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华东师范大学 · 亚洲数学教育中心

Ecological, epistemological and existential challenges of integrating digital tools into school mathematics

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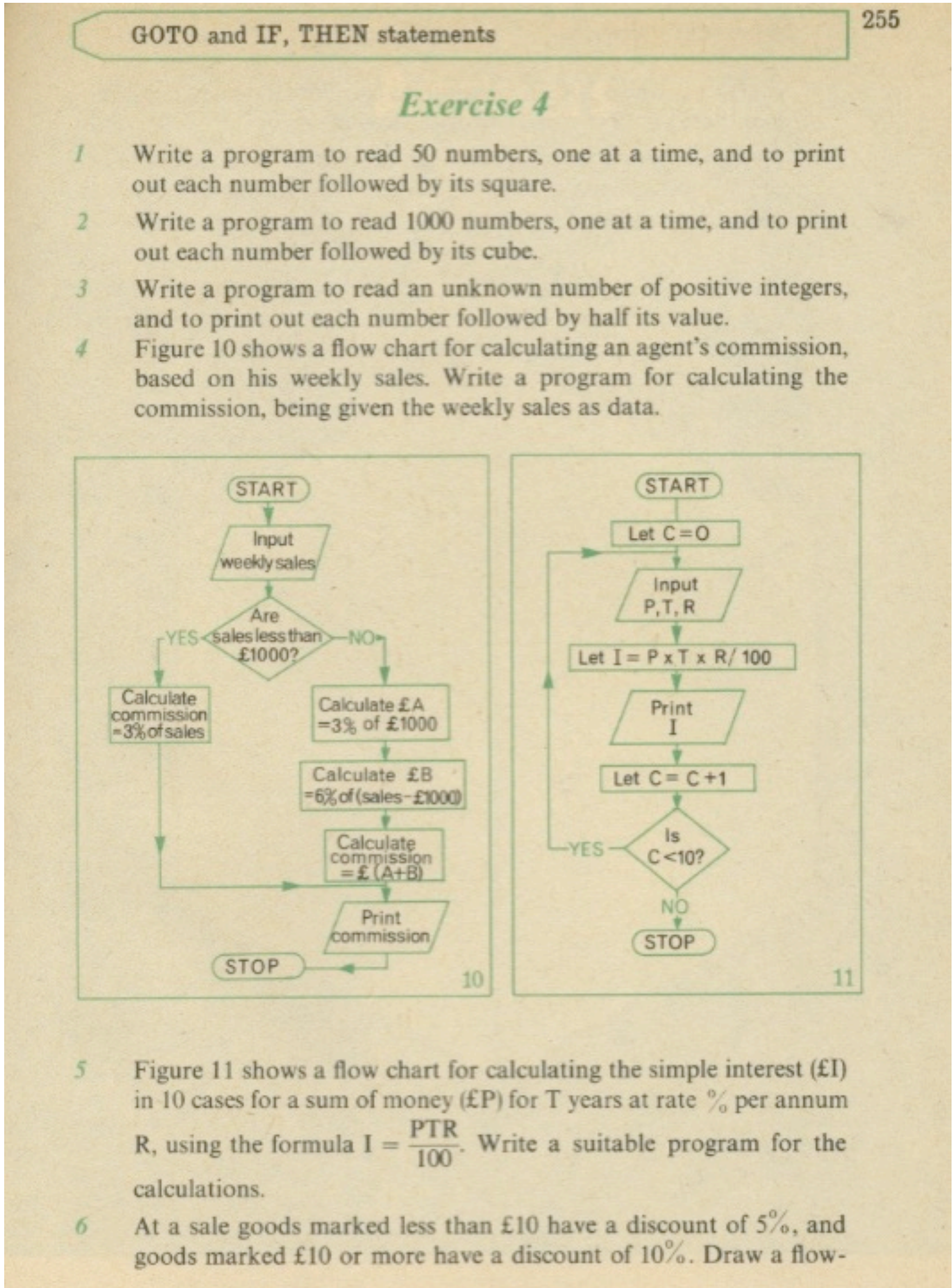
Scotland 1973

- I embarked on my teaching career, just as an updated edition of the [national] mathematics textbook series appeared.
- I imagined that, long before I retired, use of digital computational tools would become integral to school mathematics.
- How wrong I have been proved!


