME9 (pp. 1891–1897).

Benz, C. (2010). “Numbers are actually not bad”: Attitudes of people working in German kindergarten about mathematics in kindergarten. CERME6 (pp. 2547–2556).

Björklund, C. (2017). Aspects of numbers challenged in toddlers’ play and interaction. CERME10 (pp. 1821–1828).

Brandt, B., & Tiedemann, K. (2010). Learning mathematics within family discourses. CERME6 (pp. 2557–2566).

Clements, D. H., & Sarama, J. (2007). Early childhood mathematics learning. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 461–555). New York: Information Age Publishing.

Delacoeur, L. (2015). How the role of the preschool teacher affects the communication of mathematics. CERME9 (pp. 1905–1910).

Demetriou, L. (2015). The use of virtual and concrete manipulatives. CERME9 (pp. 1911–1917).

DFEE, QCA (2000). Curriculum guidance for the foundation stage. London: DFEE.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., et al. (2007). School readiness and later achievement. Developmental Psychology, 43(6), 1428–1446.

Erfjord, I., Carlsen, M., & Hundeland, P. (2015). Distributed authority and opportunities for children’s agency in mathematical activities in kindergarten. CERME9 (pp. 1918–1924).

Flottorp, V. (2011). How do children's classification appear in free play? A case study. CERME7 (pp. 1852–1861).

Hirsh-Pasek, K., Golinkoff, R. M., Berk, L. E., & Singer, D. G. (2009). A mandate for playful learning in preschool: Presenting the evidence. New York: Oxford University Press.

Hundeland, P. S., Erfjord, I., & Carlsen, M. (2013). Use of digital tools in mathematical learning: Kindergarten teachers' approaches. CERME8 (pp. 2108–2117).

Koleza, E., & Giannisi, P. (2013). Kindergarten children's reasoning about basic geometric shapes. CERME8 (pp. 2118–2127).

Ladel, S., & Kortenkamp, U. (2011). Implementation of a multi-touch environment supporting finger symbol sets. CERME7 (pp. 2278–2287).

Lange, T., & Meaney, T. (2013). iPads and mathematical play: A new kind of sandpit for young children? CERME8 (pp. 2138–2147).

Maffia, A., & Mariotti, M. A. (2015). Introduction to arithmetical expressions: A semiotic perspective. CERME9 (pp. 1947–1953).

Meaney, T. (2010). Only two more sleeps until the school holidays: Referring to quantities of things at home. CERME6 (pp. 2617–2626).

Ministry of Education, Israel (2008). Israel national mathematics preschool curriculum (INMPC). Retrieved April 7, 2009, from
http://meyda.education.gov.il/files/Tochniyot_
Limudim/KdamYesodi/Math1.pdf.

OECD (2006). Starting Strong II: Early childhood education and care. Paris: OECD Publishing. Retrieved April 7, 2016, from

Palmér, H., & Björklund, C. (2017). How do preschool teachers characterize their own mathematics teaching in terms of design and content? CERME10 (pp. 1885–1892).

Pimm, D., & Sinclair, N. (2015) "How do you make numbers?": Rhythm and turn-taking when coordinating ear, eye and hand. CERME9 (pp. 1961–1967).

Ramploud, A., & Di Paola, B. (2013). [Shuxue: Mathematics]

Take a look at China. A dialogue between cultures to approach arithmetic at first and second Italian primary classes. CERME8 (pp. 2188–2197).

Rodrigues, M., & Serrazina, L. (2015). Six years old pupils' intuitive knowledge about triangles. CERME9 (pp. 578–583).

Saebbe, P., & Mosvold, R. (2015). Asking productive mathematical questions in kindergarten. CERME9 (pp. 1982–1988).

Schuler, S., Kramer, N., Kröger, R., & Wittmann, G. (2013). Beliefs of kindergarten and primary school teachers towards mathematics teaching and learning. CERME8 (pp. 2128–2137).

Sinclair, N., & SedaghatJou, M. (2013). Finger counting and adding with TouchCounts. CERME8 (pp. 2198–2207).

Starkey, P., & Klein, A. (2000). Fostering parental support for children's mathematical development: An intervention with Head Start families. Early Education and Development, 11(5), 659–680.

Sun, X. (2013). The structures, goals and pedagogies of "variation problems" in the topic of addition and subtraction of 0–9 in Chinese textbooks and its reference books. CERME8 (pp. 2208–2217).

Tirosh, D., Tsamir, P., Tabach, M., Levenson, E., & Barkai, R. (2011). Can you take half? Kindergarten children's responses. CERME7 (pp. 1891–1902).

Tsamir, P., Tirosh, D., Levenson, E., Tabach, M., & Barkai, R. (2011). Investigating geometric knowledge and self-efficacy among abused and neglected kindergarten children. CERME7 (pp. 1902–1911).

Tubach, D. (2015). "If she had rolled five then she'd have two more": Children focusing on differences between numbers in the context of a playing environment. CERME9 (pp. 2617–2624).

van Oers, B. (2014). The roots of mathematising in young children's play. In U. Kortenkamp, B. Brandt, C. Benz, G. Krummheuer, S. Ladel, & R. Vogel (Eds.), Early mathematics learning: Selected papers of the POEM 2012 conference (pp. 111–123). New York: Springer.

Vennberg, H. (2015). Preschool class: One year to count. CERME9 (pp. 2045-2046).

Vighi, P. (2013). Game promoting early generalization and abstraction. CERME8 (pp. 2238-2247).

Vogel, R., & Jung, J. (2013). Videocoding: A methodological research approach to mathematical activities of kindergarten children. CERME8 (pp. 2248-2257).

9 Chapter 9: Mathematical potential, creativity and talent

Aizikovitsh-Udi, E., & Amit, M. (2011). Integrating theories in the promotion of critical

thinking in mathematics classrooms. CERME7 (pp. 1034–1043).

Baruch-Paz, N., Leikin, M., & Leikin, R. (2013). Memory and speed of processing in generally gifted and excelling in mathematics students. CERME8 (pp. 1146–1155).

Baruch-Paz, N., Leikin, M., & Leikin, R. (2015). Visual processing and attention abilities of general gifted and excelling in mathematics students. CERME9 (pp. 1046–1051).

Benölken, R. (2015). The impact of mathematics interest and attitudes as determinants in order to identify girls' mathematical talent. CERME9 (pp. 970–976).

Birkeland, A. (2015). Pre-service teachers' mathematical reasoning. CERME9 (pp. 977–982).

Brandl, M. (2011). High attaining versus (highly) gifted pupils in mathematics: A theoretical concept and an empirical survey. CERME7 (pp. 1044–1055).

Bureš, J., & Nováková, H. (2015). Developing students' culture of problem solving via heuristic solving strategies. CERME9 (pp. 983–988).

De Geest, E. (2013). Possibility thinking with undergraduate distance learning mathematics education students: How it is experienced. CERME8 (pp. 1166–1174).

De Vleeschouwer, M. (2010). Secondary-tertiary transition and students' difficulties: The example of duality. CERME6 (pp. 2256–2264).

Desli, D., & Zioga, M. (2015). Looking for creativity in primary school mathematical tasks. CERME9 (pp. 989–995).

Émin, V., Essonnier, N., Filho, P. L., Mercat, C., & Trgalova, J. (2015). Assigned to creativity: Didactical contract negotiation and technology. CERME9 (pp. 996–1002).

Greenes, C. (1981). Identifying the gifted student in mathematics. Arithmetic Teacher, 28, 14–18.

Ivanov, O., Ivanova, T., & Stolbov, K. (2013). Typologies of mathematical problems: From classroom experience to pedagogical conceptions. CERME8 (pp. 1175–1184).

Juter, K. (2010). Conceptual change and connections in analysis. CERME6 (pp. 2276–2285).

Karakok, G., Savic, M., Tang, G., & El Turkey, H. (2015). Mathematicians’ views on undergraduate students’ creativity. CERME9 (pp. 1003–1009).

Karp, A. (2013). Mathematical problems for the gifted: The structure of problem sets. CERME8 (pp. 1185–1194).

Karp, A., & Busev, V. (2015). Teachers of the mathematically gifted: Two case studies. CERME9 (pp. 1010–1015).

Kasuba, R. (2013). Learning with pleasure: To be or not to be? CERME8 (pp. 1195–1203).

Kattou, M., Christou, C., & Pitta-Pantazi, D. (2015). Mathematical creativity or general creativity? CERME9 (pp. 1016–1023).

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2011). Does mathematical creativity differentiate mathematical ability? CERME7 (pp. 1056–1065).

Kießwetter, K. (1992). Mathematische Begabung: Über die Komplexität der Phänomene und die Unzulänglichkeiten von Punktbewertungen. Der Mathematikunterricht, 38(1), 5–10.

Koestler, A. (1964). The act of creation. London: Hutchinson.

Kontoyianni, K., Kattou, M., Pitta-Pantazi, D., & Christou, C. (2011). Unraveling mathematical giftedness. CERME7 (pp. 1066–1074).

Krutetskii, V. A. (1976). The psychology of mathematical abilities in schoolchildren. Chicago, IL: The University of Chicago Press.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), Creativity in mathematics and the education of gifted students (pp. 129–145). Rotterdam: Sense Publishers.

Leikin, R. (2014). Giftedness and high ability in mathematics. In S. Lerman (Ed.), Encyclopedia of mathematics education. Dordrecht: Springer. Electronic Version. doi: 10.1007/s11858017-0837-9.

Leikin, R., & Elgrabli, H. (2015). Creativity and expertise – the chicken or the egg? Discovering properties of geometry figures in DGE. CERME9 (pp. 1024–1031).

Leikin, R., & Kloss, Y. (2011). Mathematical creativity of 8th and 10th grade students. CERME7 (pp. 1084–1094).

Leikin, R., Levav-Waynberg, A., & Guberman, R. (2011). Employing multiple solution tasks for the development of mathematical creativity: Two comparative studies. CERME7 (pp. 1094–1103).

Lev, M., & Leikin, R. (2013). The connection between mathematical creativity and high ability in mathematics. CERME8 (pp. 1204–1213). Ankara, Turkey: Middle East Technical University.

Levenson, E. (2011). Mathematical creativity in elementary school: Is it individual or collective? CERME7 (pp. 1104–1114).

Maj, B. (2011). Developing creative mathematical activities: Method transfer and hypotheses' formulation. CERME7 (pp. 1115–1124).

Münz, M. (2013). Mathematical creative solution processes of children with different attachment patterns. CERME8 (pp. 1214–1224).

NCTM (National Council of Teachers of Mathematics) (1995). Report of the NCTM taskforce on the mathematically promising. NCTM News Bulletin 32 (December): Special Insert, NCTM Inc., Reston, Virginia.

Nolte, M. (2013). Twice exceptional children: Mathematically gifted children in primary schools with special needs. CERME8 (pp. 1225–1234).

Nordheimer, S., & Brandl, M. (2015). Students with hearing impairment: Challenges facing the identification of mathematical giftedness. CERME9 (pp. 1032–1038).

Palha, S., Schuiterma, J., van Boxtel, C., & Peetsma, T. (2015). The effect of high versus

low guidance structured tasks on mathematical creativity.

CERME9 (pp. 1039–1045).

Pelczer, I., Singer, M., & Voica, C. (2013). Teaching highly able students in a common

class: Challenges and limits of a case-study. CERME8 (pp. 1235–1244).

Pitta-Pantazi, D., Christou, C., Kattou, M., Sophocleous, P., & Pittalis, M. (2015). Assessing

mathematically challenging problems. CERME9 (pp. 1052–1058).

Polya, G. (1973). How to solve it. Princeton, NJ: Princeton University.

Prabhu, V., & Czarnocha, B. (2014). Democratizing mathematical creativity through

Koestler Bisociation Theory. In C. Nicol, S. Oesterle, P. Liljedahl, & D. Allan (Eds.),

Proceedings of the Joint Meeting of PME 38 and PME-NA 36, Vol. 5 (pp. 1–8). Vancouver,

Canada: PME.

Renzulli, J. S. (1978). What makes giftedness? Reexamining a definition. Phi Delta Kappan,

60(3), 180–184.

Rott, B. (2015). Heuristics and mental flexibility in the problem solving processes of regular

and gifted fifth and sixth graders. CERME9 (pp. 1059–1065).

Safuanov, I., Atanasyan, S., & Ovsyannikova, I. (2015). Exploratory learning in the mathematical classroom (open-ended approach). CERME9 (pp. 1097–1098).