GOTO and IF, THEN statements 255

Exercise 4

1 Write a program to read 50 numbers, one at a time, and to print out each number followed by its square.

2 Write a program to read 1000 numbers, one at a time, and to print out each number followed by its cube.

3 Write a program to read an unknown number of positive integers, and to print out each number followed by half its value.

4 Figure 10 shows a flow chart for calculating an agent's commission, based on his weekly sales. Write a program for calculating the commission, being given the weekly sales as data.

5 Figure 11 shows a flow chart for calculating the simple interest (£I) in 10 cases for a sum of money (£P) for T years at rate % per annum R, using the formula $I = \frac{PTR}{100}$. Write a suitable program for the calculations.

6 At a sale goods marked less than £10 have a discount of 5%, and goods marked £10 or more have a discount of 10%. Draw a flow-

Motivation for the talk

- The use of digital computational tools is now commonplace in mathematical practice outside school.
- But, despite half a century of sustained advocacy and effort, the degree to which the use of such tools has become integral to school mathematics remains limited.
- This talk will examine three fundamental areas of challenge to the integration of such tools into school mathematics.
- The evidence and research that I will draw on has mainly been conducted in Western countries – Britain in particular.
- Yet, it seems plausible that, given their basic character, these same challenges are also relevant to Asian countries.

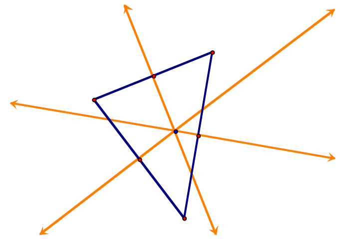
Aspects of challenge

- **Ecological: relating to the surrounding infrastructural and logistic features which structure schooling**
  - Adapting infrastructure, organisation and expertise related to the practice of school mathematics so as to make the use of digital tools viable and support associated changes in practice.
- **Epistemological: relating to the underpinning systems of knowledge which guide schooling**
  - Expanding systems of disciplinary/didactical knowledge so as to guide the use of digital tools within school mathematics and inform its associated evolution as a subject.
- **Existential: relating to the defining subject identities which shape participation in schooling**
  - Developing value-based subject representations and identities (both of the discipline and the person) which harmonise with the use of digital tools within school mathematics.

Ecological challenges

- Established classroom practice takes place in a smooth and efficient manner thanks to a highly evolved material/social organisation and participant expertise which underpin it.
- Inasmuch as the introduction of digital computational tools perturbs the order of this system, it needs to be adapted, and organisation and expertise to be developed accordingly.
- The Structuring Features of Classroom Practice framework (Ruthven 2009) was devised by drawing together disparate ideas – originally developed to understand such issues prior to the arrival of digital computational tools – in the light of early studies of the introduction of such tools.
- I will sketch each component of the framework, and illustrate it with some aspects of the experience of one teacher.

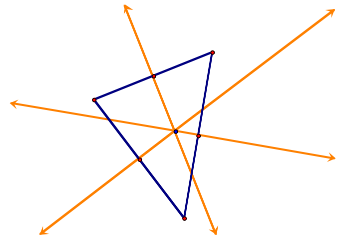
Working environment



Use of digital resources often involves changes in the working environment of lessons in terms of room location, physical layout, and class organisation, requiring modification of the routines which enable lessons to flow smoothly.

- Starting sessions in the classroom avoided disrupting established routines for launching lessons, providing an environment more conducive to maintaining student attention *“without the distraction of computers in front of them.”*
- Moving to the computer suite, routines were being established for starting up (opening workstation, logging on to network, accessing resources, maximising windows) and closing down (saving files having named them appropriately, printing work having made it findable in output from the shared printer).

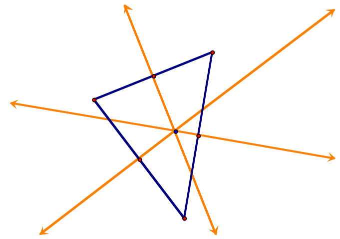
Resource system



While new technologies broaden the range of tools and resources available to support school mathematics, they present the challenge of building a coherent resource system of compatible elements that function in a complementary manner, and which participants are capable of using effectively.

- The teacher saw work with dynamic software as complementing established construction work with classical manual tools, but felt that they lacked congruence where manual techniques had no computer counterpart.
- The teacher was concerned that students were spending too much time on cosmetic aspects of presentation. By showing to what degree and for what purpose it was legitimate to *"slightly adjust the font and change the colours a little bit, to emphasise the maths, not to make it just look pretty"*, he was establishing norms for using the new tool.

Activity structure



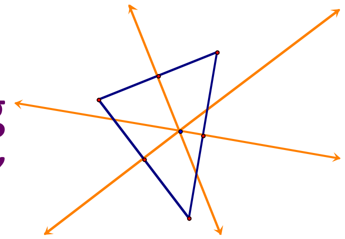
Innovation may call for adaptation of the established repertoire of activity formats that frame the action and interaction of participants during particular types of classroom episode, and that combine to create prototypical activity structures or cycles for particular styles of lesson.

- The lesson combined *"a bit of whole class, a bit of individual work and some exploration"*; a structure that the teacher wanted *"to pursue because it was the first time [he]'d done something that involved all those different aspects."*
- The teacher noted how the digital environment shaped student activity and his own interactions with them:
  - Identifying and resolving bugs in their dynamic constructions.
  - Revising their statements (which they were more willing to do) in (readily modifiable) text-boxes

Curriculum script

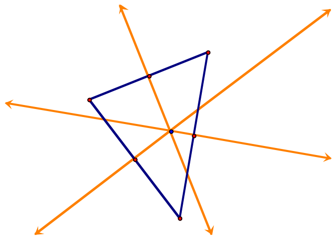
Incorporating new tools into lessons requires teachers to develop their curriculum script for a topic.

This loosely-ordered model of goals, resources and actions for teaching the topic interweaves ideas to be developed, tasks to be undertaken, activity formats to be used, and student difficulties to be anticipated, guiding the teacher in devising a lesson agenda and enacting it in a flexible way.



- The teacher was gaining knowledge of *"unusual"* and *"awkward"* aspects of software operation liable to *"cause a bit of confusion"*, as well as how to turn these to advantage in forming target mathematical ideas.
- Equally, he was developing ways of helping students appreciate geometrical significance through dragging the dynamic figure.
- When he asked about the centre's position when the triangle was dragged to become right angled, he *"was just expecting them to say it was on the line"*, not anticipating what a student pointed out: that *"it was exactly on that centre point."*

Time economy



Introduction of new technologies may influence the time economy within which teachers operate, affecting the 'rate' at which the physical time available for classroom activity can be converted into a 'didactic time' measured in terms of the advance of knowledge.

- Because this teacher viewed the software as a way of engaging students in disciplined interaction with a geometric system, he was willing to spend time to make them aware of the construction process underlying dynamic figures, by *"actually put[ting] it together in front of the students so they can see where it's coming from."*
- This teacher linked management of time to stages of investigation: *"the process of exploring something, then discussing it in a quite focused way as a group, and then writing it up"*, in which students moved from being *"vaguely aware of different properties"* to being able to *"actually write down what they think they've learned."*

Epistemological challenges

- The mathematical representations and actions provided by digital computational tools may diverge in important respects from those associated with traditional inscription by hand.
- This calls for mathematical didactical analysis to establish a coherent intellectual framework covering the digital and the traditional, and to establish appropriate curricular sequences
- Equally, many types of digital computational tool are still at a relatively early stage in their evolution, with significant differences of design between alternative tools of similar type, and between successive generations of a particular tool.
- This variability and ephemerality increases the demands made of users, and adds to the complexity of establishing stable mathematical didactical analyses.