Sarrazy, B., & Novotná, J. (2011). Didactical vs. mathematical modelling of the notion competence in mathematics education: Case of 9–10-year-old pupils' problem solving. CERME7 (pp. 1125–1132).

Sarrazy, B., & Novotná, J. (2013). Mathematical creativity and highly able students: What can teachers do? CERME8 (pp. 1245–1253).

Schindler, M., & Joklitschke, J. (2015). Designing tasks for mathematically talented students – according to their abilities. CERME9 (pp. 1066–1072).

Singer, F. M., Pelczer, I., & Voica, C. (2011). Problem posing and modification as a criterion of mathematical creativity. CERME7 (pp. 1133–1142). Rzeszów, Poland: University of Rzeszów.

Singer, F. M., Pelczer, I., & Voica, C. (2015). Problem posing: Students between driven creativity and mathematical failure. CERME9 (pp. 1073–1079).

Sinitsky, I. (2015). What can we learn from pre-service teachers' beliefs on – and dealing with – creativity stimulating activities? CERME9 (pp. 1080–1086).

Sophocleous, P., & Pitta-Pantazi, D. (2011). Creativity in three-dimensional geometry: How can an interactive 3D-geometry software environment enhance it? CERME7 (pp. 1143–1153).

Szabo, A. (2015). Mathematical problem-solving by high achieving students: Interaction of mathematical abilities and the role of the mathematical memory. CERME9 (pp. 1087–1093).

Tall, D. (1991). Advanced mathematical thinking. Dordrecht: Kluwer.

Torrance, E. P. (1974). Torrance Tests of Creative Thinking. Bensenville, IL: Scholastic Testing Service.

Vale, I., & Pimentel, T. (2011). Mathematical challenging tasks in elementary grades. CERME7 (pp. 1154–1164).

Voica, C., & Pelczer, I. (2010). Problem posing by novice and experts: Comparison between students and teachers. CERME6 (pp. 2656–2665).

Waisman, I., Leikin, M., Shaul, S., & Leikin, R. (2013). Brain potentials during solving area-related problems: Effects of giftedness and excellence in mathematics. CERME8 (pp. 1254–1263).

10 Chapter 10: Affect and mathematical thinking: exploring developments, trends, and future directions

Antognazza, D., Di Martino, P., Pellandini, A., & Sbaragli, S. (2015). The flow of emotions in primary school problem solving. CERME9 (pp. 1116–1122).

Athanasiou, C., & Philippou, G. (2010). The effects of changes in the perceived classroom social culture on motivation in mathematics across transitions. CERME6 (pp. 114–123).

Barnes, A. (2015). Improving children’s perseverance in mathematical reasoning: Creating conditions for productive interplay between cognition and affect. CERME9 (pp. 1131–1138).

Di Martino, P. (2010). “Maths and me”: software analysis of narrative data about attitude towards math. CERME6 (pp. 54–63).

Di Martino, P., & Zan, R. (2001). Attitude toward mathematics: Some theoretical issues. In M. van den Heuvel-Panhuizen (Ed.), Proceedings of the 25th IGPME Conference (Vol. 3, pp. 351–358). Utrecht, the Netherlands.

Di Martino, P., Gómez-Chacón, I., Liljedahl, P., Morselli, F., Pantziara, M., & Schukajlow, S. (2015). Introduction to the papers of TWG08: Affect and mathematical thinking. CERME9 (pp. 1104–1107).

Elliot, A. (1999). Approach and avoidance motivation and achievement goals. Educational Psychologist, 34, 169–189.

Evans, J. (2004). Methods and findings in research on affect and emotion in mathematics education. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG2/TG2_evans_cerme3.pdf.

Evans, J., Hannula, M., Philippou, G., & Zan, R. (2004). Introduction to the papers of the thematic working group Affect and Mathematical Thinking. CERME3: www.mathematik.

Furinghetti, F., & Morselli, F. (2006). Reflections on creativity: The case of a good problem solver. CERME4 (pp. 184–193).

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), Beliefs: A hidden variable in mathematics education? (pp. 39–57). Dordrecht: Kluwer.

Gagatsis, A., Panaoura, A., Deliyianni, E., & Elia, I. (2010). Students' beliefs about the use of representations in the learning of fractions. CERME6 (pp. 64–73).

Gómez-Chacón, I. M. (2006). Affect, mathematical thinking and intercultural learning: A study on educational practice. CERME4 (pp. 194–204).

Hannula, M. S. (2010). The effect of achievement, gender and classroom context on upper secondary students' mathematical beliefs. CERME6 (pp. 34–41).

Hannula, M. S. (2011). The structure and dynamics of affect in mathematical thinking and learning. CERME7 (pp. 34–60).

Hannula, M. S. (2012). Exploring new dimensions of mathematics related affect: Embodied and social theories. Research in Mathematics Education, 14(2), 137–161.

Hannula, M. S., & Moreno-Esteva, G. (2017). Identifying subgroups of CERME affect research papers. CERME10 (pp. 1098–1105).

Hannula, M. S., Op 't Eynde, P., Schlöglmann, W., & Wedege, T. (2007). Affect and mathematical thinking. CERME5 (pp. 202–208).

Hannula, M. S., Pantziara, M., Wæge, K., & Schlöglmann, W. (2010). Introduction: Multimethod approaches to the multidimensional affect in mathematics education. CERME6 (pp. 28–33).

Kaasila, R., Hannula, M. S., Laine, A., & Pehkonen, E. (2006). Autobiographical narratives, identity and view of mathematics. CERME4 (pp. 215–224).

Kapetanas, E., & Zachariades, T. (2007). Students' beliefs and attitudes concerning mathematics and their effect on mathematical ability. CERME5 (pp. 258–267).

Kleanthous, E., & Williams, J. (2011). Students' dispositions to study further mathematics in higher education: The effect of students' mathematics self-efficacy. CERME7 (pp. 1229–1238).

Lee, C., & Johnston-Wilder, S. (2011). The pupils' voice in creating a mathematically resilient community of learners. CERME7 (pp. 1189–1198).

Lewis, G. (2015). Patterns of motivation and emotion in mathematics classrooms. CERME9 (pp. 1216–1222).

Liljedahl, P. (2017). On the edges of flow: Student engagement in problem solving. CERME10 (pp. 1146–1153).

Liljedahl, P., & Hannula, M. S. (2016). Research on mathematics-related affect in PME 2005–2015. In A. Gutierrez, G. C. Leder, & P. Boero (Eds.), The second handbook of research on the psychology of mathematics education (pp. 417–446). Rotterdam: Sense Publisher.

Liljedahl, P., Rolka, K., & Rösken, B. (2007). Belief change as conceptual change. CERME5 (pp. 278–287).

Marcou, A., & Lerman, S. (2007). Changes in students' motivational beliefs and performance in a self-regulated mathematical problem-solving environment. CERME5 (pp. 288–297).

Martínez-Sierra, G. (2015). Students' emotional experiences in high school mathematics classroom. CERME9 (pp. 1181–1187).

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), Handbook of research on mathematics learning and teaching (pp. 575–596). New York: MacMillan.

Moscucci, M., Piccione, M., Rinaldi, M. G., & Simoni, S. (2006). Mathematical discomfort and school drop-out in Italy. CERME4 (pp. 245–254).

Mosvold, R., Fauskanger, J., Bjuland, R., & Jakobsen, A. (2011). Using content analysis to investigate student teachers' beliefs about pupils. CERME7 (pp. 1389–1398).

Nicolaidou, M., & Philippou, G. (2004). Attitudes towards mathematics, self-efficacy and achievement in problem-solving. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG2/TG2_nicolaidou_cerme3.pdf.

Nicolaou, A. A., & Philippou, G. N. (2007). Efficacy beliefs, problem posing, and mathematics achievement. CERME5 (pp. 308–317).

Op 't Eynde, P., De Corte, E., Mercken, I. (2007).

Students’ self-regulation of emotions in mathematics learning. CERME5, (pp. 318-3280).

Panaoura, A., & Philippou, G. (2006). The measurement of young pupils’ metacognitive ability in mathematics: The case of self-representation and self-evaluation. CERME4 (pp. 255–264).

Panaoura, A., Deliyianni, E., Gagatsis, A., & Elia, I. (2011). Self-beliefs about using representations while solving geometrical problems. CERME7 (pp. 1167–1178).

Pantziara, M., & Philippou, G. (2010). Endorsing motivation: Identification of instructional practices. CERME6 (pp. 106–113).

Pantziara, M., & Philippou, G. (2011). Fear of failure in mathematics: What are the sources? CERME7 (pp. 1269–1278).

Pantziara, M., Pitta-Pantazi, D., & Philippou, G. (2007). Is motivation analogous to cognition? CERME5 (pp. 339–348).

Pantziara, M., Wæge, K., Di Martino, P., & Rösken-Winter, B. (2013). Introduction to the papers of the thematic working group Affect and Mathematical Thinking. CERME8 (pp. 1272–1278).

Pekrun, R., Frenzel, A., Goetz, T., & Perry, R. P. (2007). The control-value theory of achievement emotions: An integrative approach to emotions in education. In P. A. Schutz & R. Pekrun (Eds.), Emotion in education (pp. 13–36), San Diego, CA: Academic Press.

Rösken, B., Hannula, M. S., Pehkonen, E., Kaasila, R., & Laine, A. (2007). Identifying dimensions of students’ view of mathematics. CERME5 (pp. 349–358).

Sánchez Aguilar, M., Romo Vázquez, A., Rosas Mendoza, A., Molina Zavaleta, J. G., & Castañeda Alonso, A. (2013). Factors motivating the choice of mathematics as a career among Mexican female students. CERME8 (pp. 1409–1418).

Schlöglmann, W. (2004). Can neuroscience help us better understand affective reactions in

mathematics learning? CERME3: http://www.mathematik.uni-dortmund.de/~erme/

CERME3/Groups/TG2/TG2_schloeglmann_cerme3.pdf.

Schukajlow, S. (2015). Is boredom important for students' performance? CERME9 (pp. 1273–1279).

Schukajlow, S., & Krug, A. (2013). Uncertainty orientation, preference for solving task with multiple solutions and modelling. CERME8 (pp. 1428–1437).

Sørlie Street, K., Malmberg, L., & Stylianides, G. (2017). Self-efficacy and mathematics performance: Reciprocal relationships. CERME10 (pp. 1186–1193).

Stylianides, G., & Stylianides, A. (2011). An intervention on students' problem-solving beliefs. CERME7 (pp. 1209–1218).

Tuohilampi, L. (2011). An examination of the connections between self-discrepancies and effort, enjoyment and grades in mathematics. CERME7 (pp. 1239–1248).

Tuohilampi, L., Näveri, L., & Laine, A. (2015). The restricted yet crucial impact of an intervention on pupils' mathematics-related affect. CERME9 (pp. 1287–1293).

Vankúš, P. (2007). Influence of didactical games on pupils' attitudes towards mathematics and process of its teaching. CERME5 (pp. 369–378).

Viitala, H. (2015). Emma's mathematical thinking, problem solving and affect. CERME9 (pp. 1294–1300).

Vollstedt, M. (2010). "After I do more exercise, I won't feel scared anymore": Examples of personal meaning from Hong Kong. CERME6 (pp. 124–135).

Wæge, K. (2007). Intrinsic and extrinsic motivation versus social and instrumental rationale for learning mathematics. CERME5 (pp. 379–388).

Wæge, K. (2010). Students' motivation for learning mathematics in terms of needs and goals. CERME6 (pp. 84–93).

Zan, R., & Di Martino, P. (2004). The role of affect in the research on affect: The case of 'attitude'. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG2/TG2_zan_cerme3.pdf.

Zan, R., & Poli, P. (1999). Winning beliefs in mathematical problem solving. CERME1 (pp. 97–104).

11 Chapter 11: Technology and resources in mathematics education

Abboud, M., & Rogalski, J. (2017). Real uses of ICT in classrooms: Tensions and disturbances in the mathematics teacher's activity. CERME10 (pp. 2334–2341).

Abboud-Blanchard, M., & Vandebrouck, F. (2013). Geneses of technology uses: A theoretical model to study the development of teachers' practices in technology environments. CERME8 (pp. 2504–2513).

Artigue, M., Bosch, M., & Gascón, J. (2011). Research praxeologies and networking theories. CERME7 (pp. 2381–2390).

Assude, T. (2007). Teacher's practices and degree of ICT integration. CERME5 (pp. 1339–1348).