Dynamic geometry systems

- Mackrell (2011) found considerable diversity in basic features of the most commonly used dynamic geometry systems:
  - Different repertoires of tools and organisation of them
  - Different styles of interface and modes of interaction
  - Different and inconsistent order of selecting action and object
  - Differing modes of behaviour of figures under dragging
- *“This diversity is an indication that creating [a] program is not simply a matter of representing the conventions of static Euclidean geometry on a screen, but is dependent on the epistemology of the designer and is influenced by both cultural conventions and pedagogical considerations.”*
- Mackrell comments on the limited volume of research on the impact of such design decisions, even for well-known ones such as selection order and the draggability of objects.

Dragging dynamic figures

- Arzarello et al. (2002) identified a variety of ways in which dragging may be used.
- Baccaglini-Frank (2019) argues that *"the discussion is still open on how to link phenomena experienced in a DGE with their interpretations in the Euclidean world."*
- We need a mathematical theory of dragging, plus didactical analyses of full curricular sequences (rather than isolated tasks) which incorporate use of dragging.

- *Wandering dragging*: moving the basic points on the screen randomly, without a plan, in order to discover interesting configurations or regularities in the drawings.

Bound dragging: moving a semi-dragable[2] point (it is already linked to an object).

Guided dragging: dragging the basic points of a drawing in order to give it a particular shape.

Dummy locus dragging: moving a basic point so that the drawing *keeps* a discovered property; the point which is moved follows a path, even if the users do not realise this: the locus is not visible and does not 'speak' to the students, who do not always realise that they are dragging along a locus.

Line dragging: drawing new points along a line in order to keep the regularity of the figure.

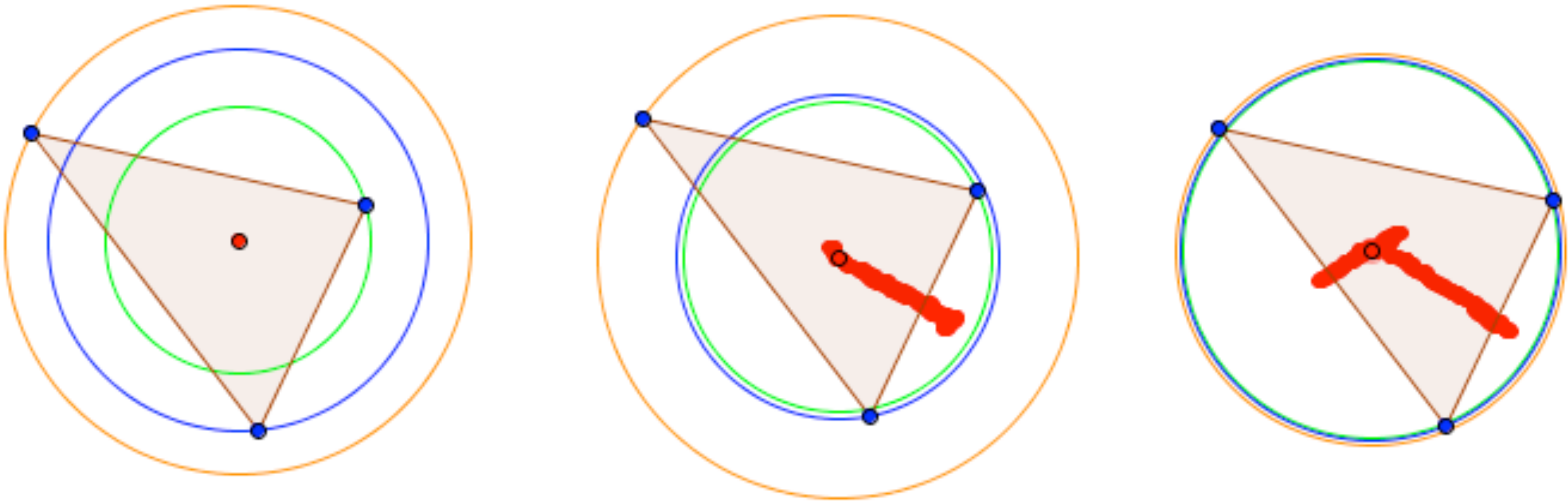
Linked dragging: linking a point to an object and moving it onto that object.