Attorps, I., Björk, K., Radic, M., & Viirman, O. (2013). Teaching inverse functions at tertiary level. CERME8 (pp. 2524–2533).

Balacheff, N. (1993). La transposition informatique, un nouveau problème pour la didactique. In M. Artigue et al. (Eds.), Vingt ans de didactique des mathématiques en France (pp. 364–370). Grenoble: La Pensée Sauvage éditions.

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), Handbook of international research in mathematics education (2nd ed.) (pp. 746–783). Mahwah, NJ: Lawrence Erlbaum.

Barzel, B., Drijvers, P., Maschietto, M., & Trouche, L. (2006). Tools and technologies in mathematical didactics. CERME4 (pp. 927–938).

Belousova, L., & Byelyavtseva, T. (1999). Training explorations on numerical methods course using technology. CERME1 (pp. 201–208).

Brousseau, G. (1997). Theory of didactical situations in mathematics. Dordrecht: Kluwer Academic Publishers.

Cerulli, M. (2002). Introducing pupils to theoretical reasoning: The case of algebra. CERME2 (pp. 139–151).

Churchhouse, R. F. (Ed.) (1986). The influence of computers and informatics on mathematics and its teaching. ICMI Study Series. Cambridge: Cambridge University Press.

Clark-Wilson, A. (2013). How teachers learn to use complex new technologies in secondary mathematics classrooms: The notion of the hiccup. CERME8 (pp. 2544–2553).

Clark-Wilson, A., Hoyles, C., & Noss, R. (2015). Scaling mathematics teachers' professional development in relation to technology: Probing the fidelity of implementation through landmark activities. CERME9 (pp. 2333–2339).

Dreyfus, T., Hillel, J., & Sierpinska, A. (1999). Cabri based linear algebra: Transformations. CERME1 (pp. 209–221).

Drijvers, P. (2011). Teachers transforming resources into orchestrations. CERME7 (pp. 2178–2187).

Drijvers, P., Doorman, M., Boon, P., & van Gisbergen, S. (2010). Instrumental orchestration: Theory and practice. CERME6 (pp. 1349–1358).

Emprin, F. (2007). Analysis of teacher education in mathematics and ICT. CERME5 (pp. 1399–1408).

Engeström, Y. (1999) Activity theory and individual social transformation. In Y. Engeström et al. (Eds.), Perspectives on activity theory (pp. 19–38). Cambridge: Cambridge University Press.

Fredriksen, H. (2013). Mathematics teaching on the web for student teachers: Action research in practice. CERME8 (pp. 2554–2563).

Fuglestad, A. B. (2011). Challenges teachers face with integrating ICT with an inquiry approach in mathematics. CERME7 (pp. 2328–2338).

Gallopin, P., & Zuccheri, L. (2002). A didactical experience carried out using, at the same time, two different tools: A conceptual one and a technological one. CERME2 (pp. 152–162).

Gélis, J.-M., & Lenne, D. (1999). Integration of learning capabilities into a CAS: The SUITES environment as example. CERME1 (pp. 222–232).

Geraniou, I., & Mavrikis, M. (2017). Investigating the integration of a digital resource in the mathematics classroom: The case of a creative electronic book on reflection. CERME10 (pp. 2555–2562).

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? Educational Studies in Mathematics, 71(3), 199–218.

Gustafsson, P. (2017). Exploring a framework for technology integration in the mathematics classroom. CERME10 (pp. 2374–2381).

Gutiérrez, A., Laborde, C., Noss, R., & Rakov, S. (1999). Tools and technologies. CERME1 (pp. 183–188).

Haspekian, M. (2011). The co-construction of a mathematical and a didactical instrument. CERME7 (pp. 2298–2307).

Hoyles, C., & Lagrange, J.-B. (Eds.) (2010). Mathematics education and technology: Rethinking the terrain. The 17th ICMI Study. New York: Springer.

Jan čařík, A., & Novotná, J. (2015). Scaffolding in e-learning course for gifted children. CERME9 (pp. 2354–2360).

Jaworski, B. (2005). Learning communities in mathematics: Creating an inquiry community between teachers and didacticians. Research in Mathematics Education, 7(1), 101–119.

Jones, K. (1999). Student interpretations of a dynamic geometry environment. CERME1 (pp. 245–258).

Jones, K., & Lagrange, J.-B. (2004). Tools and technologies in mathematical didactics: Research findings and future directions. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG9/TG9_introduction_cerme3.pdf.

Khan Academy. (n.d.) www.khanacademy.org [accessed 3 October 2017].

Kynigos, C., Bardini, C., Barzel, B., & Maschietto, M. (2007). Tools and technologies in mathematical didactics. CERME5 (pp. 1499–1508).

Lagrange, J.-B., & Psycharis, G. (2013). Exploring the potential of computer environments for the teaching and learning of functions: A double analysis from two traditions of research. CERME8 (pp. 2624–2633).

Lakoff, G., & Nuñes, R. (2000). Where mathematics comes from: How the embodied mind brings mathematics into being. New York: Basic Books.

Lavicza, Z., Juhos, I., Koren, B., Fenyvesi, K., Csapodi, C., et al. (2015). Integrating technology into primary and secondary school teaching to enhance mathematics education in Hungary. CERME9 (pp. 2430–2431).

Leung, A., & Baccaglini-Frank, A. (2016). Digital technologies in designing mathematics education tasks: Potential and pitfalls (Mathematics Education in the Digital Era, Vol. 8). Dordrecht: Springer.

Mackrell, K., Maschietto, M., & Soury-Lavergne, S. (2013). Theory of didactical situations and instrumental genesis for the design of a Cabri elem book. CERME8 (pp. 2654–2663).

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for integrating technology in teacher knowledge. Teacher College Records, 108(6), 1017–1054.

Monaghan, J., Trouche, L., & Borwein, J. (2016). Tools and mathematics. Dordrecht: Springer.

Noss, R., & Hoyles, C. (1996). Windows on mathematical meaning: Learning cultures and computers. Dordrecht: Kluwer Academic Publisher.

Olivero, F., & Robutti, O. (2002). An exploratory study of students' measurement activity in a dynamic geometry environment. CERME2 (pp. 215–226).

Polanyi, M. (1966). The tacit dimension. Chicago, IL: The University of Chicago Press.

Rabardel, P. (2002). People and technology: A cognitive approach to contemporary instruments.

Université Paris 8.
https://halshs.archives-ouvertes.fr/file/index/docid/1020705/filename/

people_and_technology.pdf [accessed 18 December 2016].

Robert, A., & Rogalski, J. (2005). A cross-analysis of the mathematics teacher's activity:

An example in a French 10th-grade class. Educational Studies in Mathematics, 59(1-3), 269–298.

Rocha, H. (2015). Knowledge for teaching mathematics with technology and the search for a suitable viewing window to represent function. CERME9 (pp. 2403–2409).

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. Education et Didactique, 3(1), 131–149.

Steinbring, H. (2006). What makes a sign a mathematical sign? An epistemological perspective on mathematical interaction. Educational Studies in Mathematics, 61(1–2), 133–162.

Tall, D. (1986). Using the computer as an environment for building and testing mathematical concepts: A tribute to Richard Skemp. In Papers in Honour of Richard Skemp (pp. 21–36). Warwick.

Trgalová, J., Jahn, A. P., & Soury-Lavergne, S. (2010). Quality process for dynamic geometry resources: The Intergeo project. CERME6 (pp. 1161–1170).

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. International Journal of Computers for Mathematics Learning, 9(3), 281–307.

Watson, A., & Ohtani, M. (2015). Task design in mathematics education: An ICMI Study 22. Cham: Springer International Publishing.

Weigand, H.-G. (2008). Teaching with a symbolic calculator in 10th grade: Evaluation of a one year project. International Journal for Technology in Mathematics Education, 15(1), 19–32.

Wenger, E. (1998). Communities of practice: Learning, meaning, identity. New York: Cambridge University Press.

12 Chapter 12: Classroom practice and teachers' knowledge, beliefs and identity

Abboud-Blanchard, M., Cazes, C., & Vandebrouck, F. (2007). Teachers' activity in exercises- based lessons: Some case studies. CERME5 (pp. 1827–1836).

Ainley, J., & Luntley, M. (2006). What teachers know: The knowledge bases of classroom practice. CERME4 (pp. 1410–1419).

Asami-Johansson, Y. (2011). A study of a problem solving oriented lesson structure in mathematics in Japan. CERME7 (pp. 2549–2558).

Aslan-Tutak, F., & Ertas, F. G. (2013). Practices to enhance preservice secondary teachers' specialized content knowledge. CERME8 (pp. 2917–2926).

Azcárate, P., Cardeñoso, J. M., & Serradó, A. (2006). The learning portfolio as an assessment strategy in teacher education. CERME4 (pp. 1430–1439).

Badillo, E., Figueiras, L., Font, V., & Martinez, M. (2013). Visualizing and comparing teachers' mathematical practices. CERME8 (pp. 2927–2935).

Baldry, F. (2017). Analyzing a teacher's orchestration of mathematics in a 'typical' classroom. CERME10 (pp. 3041–3048).

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389–407.

Ba ş, S., Didis, M. G., Erbas, A. K., Cetinkaya, B., Cakıroglu, E., & Alacacı, C. (2008). Teachers as investigators of students' written work: Does this approach provide an opportunity for professional development? CERME8 (pp. 2936–2945).

Berg, C. V. (2011). Adopting an inquiry approach to teaching practice: The case of a primary school teacher. CERME7 (pp. 2580–2589).

Boaler, J. (2003). Studying and capturing the complexity of practice: The case of the dance of agency. In N. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), Proceedings of the 27th PME International Conference, 1, 3–16.

Bosse, M., & Törner, G. (2015). Teachers' professional development in terms of identity development: A shift in perspective on mathematics teachers' learning. CERME9 (pp. 2769–2775).

Carreño, E., Ribeiro, C. M., & Climent, N. (2013). Specialized and horizon content knowledge: Discussing prospective teachers' knowledge on polygons. CERME8 (pp. 2966–2975).

Carrillo, J., Coriat, M., & Oliveira, H. (1999). Teacher education and investigations into teachers' knowledge. CERME 1 (III) (pp. 99–145).

Caseiro, A., da Ponte, J. P., & Monteiro, C. (2015). Elementary teacher practice in project work involving statistics. CERME9 (pp. 2995–3001).

Clivaz, S. (2013). Teaching multidigit multiplication: Combining multiple frameworks to analyze a class episode. CERME8 (pp. 2995–3005).

Corcoran, D. (2007). "You don't need a tables book when you have butter beans!" Is there a need for mathematics pedagogy here? CERME5 (pp. 1856–1865).

Coulange, L. (2006). Teacher's activity and knowledge: The teaching of setting up equations. CERME4 (pp. 1462–1472).

Cusi, A., & Malara, N. A. (2013). A theoretical construct to analyze the teacher's role during introductory activities to algebraic modelling. CERME8 (pp. 3015–3024).

da Ponte, J. P. (1999). Introduction: Teachers' beliefs and conceptions as a fundamental topic in teacher education. CERME1 (I) (pp. 43–49).

da Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutierrez & P. Boero (Eds.), Handbook of research on the psychology of mathematics education: Past, present and future (pp. 461–494). Rotterdam: Sense.

da Ponte, J. P., & Oliveira, H. (2002). Information technologies and the development of professional knowledge and identity in teacher education. CERME2 (pp. 310–321).

da Ponte, J. P., & Quaresma, M. (2015). Conducting mathematical discussions as a feature of teachers' professional practice. CERME9 (pp. 3100–3106).

Drageset, O. G. (2013). Using redirecting, progressing and focusing actions to characterize teachers’ practice. CERME8 (pp. 3025–3034).

Drageset, O. G. (2015). Teachers’ response to unexplained answers. CERME9 (pp. 3009–3014).

Ebbelind, A. (2013). Disentangling prospect teacher’s participation during teacher education. CERME8 (pp. 3045–3054).

Ebbelind, A. (2015). Systemic Functional Linguistics as a methodological tool when researching Patterns of Participation. CERME9 (pp. 3185–3191).

Ellis, M. W., & Berry III, R. Q. (2005). The paradigm shift in mathematics education: Explanations and implications of reforming conceptions of teaching and learning. The Mathematics Educator, 15(1), 7–17.

Espinoza-Vasquez, G., Zakaryan, D., & Carrillo Yañez, J. (2017). Use of analogies in teaching the concept of function: Relation between Knowledge of Topics and Knowledge of Mathematics Teaching. CERME10 (pp. 3288–3295).

Felmer, P., Perdomo-Díaz, J., Giaconi, V., & Espinoza, C. G. (2015). Problem solving teaching practices: Observer and teacher’s view. CERME9 (pp. 3022–3028).

Fernández, S., Figueiras, L., Deulofeu, J., & Martínez, M. (2011). Re-defining HCK to approach transition. CERME7 (pp. 2640–2649).

Ferreira, R. A. T. (2007). The teaching modes: A conceptual framework for teacher education. CERME5 (pp. 1994–2003).

Fives, H., & Buehl, M. M. (2012). Spring cleaning for the messy construct of teachers’ beliefs: What are they? Which have been examined? What can they tell us? In K. R. Harris, S. Graham, & T. Urdan (Eds.), APA educational psychology handbook. Vol. 2: Individual differences and cultural and contextual factors (pp. 471–499). Washington, DC: APA.

Flores, E., Escudero, D. I., & Carrillo, J. (2013). A theoretical review of specialised content knowledge. CERME8 (pp. 3054–3064).

Furinghetti, F., Grevholm, B., & Krainer, K. (2002).

Introduction to WG3. Teacher education between theoretical issues and practical realization. CERME2 (pp. 265–268).

Gade, S. (2015). Teacher-researcher collaboration as formative intervention and expansive learning activity. CERME9 (pp. 3029–3035).

Gee, J. P. (2000). Identity as an analytic lens for research in education. Review of Research in Education, 25(1), 99–125.

Georget, J.-P. (2007). Facilitate research activities at the primary level: Intentional communities of practice, teaching practices, exchanges about these practices. CERME 5 (pp. 1866–1875).

Grundén, H. (2017). Practice of planning for teaching in mathematics: Meaning and relations. CERME10 (pp. 3065–3072).

Gunnarsdóttir, G. H., & Pálsdóttir, G. (2015). Instructional practices in mathematics classrooms. CERME9 (pp. 3036–3042).

Helmerich, M. A. (2013). Competence in reflecting: An answer to uncertainty in areas of tension in teaching and learning processes and teachers profession. CERME8 (pp. 3095–3104).

Hodgen, J. (2004). Reflection, identity, and belief change in primary mathematics. CERME 3:

Holland, D., Skinner, D., Lachicotte Jr, W., & Cain, C. (1998). Identity and agency in cultural worlds. Cambridge, MA: Harvard University Press.

Huckstep, P., Rowland, T., & Thwaites, A. (2006). The knowledge quartet: Considering Chloe. CERME4 (pp. 1568–1578).

Jakobsen, A., Thames, M. H., & Ribeiro, C. M. (2013). Delineating issues related to horizon content knowledge for mathematics teaching. CERME8 (pp. 3125–3134).

Kaldrimidou, M., Sakonidis, H., & Tzekaki, M. (2011). Readings of the mathematical meaning shaped in the mathematics classroom: Exploring different lenses. CERME7 (pp. 2680–2689).

Kempe, U. R., Lövström, A., & Hellqvist, B. (2017). Making distinctions: Critical for the learning of the ‘existence’ of negative numbers? Exploring how the ‘instructional products’ from a theory informed lesson study can be shared and enhance student learning. CERME10 (pp. 3153–3160).

Kleve, B., & Solem, I. H. (2015). A contingent opportunity taken investigating in-between fractions. CERME9 (pp. 3051–3057).

Koklu, O., & Aslan-Tutak, F. (2015). ‘Responding to student ideas’ as an indicator of a teacher’s mathematical knowledge in teaching. CERME9 (pp. 3206–3212).

Krainer, K. (2002). Investigation into practice as a powerful means of promoting (student) teachers’ professional growth. CERME2 (pp. 281–291).

Kunter, M., Baumert, J., Blum, W., Klusman, U., Krauss, S., & Neubrand, M. (Eds.). (2013). Cognitive activiation in the mathematics classroom and professional competence of teachers: Results from the COACTIV project. New York: Springer.

Kuntze, S., Lerman, S., Murphy, B., Kurz-Milcke, E., Siller, H.-S., & Winbourne, P.

(2011). Professional knowledge related to Big Ideas in mathematics: An empirical study

with pre-service teachers. CERME7 (pp. 2717–2726).

Kwon, M. (2015). Supporting students’ development of mathematical explanation: A case

of explaining a definition of fraction. CERME9 (pp. 3058–3064).

Larsen, D. M.,Østergaard, C. H., & Skott, J. (2013). Patterns of participation: A framework for understanding the role of the teacher for classroom practice. CERME8 (pp. 3165–3174).

Lave, J. (1997). The culture of acquisition and the practice of learning. In D. Kirshner & J. A. Whitson (Eds.), Situated cognition: Social, semiotic, and psychological perspectives (pp. 17–35). Mahwah, NJ: Lawrence Erlbaum.

Leder, G. C., Pehkonen, E., & Törner, G. (Eds.). (2002). Beliefs: A hidden variable in mathematics education?

Dordrecht: Kluwer.

Lerman, S. (2000). The social turn in mathematics education research: Multiple perspectives on mathematics teaching and learning. In J. Boaler (Ed.), Multiple perspectives on mathematics teaching & learning (pp. 19–44). Westport, CT: Greenwood Publishing Group.

Malara, N. A., & Navarra, G. (2015). Principles and tools for teachers' education and the assessment of their professional growth. CERME9 (pp. 2854–2860).

Mhuirí, S. N. (2017). Considering research frameworks as a tool for reflection on practices: Grain-size and levels of change. CERME10 (pp. 3113–3120).

Montes, M., & Carrillo, J. (2015). What does it mean as a teacher to 'know infinity'? The case of convergence series. CERME9 (pp. 3220–3226).

Mosvold, R., & Hoover, M. (2017). Mathematical knowledge for teaching and the teaching of mathematics. CERME10 (pp. 3105–3112).

NCTM (1998). Principles and standards for school mathematics: Discussion draft. Reston, VA: NCTM.

NCTM (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

Nowi ńska, E. (2011). A study of the differences between the surface and the deep structures of math lessons. CERME7 (pp. 2777–2786).

Nunes, C. C., & da Ponte, J. P. (2011). Teachers managing the curriculum in the context of the mathematics' subject group. CERME7 (pp. 2787–2797).

Pepin, B., & Roesken-Winter, B. (Eds.). (2015). From beliefs to dynamic affect systems: Exploring a mosaic of relationships and interactions. Cham, Switzerland: Springer.

Petrou, M. (2010). Adapting the knowledge quartet in the Cypriot mathematics classroom. CERME6 (pp. 2020–2029).

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 257–315). Charlotte, NC: NCTM & IAP.

Reid, D. A., Savard, A., Manuel, D., & Wan Jung Lin, T. (2015). Québec anglophone teachers' pedagogies: Observations from an auto-ethnography. CERME9 (pp. 3115–3121).

Ribeiro, C. M., Monteiro, R., & Carrillo, J. (2010). Professional knowledge in an improvisation episode: The importance of a cognitive model. CERME6 (pp. 2030–2029).

Rowland, T., Jared, L., & Thwaites, A. (2011). Secondary mathematics teachers' content knowledge: The case of Heidi. CERME7 (pp. 2827–2837).

Rowland, T., Thwaites, A., & Huckstep, P. (2004). Elementary teachers' mathematics content knowledge and choice of examples. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG12/TG12_Rowland_cerme3.pdf.

Rowland, T., Turner, F., & Thwaites, A. (2013). Developing mathematics teacher education practice as a consequence of research. CERME8 (pp. 3227–3236).

Samková, L., & Hošpesová, A. (2015). Using Concept Cartoons to investigate future teachers' knowledge. CERME9 (pp. 2141–3247).

Sayers, J. (2013). The influence of early childhood mathematical experiences on teachers' beliefs and practice. CERME8 (pp. 3247–3256).

Schoenfeld, A. (2000). Models of the teaching process. Journal of Mathematical Behavior, 18(3), 243–261.

Schueler, S., Roesken-Winter, B., Weißenrieder, J., Lambert, A., & Römer, M. (2015). Characteristics of out-of-field teaching: Teacher beliefs and competencies. CERME9 (pp. 3254–3261).

Sfard, A. (2005). What could be more practical than good research? Educational Studies in Mathematics, 58(3), 393–413.

Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses, and mathematizing. Cambridge, UK: Cambridge University Press.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15(2), 4–14.

Skott, J. (2015). The promises, problems, and prospects of research on teachers' beliefs. In H. Fives & M. G. Gill (Eds.), International handbook of research on teachers' beliefs (pp. 13–30). New York: Routledge.

Stylianides, G. J., & Stylianides, A. J. (2010). The mathematical preparation of teachers: A focus on tasks. CERME6 (pp. 1931–1940).

Taylan, R. D. (2015). Characterizing a highly-accomplished teacher's instructional actions in response to students' mathematical thinking. CERME9 (pp. 3136–3143).

Turner, F. (2007). The mathematics content knowledge of beginning teachers: The case of Amy. CERME5 (pp. 2004–2013).

Turner, F. (2011). Differences in the propositional knowledge and the knowledge in practice of beginning primary school teachers. CERME7 (pp. 2898–2907).

Tzekaki, M., Kaldrimidou, M., & Sakonidis, X. (2002). Reflections on teachers' practices in dealing with pupils' mathematical errors. CERME2 (pp. 322–332).

Vasco, D., Climent, N., Escudero-Ávila, & Flores-Medrano, E. (2015). The characterisation of the specialised knowledge of a university lecturer in linear algebra. CERME9 (pp. 3283–3288).

Velez, I., & da Ponte, J. P. (2015). Promoting the understanding of graph representations by grade 3 students. CERME9 (pp. 3143–3149).

Wenger, E. (1998). Communities of practice: Learning, meaning, and identity. Cambridge, UK: Cambridge University Press.

Wester, R., Wernberg, A., & Meaney, T. (2015). Students' perceptions of Norms in a reformed classroom. CERME9 (pp. 3150–3156).

Wittmann, G., Schuler, S., & Levin, A. (2015). To what extent can kindergarten teachers and primary school teachers initiate and foster learning mathematics in typical situations? CERME9 (pp. 3289–3296).

Zehetmeier, S., & Krainer, K. (2011). How to promote sustainable professional development. CERME7 (pp. 2868–2877).

13 Chapter 13: Mathematics teacher education and professional development

Aizikovitsh-Udi, E., Clarke, D., & Star, J. (2013). Good questions or good questioning: an essential issue for effective teaching. CERME8 (pp. 2908–2916).

Berg, C. V. (2007). Expressing generality: Focus on teachers' use of algebraic notation. CERME5 (pp. 1837–1846).

Berg, C. V. (2011). Adopting an inquiry approach to teaching practice: The case of a primary school teacher. CERME7 (pp. 2580–2589).

Bräuning, K., & Nührenbörger, M. (2010). Teachers' reflections of their own mathematics teaching processes. CERME6 (pp. 944–953).

Brown, L. (2006). From practices to theories to practices . . . in learning to teach mathematics and learning mathematics. CERME4 (pp. 1451–1461).

Carrillo, J., Climent, N., & Muñoz, C. (2006). The transition from initial training to the immersion in practice: The case of a mathematics primary teacher. CERME4 (pp. 1526–1536).

Carrillo, J., Coriat, M., & Oliveira, H. (1999). Teacher education and investigations into teacher's knowledge. CERME1 (pp. 99–146).

Climent, N., & Carrillo, J. (2002). Developing and researching professional knowledge with primary teachers. CERME2 (pp. 269–280).

Cusi, A., & Malara, N. A. (2011). Analysis of the teacher's role in an approach to algebra as a tool for thinking: Problems pointed out during laboratorial activities with perspective teachers. CERME7 (pp. 2619–2629).

da Ponte, J. P., Serrazina, L., & Fonseca, H. (2004). Professionals investigate their own practice. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG11/TG11_Ponte_cerme3.pdf.

Dewey, J. (1910). How we think. Boston, MA: D. C. Heath & Co. Publishers.

Dias, P., & Santos, L. (2015). An assessment practice that teacher José uses to promote selfassessment of mathematics learning. CERME9 (pp. 3002-3008).

García, M., Sánchez, V., Escudero, P., & Llinares, S. (2004). The dialectic relationship between theory and practice in MTE. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG11/TG11_Garcia_cerme3.pdf.