Dragging test: moving dragable or semi-dragable points in order to see whether the drawing keeps the initial properties. If so, then the figure passes the test; if not, the drawing was not constructed according to the geometric properties you wanted it to have.

We have observed that students exploited these different dragging modalities in order to achieve different aims, such as exploring, conjecturing, validating, justifying. For example, *wandering* and *guided dragging* were generally used in the discovering phase, *dummy locus dragging* marked the construction of a conjecture, *dragging test* was mainly used to test a conjecture.

Progression in dynamic geometry scenarios

- Laborde (2001) identified a progression in types of curricular scenario making use of dynamic geometry software:
  1. Facilitates material aspects of familiar task: e.g. constructing a diagram showing the triangle and perpendicular bisectors.
  2. Assists mathematical analysis of familiar task: e.g. through dragging the triangle to identify the concurrence of perpendicular bisectors as an invariant property.
  3. Substantively modifies a familiar task: e.g. through dragging to identify a variable property of the triangle which determines the positioning of the circumcentre as internal or external.
  4. Creates task which could not be posed without dynamic software: e.g. a task in which dragging is used to identify the conditions under which circles with a common free centre through each of the vertices of the triangle are concurrent.



But tools differ in complexity and stability

- Dynamic geometry systems are relatively complex digital computational tools, which radically augment available mathematical representations and actions, and which have not yet achieved stability in design.
- By contrast, the arithmetic calculator is much less complex, employs broadly familiar mathematical representations and actions, and has achieved relative stability in design.
- Its use in primary mathematics was the subject of extensive developmental work through the ‘calculator aware’ number (CAN) project which influenced the English National Curriculum established in 1989 (Shuard et al. 1992)

Curriculum specification for developing proficiency in use of an arithmetic calculator

CALCULATIONS		Using a calculator
Pupils should be taught to:	**As outcomes, Year 5 pupils should, for example:**	**As outcomes, Year 6 pupils should, for example:**
Develop calculator skills and use a calculator effectively	Use, read and write, spelling correctly: *calculator, display, key, enter, clear, constant...* Know how to: • clear the display before starting a calculation; • use the [+], [–], [×] and [÷] keys, the [=] key and decimal point to calculate with realistic data; • change an accidental wrong entry by using the [clear entry] key; • recognise a negative number output; • key in and interpret money calculations: for example, key in £4.35 + £3.85 as 4.35 [+] 3.85 [=], and interpret the outcome of 8.2 as £8.20; key in £6.30 + 85p as 6.3 [+] 0.85 [=], recognising that '0.' signals no pounds and only pence (alternatively, change money to pence and divide final answer by 100 to convert back to pounds); • begin to select the correct key sequence to carry out calculations involving more than one step: for example, 8 × (37 + 58); • know, for example, that a number such as 81.75 lies between 81 and 82; • interpret a rounding error such as 6.9999999 as 7; • have a feel for the approximate size of an answer, and check it by performing the inverse calculation or by clearing and repeating the calculation.	Use, read and write, spelling correctly: *calculator, display, key, enter, clear, constant... recurring* Know how to: • use the [clear] and [clear entry] keys, all operation keys, the [=] key and decimal point, to calculate with realistic data; • recognise a negative number output and use the [sign change] key where appropriate; • key in and interpret the outcome of calculations involving sums of money; • key in fractions, recognise the equivalent decimal form, and use this to compare and order fractions; • read the display of, say, 0.3333333 as 'point three recurring', know that it represents one third, and that 0.6666666 represents two thirds; • start to use the memory and select the correct key sequence to carry out calculations involving more than one operation including brackets: for example, (23 + 41) × (87 + 48); • have a feel for the approximate size of an answer after a calculation, and check it appropriately.