Georget, J. (2007). Facilitate research activities at the primary level: Intentional communities of practice, teaching practices, exchanges about these practices. CERME5 (pp. 1866–1975).

Goffree, F., & Oonk, W. (1999). A digital representation of “full practice” in teacher education: The MILE project. CERME1 (pp. 187–199).

Guerreiro, A., & Serrazina, L. (2010). Communication as social interaction primary school teacher practices. CERME6 (pp. 1744–1750).

Gunnarsdóttir, G., & Pálsdóttir, G. (2011). Lesson study in teacher education: A tool to establish a learning community. CERME7 (pp. 2660-2669).

Gunnarsdóttir, G., & Pálsdóttir, G. (2013). New teachers’ ideas on professional development. CERME8 (pp. 3085–3094).

Helmerich, M. (2013). Competence in reflecting: An answer to uncertainty in areas of tension in teaching and learning processes and teachers profession. CERME8 (pp. 3095–9104).

Hodgen, J. (2004). Reflection, identity and belief change in primary mathematics. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG12/TG12_Hodgen_cerme3.pdf.

Hošpesová, A., & Tichá, M. (2006). Developing mathematics teacher’s competence. CERME4 (pp. 1483–1493).

Hošpesová, A., Tichá, M., & Machá čková, J. (2007). Differences and similarities in (qualified) pedagogical reflection. CERME5 (pp. 1906–1915).

Jaworski, B. (1999). Teacher education through teachers’ investigation into their own practice. CERME1 (pp. 201–221).

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: Towards a theoretical framework based on co-learning partnerships. Educational Studies in Mathematics, 54, 249–282.

Jaworski, B. (2004). Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development. In Proceedings of the 28th PME Conference (Volume I, pp. 17–32). Bergen: Bergen University College.

Jaworski, B. (2007). Theoretical perspectives as a basis for research in LCM and ICTML. In B. Jaworski, A. B. Fuglestad, R. Bjuland, T. Breiteig, S. Goodchild, & B. Grevholm (Eds.), Læringsfellesskapimatematikk – learning communities in mathematics. Bergen: Caspar Forlag.

Jaworski, B. (2008). Building and sustaining inquiry communities in mathematics teaching development. In K. Krainer & T. Wood (Eds.), Participants in MTE (pp. 309–330). Rotterdam: Sense Publishers.

Jaworski, B. (2014). Communities of inquiry in mathematics teacher education. In S. Lerman (Ed.), Encyclopedia of Mathematics Education (pp. 76–78). New York: Springer.

Joubert, M., Back, J., De Geest, E., Hirst, C., & Sutherland, R. (2010). PD for teachers of mathematics: Opportunities and change. CERME6 (pp. 1761–1770).

Karsenty, R., Arcavi, A., & Nurickp, Y. (2015). Video-based peer discussions as sources for knowledge growth of secondary teachers. CERME9 (pp. 2825–2832).

Krainer, K. (1996). Some considerations on problems and perspectives of in service MTE. In C. Alsina et al. (Eds.), 8th International Congress on Mathematics Education: Selected Lectures (pp. 303–321). Sevilla: SAEM Thales.

Krainer, K. (2002). Investigation into practice as a powerful means of promoting (student) teachers' professional growth. CERME2 (pp. 281–291).

Lajoie, C., & Maheux, J. F. (2013). Richness and complexity of teaching division: Prospective elementary teachers' roleplaying on a division with remainder. CERME8 (pp. 3155–3164).

Lin, F.-L., & Rowland, T. (2016). Pre-service and in-service mathematics teachers' knowledge and professional development. In Á. Gutiérrez, G. C. Leder & P. Boero

(Eds.), The second handbook of research on the psychology of mathematics education (pp. 483–520). Rotterdam: Sense Publishers.

Llinares, S., & Krainer, K. (2006). Mathematics (student) teachers and teacher educators as learners. In A. Guriérrez & P. Boero (Eds.), Handbook of research on the psychology of mathematics education (pp. 429–459). Rotterdam: Sense Publishers.

Malara, N. A., & Navarra, G. (2007). A task aimed at leading teachers to promoting a constructive early algebra approach. CERME5 (pp. 1925–1934).

Martignone, F. (2015). A development over time of the researchers' meta-didactical praxeologies. CERME9 (pp. 2867–2873).

Martinho, H., & da Ponte, J. P. (2010). A collaborative project as a learning opportunity for mathematics teachers. CERME6 (pp. 1961–1970).

Muñoz-Catalán, M. C., Carrillo, J., & Climent, N. (2007). The PD of a novice teacher in a collaborative context: An analysis of classroom practice. CERME5 (pp. 1935–1944).

Muñoz-Catalán, M. C., Carrillo, J., & Climent, N. (2010). Analysis of interactions in a collaborative context of professional development. CERME6 (pp. 2010–2019).

Nilsson, P., Sollervall, H., & Milrad, M. (2010). Collaborative design of mathematical activities for learning in an outdoor setting. CERME6 (pp. 1101–1110).

Nunes, C., & da Ponte, J. P. (2010). Curriculum management in the context of a mathematics subject group. CERME6 (pp. 1714–1723).

Nunes, C., & da Ponte, J. P. (2011). Teachers managing the curriculum in the context of the mathematics subject group. CERME7 (pp. 2787–2797).

Oliveira, H., & Henriques, A. (2015). Characterizing one teacher's participation in a developmental research project. CERME9 (pp. 2881–2887).

Pesci, A. (2007). From studies of cooperative learning practices towards a model of intervention on mathematics teachers. CERME5 (pp.1945–1954).

Pesci, A. (2010). Developing mathematics teachers' education through personal reflection and collaborative inquiry: Which kinds of tasks? CERME6 (pp. 1981–1990).

Reinup, R. (2010). Developing of mathematics teachers' community: Five groups, five different ways. CERME6 (pp. 1831–1840).

Ribeiro, C. M., & Carrillo, J. (2011). Knowing mathematics as a teacher. CERME7 (pp. 2817–2826).

Ribeiro, C. M., Aslan-Tutak, F., Charalambous, C., & Meinke, J. (2015). Introduction to the papers of TWG20: Mathematics teacher knowledge, beliefs, and identity: Some reflections on the current state of the art. CERME9 (pp. 3177–3183).

Ribeiro, C. M., Monteiro, R., & Carrillo, J. (2010). Professional knowledge in an improvisation episode: The importance of a cognitive model. CERME6 (pp. 2030–2039).

Sánchez, M. (2011). Concepts from mathematics education research as a trigger for mathematics teachers' reflections. CERME7 (pp. 2878–2887).

Santos, L. (2006). The portfolio in teacher education. CERME4 (pp. 1579–1588).

Santos, L., & Bento, A. (2007). The project work and the collaboration on the initial teacher training. CERME5 (pp. 1974–1983).

Santos, L., Carrillo, J., Hošpesová, A., & Blanchard, M. (2010). From a study of teaching practices to issues in teacher education. CERME6 (pp. 1688–1691).

Scherer, P., & Steinbring, H. (2004). The professionalisation of mathematics teachers' knowledge: Teachers commonly reflect feedbacks to their own instruction activity. CERME3:

Schön, D. A. (1983). The reflective practitioner. New York: Basic Books.

Seeger, F., & Steinbring, H. (Eds.) (1992). The dialogue between theory and practice in mathematics education: Overcoming the broadcast metaphor. Proceedings of the Fourth Conference on Systematic Cooperation between Theory and Practice in Mathematics Education (SCTP). Brakel. (IDM Materialien und Studien 38). Bielefeld: IDM Universität

Bielefeld.

Skott, J. (2006). The role of the practice of theorising practice. CERME4 (pp. 1598–1608).

Slavík, J. (2004). Profesionální reflexe a interpretace výuky jako prost ředníkmeziteorií a praxí. (In Czech: Professional reflection and interpretation of education as a mediator between theory and practice.) In Konference Oborové didaktiky v pregarduálním u čitelském studiu. Brno: PdF MUNI.

Somayajulu, R. V. (2013). Capturing pre-service teachers' mathematical knowledge for teaching. CERME8 (pp. 3267–3276).

Tichá, M., & Hošpesová, A. (2011). Teacher competences prerequisite to natural differentiation. CERME7 (pp. 2888–2897).

Van Bommel, J., & Liljekvist, Y. (2015). Facebook and mathematics teachers' professional development: Informing our community. CERME9 (pp. 2930–2936).

Verhoef, N. C., & Terlouw, C. (2007). Training mathematics teachers in a community of learners (COL). CERME5 (pp. 2014–2023).

Wenger, E. (1998). Communities of practice: Learning, meaning and identity. Cambridge, UK: Cambridge University Press.

Witterholt, M., & Goedhart, M. (2010). The learning of mathematics teachers working in

a peer group. CERME6 (pp. 1991–1999).

Wittmann, E. Ch. (2001). Developing mathematics education in a systemic process.

Educational Studies in Mathematics, 48, 1–20.

Zehetmeier, S. (2010). The sustainability of professional development. CERME6 (pp. 1951–1960).

Zehetmeier, S., Bosse, M., Brown, L., Hošpesová, A., Malara, N., & Rösken-Winter, B. (2015). Introduction to the papers of TWG18: MTE and professional development. CERME9 (pp. 2730–2732).

14 Chapter 14: Mathematics education and language: lessons and directions from two decades of research

Albano, G., Coppola, C., & Pacelli, T. (2015). Reading data from graphs: A study on the

role of language. CERME9 (pp. 1326–1332).

Austin, J. L., & Howson, A. G. (1979). Language and mathematical education. Educational Studies in Mathematics, 10(2), 161–197.

Barwell, R. (2015). Linguistic stratification in a multilingual mathematics classroom. CERME9 (pp. 1333–1339).

Bjuland, R., Cestari, M. L., & Borgersen, H. E. (2007). Pupils’ mathematical reasoning expressed through gesture and discourse. CERME5 (pp. 1129–1139).

Chronaki, A., Mountzouri, G., Zaharaki, M., & Planas, N. (2015). Number words in ‘other’ languages: The case of little Mariah. CERME9 (pp. 1347–1353).

Consogno, V. (2006). The semantic-transformational function of written verbal language in mathematics. CERME4 (pp. 810–820).

Edwards, J.-A. (2007). The language of friendship: Developing sociomathematical norms in the secondary school classroom. CERME5 (pp. 1190–1199).

Erath, K., & Prediger, S. (2015). Diverse epistemic participation profiles in socially established explaining practices. CERME9 (pp. 1374–1381).

Fetzer, M., & Tiedemann, K. (2015). The interplay of language and objects in the mathematics classroom. CERME9 (pp. 1387–1392).

Jung, J., & Schütte, M. (2015). Discourses in kindergarten and how they prepare for future decontextualised learning of mathematics. CERME9 (pp. 1414–1420).

Krause, C. (2017). DeafMath: Exploring the influence of sign language on mathematical conceptualisation. CERME10 (pp. 1316–1323).

Krummheuer, G. (1999). The narrative character of argumentative mathematics classroom interaction in primary education. CERME1 (pp. 331–341).

Misfeldt, M. (2007). Idea generation during mathematical writing: Hard work or a process of discovery? CERME5 (pp. 1240–1249).

Morgan, C. (2013). Language and mathematics: A field without boundaries. CERME8 (pp. 50–67).

Morgan, C., & Alshwaikh, J. (2010). Mathematical activity in a multi-semiotic environment. CERME6 (pp. 993–1002).

Ní Ríordáin, M. (2013). A comparison of Irish and English language features and the potential impact on mathematical processing. CERME8 (pp. 1576–1585).

Nührenbörger, M., & Steinbring, H. (2007). Students' mathematical interactions and teachers' reflections on their own interventions. CERME5 (pp. 1250–1269).

Pimm, D. (1987). Speaking mathematically: Communication in mathematics classrooms. London: Routledge.

Poisard, C., Ní Ríordáin, M., & Le Pipec, E. (2015). Mathematics education in bilingual contexts. CERME9 (pp. 1468–1474).

Price, A. J. (1999). It is not just about mathematics, it is about life. CERME1 (pp. 364–374).

Rønning, F. (2010). Tensions between an everyday solution and a school solution to a measuring problem. CERME6 (pp. 1013–1022).

Rowland, T. (2002). Pragmatic perspectives on mathematics discourse. CERME2 (pp. 408–419).

Schreiber, C. (2006). Semiotic processes in a mathematical internet-chat. CERME4 (pp. 903–912).

Schütte, M. (2006). Interaction structures in primary school mathematics with multilingual student body. CERME4 (pp. 913–923).