Department for Education and Employment (1999). Framework for teaching mathematics from Reception to Year 6

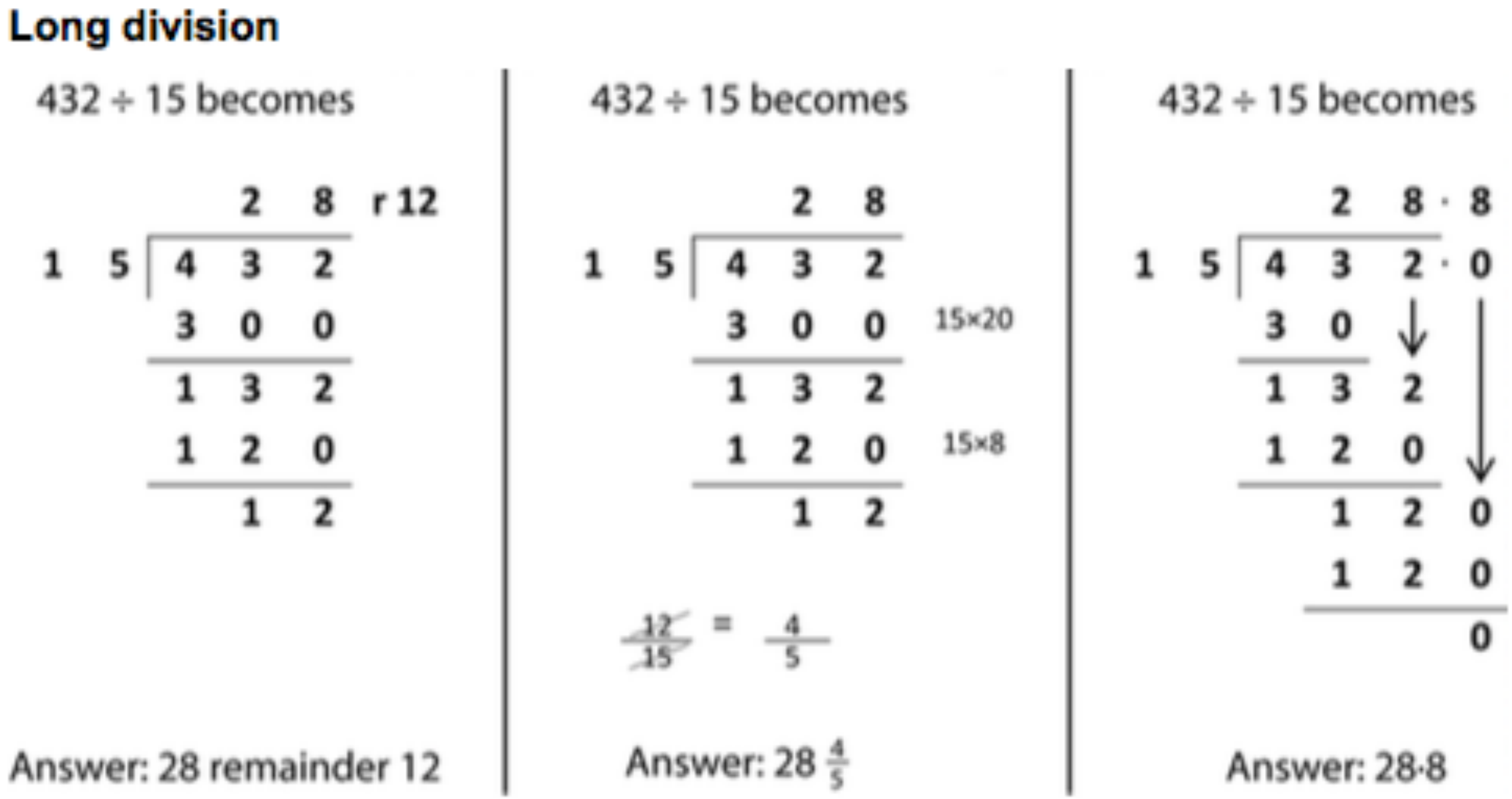
Calculator-aware number curriculum

- Despite apparent congruence with established mathematical operations, introducing the arithmetical calculator to primary school mathematics had implications for curricular sequences:
  - Using, and experimenting with, the calculator led to children encountering negative numbers and decimal fractions much earlier than in the traditional curriculum.
  - Availability of a calculator made it more possible for children to tackle problems using realistic data from their everyday surroundings, and to do so from an early stage.
  - Ease of computation with a calculator made methods of problem solving based on trial and improvement much more feasible.

From ‘calculator aware’ to ‘calculator beware’

- The influence of the CAN project on the English National Curriculum can be seen in its inclusion, from the start, of an evolving section on ‘calculator methods’ (c.f. DfEE 1999).
- More recently, however, such matters have disappeared from an increasingly ‘calculator beware’ curriculum, in which the only reference to calculators is as follows:
  - *Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of key stage 2 to support pupils’ conceptual under-standing and exploration of more complex number problems, if written and mental arithmetic are secure.* (DfE 2013).
- Moreover, while this revised English National Curriculum introduced detailed specification of standard written methods of calculation, it provided no parallel calculator methods.

Written methods of division specified in the English primary mathematics curriculum



Department for Education (2013). Mathematics programmes of study: key stages 1 and 2

Calculator methods of division ***not*** specified in the English primary mathematics curriculum

432 ÷ 15 becomes

432 ÷ 15

28.8

28 x 15

420