Stamou, A. G., & Chronaki, A. (2007). Writing mathematics through dominant discourses: The case of a Greek school mathematics magazine. CERME5 (pp. 1311–1320).

Steinbring, H. (2005). The construction of new mathematical knowledge in classroom interaction: An epistemological perspective. New York: Springer.

Tatsis, K. (2011). Language as a shaping identity tool: The case of in-service Greek teachers. CERME7 (pp. 1376–1385).

15 Chapter 15: Diversity in mathematics education

Abreu, G. de, (2006). Cultural identities in the multi-ethnic mathematical classroom.

CERME4 (pp. 1131–1140).

Abreu, G. de (2014). Cultural diversity in mathematics education. In S. Lerman (Ed.),

Encyclopedia of mathematics education (pp. 125–129). Dordrecht: Springer.

Abreu, G. de, César, M., Gorgorió, N., & Valero, P. (2006). Issues and challenges in

researching mathematics education in multicultural settings. CERME4 (pp. 1125–1130).

Abreu, G. de, & Crafter, S. (2016). Mathematics learning in and out of school: Towards

continuity or discontinuity? In L. English & D. Kirshner (Eds.), Handbook of international

research in mathematics education (3rd ed.) (pp. 395–415). London, UK: Routledge.

Abreu, G. de, Crafter, S., Gorgorió, N., & Prat, M. (2013). Understanding immigrant stu

dents' transitions as mathematical learners from a dialogical self-perspective. CERME8

(pp. 1648–1655).

Abreu, G. de, & Gorgorió, N. (2007). Social representations and multicultural mathematics teaching and learning. CERME5 (pp. 1159–1566).

Alrø, H., Skovsmose, O., & Valero, P. (2004). Communication, conflict and mathematics education in the multicultural classroom. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG10/TG10_Alro_cerme3.pdf.

Alrø, H., Skovsmose, O., & Valero, P. (2006). Culture, diversity and conflict in landscapes of mathematics learning. CERME4 (pp. 1141–1152).

Andersson, A. (2011). Interplays between context and students' achievement of agency. CERME7 (pp. 1399–1408).

Andersson, A., & Norén, E. (2011). Agency in mathematics education. CERME7 (pp. 1389–1398).

Arzarello, F., Dorier, J.-L., Hefendehl-Hebeker, L., & Turnau, S. (1999). Mathematics as a cultural product. CERME1 (pp. 70–77).

Bagger, A. (2015). Pressures and positions of need during the Swedish third-grade national test in mathematics. CERME9 (pp. 1558–1563).

Bishop, A. (1988). Mathematical enculturation: A cultural perspective on mathematics education. Dordrecht: Kluwer.

Bishop, A. J., Clarkson, P., FitzSimons, G., & Seah, W. T. (2002). Studying values in mathematics education: Aspects of the VAMP Project. CERME2 (pp. 368–376).

Black, L., Solomon, Y., & Radovic, D. (2015). Mathematics as caring: The role of 'others' in a mathematical identity. CERME9 (pp. 1564–1570).

Boistrup, L. B., Bohlmann, N., Díez-Palomar, J., Kollosche, D., & Meaney, T. (2017). Introduction to the papers of TWG10: Diversity and mathematics education – social, cultural and political challenges. CERME10 (pp. 1301–1304).

Boistrup, L., & Keogh, J. (2017). The context of workplaces as part of mathematics education in vocational studies: Institutional norms and (lack of) authenticity. CERME10 (pp. 1337–1344).

Boistrup, L. B., Meaney, T., Mesquita, M., & Straehler-Pohl, H. (2015). Introduction to the papers of TWG10: Diversity and mathematics education – social, cultural and political challenges. CERME9 (pp. 1534–1537).

César, M., & Favilli, F. (2006). Diversity seen through teachers' eyes: Discourses about multicultural classes. CERME4 (pp. 1153–1164).

Cole, M. (1996). Cultural psychology. Cambridge, MA: The Belknap Press of Harvard University Press.

Crafter, S. (2010). Parental resources for understanding mathematical achievement in multiethnic settings. CERME6 (pp. 1453–1461).

Crafter, S., & Abreu, G. de. (2011). Teachers' discussions about parental use of implicit and explicit mathematics in the home. CERME7 (pp. 1419–1429).

D'Ambrosio, U. (1985). Socio-cultural bases for mathematics education. Campinas, Brasil: Unicamp.

De Haan, M., & Elbers, E. (2008). Diversity in the construction of modes of collaboration in multiethnic classrooms. In B. van Oers, W. Wardekker, E. Elbers, & R. van der Veer (Eds.), The transformation of learning: Advances in cultural-historical activity (pp. 219–241). Cambridge, UK: Cambridge University Press.

Díez-Palomar, J., & Ortin, S. T. (2011). Socio-cultural roots of the attribution process in family mathematics education. CERME7 (pp. 1491–1500).

Do ğan, O., & Haser, Ç. (2013). The gap between mathematics education and low income students' real life: A case from Turkey. CERME8 (pp. 1697–1704).

Domite, M. do C., & Pais, A. S. (2010). Understanding ethnomathematics from its criticisms and contradictions. CERME6 (pp. 1473–1483).

Eikset, A., Fosse, T., Lange, T., Lie, J., Lossius, M. H., Meaney, T., & Severina, E. (2017).

(Wanting to do) Ethical research in a shifting context. CERME10 (pp. 1353–1360).

Elbers, E., & de Haan, M. (2004). The construction of word meaning in a multicultural

classroom: Talk and collaboration during mathematics lessons. CERME3: www.

mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG10/TG10_Elbers_

cerme3.pdf.

Favilli, F., Oliveras, M. L., & César, M. (2004). Maths teachers in multicultural classes:

Findings from a Southern European project. CERME3: www.mathematik.uni-dortmund.

de/~erme/CERME3/Groups/TG10/TG10_Favilli_cerme3.pdf

François, F., & Pinxten, R. (2013). Multimathemacy. CERME8 (pp. 1735–1743).

Fyhn, A., Meaney, T., Nystad, K., & Nutti, Y. (2017). How Sámi teachers' development of

a teaching unit influences their self-determination. CERME10 (pp. 1361–1368).

Gebremichael, A. T., Goodchild, S., & Nygaard, O. (2011). Students' perceptions about the

relevance of mathematics in an Ethiopian preparatory school. CERME7 (pp. 1430–1439).

Gellert, U., & Straehler-Pohl, H. (2011). Differential access to vertical discourse: Managing

diversity in a secondary mathematics classroom. CERME7 (pp. 1440–1449).

Gorgorió, N., Planas, N., & Vilella, X. (2002). Immigrant children learning mathematics in

mainstream schools. In G. de Abreu, A. Bishop, & N. Presmeg (Eds.), Transitions between

contexts of mathematical practice (pp. 23–52). Dordrecht: Kluwer.

Gorgorió, N., & Prat, M. (2011). Mathematics teachers' social representations and identities

made available to immigrant students. CERME7 (pp. 1450–1459).

Hauge, K. H., Sørngård, M. A., Vethe, T. I., Bringeland, T. A., Hagen, A. A., & Sumstad,

M. S. (2015). Critical reflections on temperature change. CERME9 (pp. 1577–1583).

Johansson, M. C., & Boistrup, L. B. (2013). It is a matter of blueness or redness: Adults'

mathematics containing competences in work. CERME8 (pp. 1744–1753).

Kaiser, G. (2004). Learning mathematics within the context of linguistic and cultural

diversity: An empirical study. CERME3: www.mathematik.uni-dortmund.de/~erme/

CERME3/Groups/TG10/TG10_Kaiser_cerme3.pdf.

Kilpatrick, J. (1999). Ich bin Europäisch. CERME1 (pp. 49–68).

Krummheuer, G. (1999). Introduction (Social interactions in mathematical learning situa

tions). CERME1 (pp. 305–307).

Krummheuer, G. (2002). The comparative analysis in interpretative classroom research in

mathematics education. CERME2 (pp. 339–346).

Lange, T., & Meaney, T. (2011). Becoming disadvantaged: Public discourse around national

testing. CERME7 (pp. 1470–1480).

Montecino, A., & Valero, P. (2015). Statements and discourses about the mathematics

teacher: The research subjectivation. CERME9 (pp. 1617–1623).

Mukhopadhyay, S., & Greer, B. (2015). Cultural responsiveness and its role in humanizing

mathematics education. CERME9 (pp. 1624–1629).

Newton, R., & Abreu, G. de (2011). Parent-child interactions on primary school-related

mathematics. CERME7 (pp. 1481–1490).

Newton, R., & Abreu, G. de (2013). The dialogical mathematical ‘self’. CERME8 (pp. 1784–1791).

Pais, A., Crafter, S., Straehler-Pohl, H., & Mesquita, M. (2013). Introduction to the papers and posters of WG10: Cultural diversity and mathematics education. CERME8 (pp. 1820–1824).

Parra-Sánchez, A. (2015). Dialogues in ethnomathematics. CERME9 (pp. 1644–1650).

Radovic, D., Black, L., Salas, C., & Williams, J. (2015). The intersection of girls' mathematics and peer group positionings in a mathematics classroom. CERME9 (pp. 1651–1657).

Rangnes, T. E. (2011). Moving between school and company. CERME7 (pp. 1501–1510).

Roos, H. (2017). Diversity in an inclusive mathematics classroom: A student perspective. CERME10 (pp. 1433–1440).

Seah, W. T., Davis, E. K., & Carr, M. (2017). School mathematics education through the eyes of students in Ghana: Extrinsic and intrinsic valuing. CERME10 (pp. 1441–1448).

Skovsmose, O. (2014). Critical mathematics education. Dordrecht: Springer.

Stathopoulou, C., François, K., & Moreira, D. (2011). Ethnomathematics in European context. CERME7 (pp. 1511–1520).

Stentoft, D. (2007). Multiple identities in the mathematics classroom: A theoretical perspective. CERME5 (pp. 1597–1608).

Straehler-Pohl, H., & Pais, A. (2013). To participate or not participate? That is not the question! CERME8 (pp. 1794–1803).

Turvill, R. (2015). Number sense as a sorting mechanism in primary mathematics education. CERME9 (pp. 1658–1663).

Valero, P. (2013). Mathematics for all and the promise of a bright future. CERME8 (pp. 1804–1813).

Valero, P., Crafter, S., Gellert, U., & Gorgorió, N. (2011). Introduction to the papers of WG 10: Discussing diversity in mathematics education from social, cultural and political perspectives. CERME7 (pp. 1386–1388).

16 Chapter 16: Comparative studies in mathematics education

An, T., Mintos, A., & Yigit, M. (2013). A cross-national standards analysis: Quadratic equations and functions. CERME8 (pp. 1825–1834).

Andrews, P. (2010). Comparing Hungarian and English mathematics teachers' professional motivations. CERME6 (pp. 2452–2462).

Andrews, P. (2011). The teaching of linear equations: Comparing effective teachers from three high achieving European countries. CERME7 (pp. 1555–1564).

Andrews, P., Sayers, J., & Marschall, G. (2015). Developing foundational number sense: Number line examples from Poland and Russia. CERME9 (pp. 1681–1687).

Andrews, P., & Xenofontos, C. (2017). Beginning teachers' perspectives on linear equations: A pilot quantitative comparison of Greek and Cypriot students. CERME10 (pp. 1594–1601).

Asami-Johansson, Y., Attorps, I., & Laine, A. (2017). Comparing the practices of primary school mathematics teacher education: Case studies from Japan, Finland and Sweden. CERME10 (pp. 1602–1609).

Back, J., Sayers, J., & Andrews, P. (2013). The development of foundational number sense in England and Hungary: A case study comparison. CERME8 (pp. 1735–1844).

Bjarnadóttir, K. (2007). Development of the mathematics education system in Iceland in the 1960s in comparison to three neighbouring countries. CERME5 (pp. 2403–2412).

Bofah, E. A. (2015). Reciprocal determinism between students' mathematics self-concept and achievement in an African context. CERME9 (pp. 1688–1694).

Bofah, E. A., & Hannula, M. (2011). The case of calculus: Comparative look at task representation in textbooks. CERME7 (pp. 1545–1554).

Branchetti, L., Ferretti, F., Lemmo, A., Maffia, A., Martignone, F., Matteucci, M., & Mignani, S. (2015). A longitudinal analysis of the Italian national standardized mathematics tests. CERME9 (pp. 1695–1701).