432 - 420

12

Answers: 28.8, 28R12, $28^{12}/_{15}$

Transparent but less efficient

432[STO] ÷ 15

28.8

[RCL] - 28 x 15

12

Answers: 28.8, 28R12, $28^{12}/_{15}$

Efficient but less transparent

Existential challenges

- Use of digital computational tools has the potential to (be perceived to) modify or even question established features of school mathematics (often perceived as ‘natural’).
- To the extent that such features are championed, particularly by powerful and influential groups, the introduction of these tools, or development of their use beyond a certain point, is likely to encounter reluctance or more active resistance.
- This process is mediated by the social representations in circulation: the simplifying models through which people make sense of a new and unfamiliar phenomenon by relating it to more established and familiar ones.
- The calculator has become a popular archetype around which prevalent social representations of digital computational tools in school mathematics have formed.

Representations of calculator use in primary school mathematics – an English case study

- From its inception in 1989, the English National Curriculum for Mathematics had a section on "calculator methods" for pupils aged 7 to 11 (alongside much more extensive sections on mental and written methods of calculation).
- One of the three mathematics test papers that children sat at the end of primary school allowed children to use calculators.
- In 2011, the government announced a curriculum review (which led to the use of calculators being removed).
- This provides a useful case for study of social representations of calculator use in primary school mathematics based on:
  - Statements by politicians: i.e. the responsible schools ministers
  - Comments by members of the public in a discussion on a newspaper comment board (Ruthven 2014)

Representations of calculator use in primary school mathematics in ministerial press releases

- Below are key statements from 2011 press releases from the two ministers for schools over the ensuing period:
  - *Children can become too dependent on calculators if they use them at too young an age. They shouldn't be reaching for a gadget every time they need to do a simple sum. They need to master addition, subtraction, times tables and division, using quick, reliable written methods. This rigour provides the groundwork for the more difficult maths they will come across later in their education.*
  - *We should ensure that schools equip children with the mathematical basics that allow them to succeed in life. We are in danger of producing a 'Sat-Nav' generation of students overly reliant on technology.*

Representations of calculator use in primary school mathematics on a news comment board

- Many comments depict the use of calculators by pupils as antagonistic to thought and subversive of intelligence:
  - *One of the most important things that a child learns is the ability to think. If you give them a tool that discourages that at such a young age, that aspect of their thinking will be stunted.*
  - *A child's mind needs exercise just as their body does.*
  - *Nothing clever about using a calculator to work out numbers.*
- Some portray use of calculators not only as developmentally debilitating but as a morally iniquitous avoidance of effort:
  - *Using a calculator… rots the brain, not to mention the poor ethic it instils… if they don't work out the answers with hard graft.*
  - *Going straight for the answer is the easy, cheap and wrong way to go.*

Representations of calculator use in primary school mathematics on a news comment board

- Where contributions concede that using a calculator does involve a degree of expertise, this tends to be presented as distinct from mathematics itself:
  - *Learning to work a calculator is only learning to work a calculator, not learning how to do maths.*
- A common suggestion is that access to calculators should be granted pupils only once they have become confident with number and proficient in mental or written calculation:
  - *They should learn how basic arithmetic works first, which means doing it either in their head or on paper.*
  - *They should be taught to use them, after they have developed and demonstrated a sufficient mathematical understanding to actually benefit from the use of a calculator.*

Narratives of calculator (dis)identification on a news comment board

- Some comments are salutary in showing how opposition to calculators is embedded in contributors' sense of personal worth, grounded in their own educational experiences.
- One such identity narrative from a contributor conveys a sense of personal accomplishment associated with mastery of mental and written calculation, expressed in a continuing proud refusal to use a calculator:
  - *I learned arithmetic the old-fashioned way, using a sums book, following the methods demonstrated by the teacher on the blackboard. By six, I could add and subtract up to a hundred, by eight I had long division and multiplication, and all the tables to ten… My mathematical skills took me all the way through A level into degree-level statistics, and then a (boring) first job in Health Service data analysis. I have never owned a stand-alone calculator, and I don't use the one in MacOs X.*

A narrative of calculator identification on a news comment board

- Another (atypical) identity narrative conjures up a very different type of personal history, offering a sense of how, for some pupils at least, calculators serve as a catalyst for developing interest and capability with numbers:
  - *I am really good at mental arithmetic, but as a child abhorred rote learning of times tables, couldn't see the point as I could work them out in an instant. It almost alienated me completely from maths, luckily playing with calculators… rekindled my interest in number games. So when I was older and scientific calculators starting coming in… I used to play with it, especially the functions that worked out means and standard deviations. That set me up well for the types of maths I used in later life, inferential statistics.*

How dominant representations devalorise the use of digital computational tools

- Each strand of these dominant popular representations devalorises use of digital computational tools:
  - **Cognitive self-sufficiency:** thinking ‘independent’ of digital tools *versus* unthinking ‘dependence’ or ‘over-reliance’ on such tools.
  - **Mathematical essence:** ‘purely’ mathematical mental/written methods *versus* (wholly/partially) ‘non-mathematical’ use of digital tools.
  - **Moral virtue:** ‘effortful’ use of ‘rigorous’ mental/written methods *versus* ‘lazy’ recourse to ‘slipshod’ use of digital tools.
  - **Epistemic value:** use of mental/written methods taken as exercising intelligence and developing understanding *versus* use of digital tools taken as doing neither.

Aspects of challenge

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Your thoughts?

- Are these same challenges also relevant for Asian countries?

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