Brown, R. G. (2007). Policy change, graphing calculators and ‘High stakes examinations’: A view across three examination systems. CERME5 (pp. 2413–2422).

Cabassut, R. (2007). Examples of comparative methods in the teaching of mathematics in France and in Germany. CERME5 (pp. 2423–2432).

Cabassut, R., & Ferrando, I. (2013). Modelling in French and Spanish syllabus of secondary education. CERME8 (pp. 1845–1854).

Cabassut, R., & Villette, J.-P. (2011). Exploratory data analysis of a European teacher training course on modelling. CERME7 (pp. 1565–1574).

Clarke, D. J. (2013). The validity-comparability compromise in crosscultural studies in mathematics education. CERME8 (pp. 1855–1864).

Clarke, D. J. (2015). The role of comparison in the construction and deconstruction of boundaries. CERME9 (pp. 1702–1708).

Clarke, D. J., & Xu, L. H. (2010). Spoken mathematics as a distinguishing characteristic of mathematics classrooms in different countries. CERME6 (pp. 2463–2472).

Clarke, D. J., Emanuelsson, J., Jablonka, E., & Mok, I. A. (Eds.) (2006). Making connections: Comparing mathematics classrooms around the world. Rotterdam: Sense Publishers.

Clarke, D. J., Mesiti, C., Cao, Y., & Novotna, J. (2017). The lexicon project: Examining the consequences for international comparative research of pedagogical naming systems from different cultures. CERME10 (pp. 1610–1617).

da Ponte, J. P., & Marques, S. (2007). Proportion in school mathematics textbooks: A comparative study. CERME5 (pp. 2443–2452).

Eisenmann, T., & Even, R. (2007). Types of algebraic activities in two classes taught by the same teacher. CERME5 (pp. 2433–2442).

Gosztonyi, K. (2015). The ‘New Math’ reform and pedagogical flows in Hungarian and French mathematics education. CERME9 (pp. 1709–1716).

Hannula, M., Lepik, M., Pipere, A., & Tuohilampi, L.

(2013). Mathematics teachers' beliefs in Estonia, Latvia and Finland. CERME8 (pp. 1865–1874).

Hemmi, K., Koljonen, T., Hoelgaard, L., Ahl, L., & Ryve, A. (2013). Analyzing mathematics curriculum materials in Sweden and Finland: Developing an analytical tool. CERME8 (pp. 1875–1884).

Hommel, M., & Clarke, D. J. (2015). Reflection and questioning in classrooms in different cultural settings. CERME9 (pp. 1717–1723).

Jablonka, E. (2013). Boredom in mathematics classrooms from Germany, Hong Kong and the United States. CERME8 (pp. 1885–1894).

Jablonka, E. (2015). Why look into mathematics classrooms? Rationales for comparative classroom studies in mathematics education. CERME9 (pp. 1724–1730).

Jablonka, E., & Andrews, P. (2012). CERME7: Comparative studies in mathematics education. Research in Mathematics Education, 14(2), 203–204. doi: 10.1080/14794802.2012.694291.

Jakobsen, A., Fauskanger, J., Mosvold, R., & Bjuland, R. (2011). Comparison of item performance in a Norwegian study using U.S. developed mathematical knowledge for teaching measures. CERME7 (pp. 1575–1584).

Joubert, M. (2015). The perceived causes of the (assumed) mathematics problems in England and South Africa: A social media experiment. CERME9 (pp. 1731–1737).

Kiliç, Ç. (2011). Belgian and Turkish pre-service primary school mathematics teachers' metaphorical thinking about mathematics. CERME7 (pp. 1585–1593).

Kingji-Kastrati, J., Sajka, M., & Vula, E. (2017). Comparison of Kosovan and Polish preservice teachers' knowledge of fractions. CERME10 (pp. 1618–1625).

Knipping, C., & Müller-Hill, E. (2013). The problem of detecting genuine phenomena amid a sea of noisy data. CERME8 (pp. 1895–1904).

Knutsson, M., Hemmi, K., Bergwall, A., & Ryve, A. (2013). School-based mathematics teacher education in Sweden and Finland: Characterising mentor–prospective teacher discourse. CERME8 (pp. 1905–1914).

Koljonen, T. (2017). Finnish teaching materials in the hands of a Swedish teacher: The telling case of Cecilia. CERME10 (pp. 1626–1633).

Kuzniak, A., Parzysz, B., Santos-Trigo, M., & Vivierr, L. (2011). Problem solving and open problems in teachers' training in the French and Mexican modes. CERME7 (pp. 1594–1604).

Larson, N., & Bergsten, C. (2013). Comparing mathematical work at lower and upper secondary school from the students' perspective. CERME8 (pp. 1915–1924).

Maréchal, C. (2011). What kind of teaching in different types of classes? CERME7 (pp. 1605–1613).

Modeste, S., & Rafalska, M. (2017). Algorithmics in secondary school: A comparative study between Ukraine and France. CERME10 (pp. 1634–1641).

Mulat, T., & Arcavi, A. (2010). Mathematical behaviors of successful students from a challenged ethnic minority. CERME6 (pp. 2473–2482).

Navarra, G., Malara, N. A., & Ambrus, A. (2010). A problem posed by J. Mason as a starting point for a Hungarian-Italian teaching experiment within a European project. CERME6 (pp. 2483–2494).

Nguyen, H. T. T., & Grégoire, J. (2013). Re-examining the language supports for children's mathematical understanding: A comparative study between French and Vietnamese languages. CERME8 (pp. 1925–1934).

Nilsen, H. K. (2010). A comparison of teachers' beliefs and practices in mathematics teaching at lower secondary and upper secondary school. CERME6 (pp. 2494–2503).

Nosrati, M., & Andrews, P. (2017). Ten years of mathematics education: Preparing for the supermarket? CERME10 (pp. 1642–1649).

O'Reilly, M., & Parker, N. (2013). 'Unsatisfactory saturation': A critical exploration of the notion of saturated sample sizes in qualitative research. Qualitative Research, 13(2), 190–197. doi: 10.1177/1468794112446106.

Peng, A., Sollervall, H., Stadler, E., Shang, Y., & Ma, L. (2015). Swedish and Chinese teachers' views on what constitutes a good mathematical test task: A pilot study. CERME9 (pp. 1738–1744).

Pepin, B. (2010). Mathematical tasks and learner dispositions: A comparative perspective. CERME6 (pp. 2504–2512).

Saari, J. v. R. (2010). Elite mathematics students in Finland and Washington: Access, collaboration, and hierarchy. CERME6 (pp. 2513–2522).

Saeki, A., Matszaki, A., Kawakami, T., & Lamb, J. (2015). Examining the heart of the dual modelling cycle: Japanese and Australian students advance this approach. CERME9 (pp. 1745–1751).

Sajka, M. (2017). Visual attention while reading a multiple choice task by academics and students: A comparative eye-tracking approach. CERME10 (pp. 1650–1657).

Sajka, M., & Rosiek, R. (2015). Solving a problem by students with different mathematical abilities: A comparative study using eye-tracking. CERME9 (pp. 1752–1758).

Schäfer, I., & Winkler, A. (2011). Comparing the construction of mathematical knowledge between low-achieving and high-achieving students: A case study. CERME7 (pp. 1614–1624).

Soto-Andrade, J., & Reyes-Santander, P. (2011). Conceptual metaphors and Grundvorstellungen: A case of convergence? CERME7 (pp. 1625–1635).

Tchoshanov, M., Quinones, M. C., Shakirova, K., Ibragimova, E., & Shakirova, L. (2017). Cross-national study of lower secondary mathematics teachers' content knowledge in USA and Russia. CERME10 (pp. 1658–1665).

Tillman, L. C. (2006). Researching and writing from an African-American perspective: Reflective notes on three research studies. International Journal of Qualitative Studies in Education, 19(3), 265–287.

Törnroos, J. (2007). Factors related to students' mathematical literacy in Finland and Sweden. CERME5 (pp. 2453–2462).

Tuohilampi, L., Hannula, M., Giaconi, V., Laine, A., & Näveri, L. (2013). Comparing the structures of 3rd graders' mathematics-related affect in Finland and Chile. CERME8 (pp. 1935–1944).

Vantourout, M. (2007). A comparative study of assessment activity involving 8 pre-service teachers: What referent for the assessor? CERME5 (pp. 2463–2472).

Vollstedt, M. (2007). The construction of personal meaning: A comparative case study in Hong Kong and Germany. CERME5 (pp. 2473–2482).

Vula, E., Kingji-Kastrati, J., & Podvorica, F. (2015). A comparative analysis of mathematics textbooks from Kosovo and Albania based on the topic of fractions. CERME9 (pp. 1759–1765).

Xenofontos, C. (2010). International comparative research on mathematical problem solving: Suggestions for new research directions. CERME6 (pp. 2523–2532).

Xenofontos, C., & Andrews, P. (2017). Explanations as tools for evaluating content knowledge for teaching: A cross-national pilot study in Cyprus and Greece. CERME10 (pp. 1666–1673).

Xenofontos, C., & Papadopoulos, C. (2015). The history of mathematics in the lower secondary textbook of Cyprus and Greece: Developing a common analytical framework. CERME9 (pp. 1766–1773).

17 Chapter 17: History and mathematics education

Alpaslan, M., & Haser, Ç. (2015). Selecting and preparing original sources for pre-service mathematics teacher education: The preliminary of a dissertation. In E. Barbin, U. T. Jankvist, & T. H. Kjeldsen (Eds.), History and Epistemology in Mathematics Education. Proceedings of the Seventh European Summer University ESU 7, Copenhagen, Denmark 14–18 July 2014 (pp. 451–463). Copenhagen, Denmark: Danish School of Education, Aarhus University.

Alpaslan, M., Schubring, G., & Günergun, F. (2015). 'Mebahis-i İlmiye' as the first periodical on mathematical sciences in the Ottoman Turkey. CERME9 (pp. 1783–1789).

Chorlay, R., Clark, K., Gosztonyi, K., & Lawrence, S. (2017). Introduction to the papers and posters of TWG12: History in mathematics education. CERME10 (pp. 1681–1684).

Clark, K. (2015). The contribution of history of mathematics on students' mathematical thinking competency. CERME9 (pp. 1804–1810).

da Costa, D. A. (2010). Arithmetic in primary school in Brazil: End of the nineteenth century. CERME6 (pp. 2712–2721).

Fasanelli, F., & Fauvel, J. (2006). History of the international study group on the relations between the History and Pedagogy of Mathematics: The first twenty-five years, 1976–2000. In F. Furinghetti, S. Kaijser, & C. Tzanakis (Eds.), Proceedings of HPM 2004 & ESU 4 (pp. x–xxviii). Iraklion, Greece: University of Crete.

Fauvel, J. (1990). History in the mathematics classroom: The IREM papers. Leicester, UK: The Mathematical Association.

Fauvel, J. (1991). Using history in mathematics education. For the Learning of Mathematics 11, 3–6.

Fauvel, J., & van Maanen, J. (Eds.) (2000). History in mathematics education: The ICMI Study. Dordrecht: Kluwer Academic Publishers.

Fried, M. N., & Jahnke, H. N. (2015). Otto Toeplitz's 1927 paper on the genetic method in the teaching of mathematics. Science in Context, 28, 285–295.

Furinghetti, F., Dorier, J. L., Jankvist, U. T., van Maanen, J., & Tzanakis, C. (2010). Introduction: Theory and research on the role of history in mathematics education. CERME6 (pp. 2679–2681).

Jankvist, U. T. (2012). A first attempt to identify and classify empirical studies on ‘History in mathematics education’. In B. Sriraman (Ed.), Crossroads in the history of mathematics and mathematics education (pp. 295–332). The Montana Mathematics Enthusiast Monographs 12. Charlotte, NC: Information Age Publishing.

Jankvist, U. T., Chorlay, R., Clark, K., & Lawrence, S. (2015). Introduction to the papers and posters of TWG12: History in mathematics education. CERME9 (pp. 1779–1781).

Jankvist, U. T., Clark, K., Lawrence, S., & van Maanen, J. (2013). Introduction to the papers

and posters of WG12: History in mathematics education. CERME8 (pp. 1945–1950).

Jankvist, U. T., Lawrence, S., Tzanakis, C., & van Maanen, J. (2011). Introductory report

from WG12: History in mathematics education. CERME7 (pp. 1636–1639).

Karp, A., & Schubring, G. (Eds.) (2014). Handbook on the history of mathematics education.

New York: Springer.

Katz, V. J. (Ed.) (2000). Using history to teach mathematics: An international perspective. Washington, DC: Mathematical Association of America.

Katz, V. J., & Tzanakis, C. (Eds.) (2011). Recent developments on introducing a historical dimension in mathematics education. Washington, DC: Mathematical Association of America.

Kjeldsen, T. H. (2011). Uses of history in mathematics education: Development of learning strategies and historical awareness. CERME7 (pp. 1640–1649).

Krüger, J., & van Maanen, J. (2013). Evaluation and design of mathematics curricula: Lessons from three historical cases. CERME8 (pp. 2030–2039).

Mota, C., Ralda, M. E., & Estrada, M. F. (2013). The teaching of the concept of tangent line using original sources. CERME7 (pp. 2048-2057).

NCTM Yearbook (1969). Historical topics for the mathematics classroom. Reston, VA: NCTM. (Updated edition 1989).

Niss, M. (1996). Goals of mathematics teaching. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), International handbook of mathematics education (pp. 11-48). Dordrecht: Kluwer Academic.

Niss, M., & Højgaard, T. (2011). Competencies and mathematical learning ideas and inspiration for the development of mathematics teaching and learning in Denmark. Danish edition 2002, English edition, October 2011. IMFUFA tekst no. 485. Roskilde, Denmark: Roskilde University.

Schubring, G. (2000). The first international curricular reform in mathematics and the role of Germany: A case study in the transmission of concepts. In A. Gagatsis, C. P. Constantinou, & L. Kyriakides (Eds.), Learning and assessment in mathematics and science (pp. 265-287). Nicosia, Cyprus: Department of Education, University of Cyprus.

Tzanakis, C., & Arcavi, A. (2000). Integrating history of mathematics in the classroom: An analytic survey. In J. Fauvel & J. van Maanen (Eds.), History in mathematics education: The ICMI Study (pp. 201-240). Dordrecht: Kluwer Academic Publishers.

18 Chapter 18: Theoretical perspectives and approaches in mathematics education research

Artigue, M. (2007). Digital technologies: A window on theoretical issues in mathematics

education. CERME5 (pp. 68–82).

Artigue, M., Bartolini-Bussi, M., Dreyfus, T., Gray, E., & Prediger, S. (2006). Different

theoretical perspectives and approaches in research in mathematics education. CERME4

(pp. 1239–1244).

Artigue, M., Bosch, M., Gascón, J., & Lenfant, A. (2010). Research problems emerging from

a teaching episode: A dialogue between TDS and ATD. CERME6 (pp. 1535–1544).

Artigue, M., Bosch, M., & Gascón, J. (2011). Research praxeologies and networking theories. CERME7 (pp. 2381–2390).

Arzarello, F., Bosch, M., Lenfant, A., & Prediger, S. (2007). Different theoretical perspectives in research. CERME5 (pp. 1618–1627).

Bakker, A. (2016). Networking theories as an example of boundary crossing. Educational Studies in Mathematics, 93(2), 265–273.

Bergsten, C. (2007). How do theories influence the research on teaching and learning limits of functions? CERME5 (pp. 1638–1647).

Bikner-Ahsbahs, A. (2010). Networking of theories: Why and how? CERME6 (pp. 6–15).

Bikner-Ahsbahs, A., Bakker, A., Haspekian, M., & Maracci, M. (2017). Theoretical perspectives and approaches in mathematics education research. CERME10 (pp. 2683–2690).

Bikner-Ahsbahs, A., Knipping, C., & Presmeg, N. C. (Eds.) (2015). Approaches to qualitative research in mathematics education: Examples of methodology and methods. Dordrecht: Springer.

Bikner-Ahsbahs, A., & Prediger, S. (Eds.) (2014). Networking of theories as a research practice in mathematics education. Cham: Springer.

Bingolbali, E., & Bingolbali, F. (2015). Principles of student centred teaching and implications for mathematics teaching. CERME9 (pp. 2600–2606).

Bosch, M., Chevallard, Y., & Gascón, J. (2006). Science or magic? The use of models and theories in didactics of mathematics. CERME4 (pp. 1254–1263).

Brousseau, G. (1997). Theory of Didactical Situations in mathematics. Dordrecht: Kluwer Academic Publishers Group.

Cerulli, M., Pedemonte, B., & Robotti, E. (2006). An integrated perspective to approach technology in mathematics education. CERME4 (pp. 1389–1399).

Chevallard, Y. (2000). La recherche en didactique et la formation des professeurs: problématiques, concepts, problèmes. In M. Bailleul (Ed.), Actes de la Xe École d'été de didactique des mathématiques (pp. 98–112). Caen: IUFM.

Dreyfus, T. (2010). Ways of working with different theoretical approaches in mathematics education research: An introduction. CERME6 (pp. 2–5).

Florensa, I., Bosch, M., & Gascón, J. (2015). The epistemological dimension in didactics: Two problematic issues. CERME9 (pp. 2635–2641).

Font, V., Malaspina, U., Giménez, J., & Wilhelmi, M. R. (2011). Mathematical objects through the lens of three different theoretical perspectives. CERME7 (pp. 2411–2420).

Gellert, U. (2007). Emergence or structure: A comparison of two sociological perspectives on mathematics classroom practice. CERME5 (pp. 1668–1677).

Goodchild, S. (2007). An activity theory perspective of didacticians' learning within a mathematics teaching development research project. CERME5 (pp. 1678–1687).

Hejny, M., Shiu, C., Godino, J. D., & Maier, H. (1999). Research paradigms and methodologies and their relationship to questions in mathematics education. CERME1 (pp. 211–219).

Hodgson, B. R., Kuzniak, A., & Lagrange, J.-B. (Eds.) (2016). The didactics of mathematics: Approaches and issues (A homage to Michèle Artigue). Switzerland: Springer.

Jaworski, B. (2007). Theory in developmental research in mathematics teaching and learning: Social practice theory and community of inquiry as analytical tools. CERME5 (pp. 1688–1697).

Jaworski, B., Serrazina, L., Koop, A. P., & Krainer, K. (2004). Inter-relating theory and practice in mathematics teacher education. CERME3: www.mathematik.uni-dortmund.de/~erme/CERME3/Groups/TG11/TG11_introduction_cerme3.pdf.

Kidron, I. (2015). The epistemological dimension revisited. CERME9 (pp. 2662–2667).

Kidron, I., Bikner-Ahsbahs, A., Monaghan, J., Radford, L., & Sensevy, G. (2011). Different theoretical perspectives and approaches in research in mathematics education. CERME7 (pp. 2376–2380).

Kidron, I., Bosch, M., Monaghan, J., & Radford, L. (2013). Different theoretical perspectives and approaches in research in mathematics education. CERME8 (pp. 2785–2789).

Kidron, I., Lenfant, A., Bikner-Ahsbahs, A., Artigue, M., & Dreyfus, T. (2007). Social interaction in learning processes as seen by three theoretical frameworks. CERME5 (pp. 1708–1724).

Lagrange, J.-B., & Kynigos, C. (2014). Digital technologies to teach and learn mathematics: Context and recontextualization. Educational Studies in Mathematics, 85(3), 381–403.

Lagrange, J.-B., & Monaghan, J. (2010). On the adoption of a model to interpret teachers' use of technologies in mathematics lessons. CERME6 (pp. 1605–1614).

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), Multiple perspectives on mathematics teaching and learning (pp. 19–44). Westport, CT: Ablex Publishing.

Lerman, S. (Ed.) (2014). Encyclopedia of mathematics education. Dordrecht: Springer.

Ligozat, F., & Schubauer-Leoni, M. L. (2010). The joint action theory in didactics: Why do we need it in the case of teaching and learning mathematics? CERME6 (pp. 1615–1624).

Monaghan, J. (2010). People and theories. CERME6 (pp. 16–23).

Nardi, E., Biza, I., González-Martín, A. S., Gueudet, G., & Winsløw, C. (2014). Institutional, sociocultural and discursive approaches to research in university mathematics education. Research in Mathematics Education, 16(2), 91–94.

Prediger, S., Bosch, M., Kidron, I., Monaghan, J., & Sensevy, G. (2010). Different theoretical perspectives and approaches in mathematics education research: Strategies and difficulties when connecting theories. CERME6 (pp. 1529–1534).

Prediger, S., & Ruthven, K. (2007). From teaching problems to research problems. CERME5 (pp. 1745–1754).

Radford, L. (2008). Connecting theories in mathematics education: Challenges and possibilities. ZDM – The International Journal on Mathematics Education, 40(2), 317–327.

Rodríguez, E., Bosch, M., & Gascón, J. (2007). An anthropological approach to metacognition. CERME5 (pp. 1798–1807).

Roos, H., & Palmér, H. (2015). Communities of practice: Exploring the diverse use of a theory. CERME9 (pp. 2702–2708).

Ruiz-Munzón, N., Bosch, M., & Gascón, J. (2013). Comparing approaches through a reference epistemological model: The case of algebra. CERME8 (pp. 2870–2879).

Sensevy, G. (2010). Outline of a joint action theory in didactics. CERME6 (pp.1645–1654).

von Glasersfeld, E. (1995). Radical constructivism: A way of knowing and learning. London: Falmer Press.

20 Chapter 20: Communication, cooperation and collaboration: ERME's magnificent experiment

Balacheff, N. (1990). Beyond a psychological approach: The psychology of mathematics

education. For the Learning of Mathematics, 10(3), 2–8.

Bikner-Ahsbahs, A., & Kidron, I. (2015). A cross-methodology for the networking of theories: The General Epistemic Need (GEN) as a new concept at the boundary of two theories. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), Approaches to qualitative research in mathematics education: Examples of methodology and methods (pp. 233–250). Dordrecht: Springer.

Bikner-Ahsbahs, A., Prediger, S., & Networking Theories Group (Eds.) (2014). Networking of theories as a research practice. Heidelberg/New York: Springer.

Confrey, J. (1991). Learning to listen: A student's understanding of powers of ten. In E. von Glasersfeld (Ed.), Radical constructivism in mathematics education (pp. 111–138). Dordrecht: Kluwer.

Confrey, J. (1994a). A theory of intellectual development (Part 1). Mathematical Thinking and Learning, 14(3), 2–8.

Confrey, J. (1994b). Voice and perspective: Hearing epistemological innovation in students' words. In N. Bednarz, M. Larochelle, & J. Désautels (Eds.), Revue de sciences de l'education. Special Issue: Constructivism in education, 20(1), 115–133.

Confrey, J. (1995). A theory of intellectual development (Part 2). Mathematical Thinking and Learning, 15(1), 38–48.

Conway, J. (1985). Four views of ICME-5: ICME past, ICME still to come. For the Learning of Mathematics, 5(1), 29–36.

Fried, M. (2014). Mathematics and mathematics education: Searching for common ground (Introduction). In M. Fried & T. Dreyfus (Eds.), Mathematics and mathematics education: Searching for common ground (pp. 3–22). Dordrecht: Springer.

Furinghetti, F. (2014). History of international cooperation in mathematics education. In A. Karp & G. Schubring (Eds.), Handbook on the history of mathematics education (pp. 543–564). Dordrecht: Springer.

Husserl, E. (1970). The crisis of European sciences and transcendental phenomenology. (Translated from German: original 1954.) Chicago, IL: Northwestern University Press.

Karp, A., & Schubring, G. (Eds.) (2014). Handbook on the history of mathematics education. Dordrecht: Springer.

Kidron, I., & Bikner-Ahsbahs, A. (2015). Advancing research by means of the networking of theories. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), Approaches to qualitative research in mathematics education: Examples of methodology and methods (pp. 221–232). Dordrecht: Springer.

Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? Journal for Research in Mathematics Education, 27(2), 133–150.

Presmeg, N. (1985). The role of visually mediated processes in high school mathematics: A classroom investigation. Unpublished PhD dissertation, University of Cambridge.

Presmeg, N., Radford, L., Roth, W.-M., & Kadunz, G. (2016). Semiotics in mathematics education. (ICME-13 Topical Surveys, Ed. G. Kaiser.) Hamburg: Springer Open.

Presmeg, N., Radford, L., Roth, W.-M., & Kadunz, G. (2018). Signs of signification: Semiotics in mathematics education research. (ICME-13 Topical Surveys, Ed. G. Kaiser.) Hamburg: Springer Open.

Sierpinska, A., & Kilpatrick, J. (1998). Mathematics education as a research domain: A search for identity. Dordrecht: Kluwer.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27(4), 458–477.