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Towards a calculator-aware number curriculum

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ABSTRACT**:** *While many current position papers and policy documents advocate a place for the use of calculators in elementary education, and point to their potential to enhance mathematical learning, there is continuing professional uncertainty over how to realise such aspirations. This paper reviews a pioneering effort to craft a 'calculator-aware' number curriculum. It examines pedagogical strategies based on* Diagnose-Explain-Reinforce *and* Observe-Predict-Surpass *sequences, and the wider use of calculators for purposes of* Computation-Implementing, Result-Checking, Trial-Improving *and* Structure-Modelling. *The paper then reports on research which examined the long-term impact of such a curriculum. It identifies the complexities of supporting pupils' development of personal methods of calculation, and of systematising such a curriculum to support progression in children's learning. The major long-term impact of the curriculum was on pupils' attitude to mental calculation and proficiency in it. Pupils proved more prone to calculate mentally, and more liable to adopt relatively powerful and efficient strategies. Analysis of pupils' calculator use in tackling a realistic number problem shows how effective use of the machine calls not only for mastery of operating procedures but a grasp of underlying mathematical ideas and the development of distinctive calculator methods. Finally, implications for policy and practice are suggested, emphasising that a calculator-aware approach cannot simply be improvised around a conventional curriculum.*

Key words: *Arithmetic calculators, calculation strategies, elementary school, mathematics education, number curriculum, student thinking, teaching approaches.*

POSITIONING CALCULATORS IN THE CURRICULUM

Professional bodies, such as the North American National Council of Teachers of Mathematics (NCTM), have long advocated the integration of calculators into school mathematics programmes at all grade levels, arguing that "appropriate instruction that includes calculators can extend students' understanding of mathematics and will allow all students access to rich problem-solving experiences" (NCTM, 1998). Indeed, this position can be seen as a corollary of the broader Technology Principle enunciated in the influential *Principles and Standards for School Mathematics* (NCTM, 2000), which propose that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). Nevertheless, as contributions to a focus issue of that organisation's *Dialogues* (NCTM, 1999) vividly illustrate, argument continues over the place of calculators, particularly in the elementary school classroom. The debate centres on the relationship between a range of modes of calculation –including standard written methods and informal mental strategies– and their contribution –both direct and indirect– to the development of wider mathematical proficiency.

Prominent concerns surrounding calculator use relate to students' 'reliance' or 'dependence' on technology, degenerating into 'mindless button pushing' (Mackey, 1999: p.3), serving as 'a substitute for thinking' (quoted in Mackey, 1999: p.3). Extending this theme of cognitive impoverishment, critics suggest that calculator use has led school mathematics to shift attention from '*methods* to… *answers*' (quoted in Mackey, 1999: p.3), and to neglect the contribution of written algorithms in developing mathematical concepts (Klein & Milgram, undated: p.2). Such ideas frame a widespread view that 'calculators should be used only after students ha[ve] learned how to do the relevant mathematics without them' (Ballheim, 1999: p.4). They can then be represented 'as powerful tools when used appropriately', particularly where 'problem solving is the main focus', because they 'relieve students of cumbersome computation, allowing them to concentrate on more meaningful mathematical activities' (Ballheim, 1999: p.4).

In response to such controversies, advocacy of calculator use in school mathematics has become more muted. As now articulated (NCTM, 2005), it interweaves several strands. The overarching theme is one of 'appropriate use of the calculator' as part of 'a balanced mathematics program' which develops 'basic mathematical understandings'. The position appeals first to the way in which 'technology pervades the world outside school', so that 'students will be expected to use calculators in other settings'. It then emphasises that 'teachers can capitalize on the appropriate use of this technology to expand students' mathematical understanding, not to replace it'. Finally, it asserts the continuing importance of 'written mathematical procedures' and 'estimation and mental math', arguing that these latter mental skills are 'essential both for understanding numbers and because of their usefulness outside school'.

Such responses do not confront the basic charge that calculator use in itself is cognitively impoverished, even if they represent use of the technology as mathematically enriching in other forms. This paper will examine what policy-makers and practitioners might learn in this respect from pioneering work to develop a 'calculator-aware' number curriculum (Shuard, Walsh, Goodwin & Worcester, 1991; Shuard, 1992), and from a subsequent study of its long-term influence on pupil attitude and attainment (Ruthven, Rousham & Chaplin, 1997; Ruthven 1998, 2001).

CRAFTING A 'CALCULATOR-AWARE' NUMBER CURRICULUM

A project team, aiming to create a 'calculator-aware' number (CAN) curriculum, worked in collaboration with teachers in four clusters of primary schools in England and Wales to develop a new approach to the teaching of number, based on the following principles (Shuard et al. 1991: p. 7):

- classroom activities should be practical and investigational, emphasising the development and use of language, and ranging across the whole curriculum;
- encouragement should be given to exploring and investigating 'how numbers work';
- the importance of mental calculation should be emphasised; children should be encouraged to share their methods with others;
- children should always have a calculator available; the choice as to whether to use it should be the child's not the teacher's; and
- traditional written methods of calculation should not be taught; children should use a calculator for those calculations which they could not do mentally.

This was a 'calculator-aware' number curriculum in several sense. First, as the last two principles indicate, it acknowledged the way in which electronic calculation devices were rapidly displacing written methods outside school. Correspondingly, within project schools, electronic calculators were to replace written methods as the computational means of secure resort. Given the degree to which the established number curriculum centred on the development of standard written methods of calculation, this was a radical change, not simply removing content and releasing time, but removing a major organising strand of the curriculum. Equally –as the first two principles indicate– this new curriculum recognised that numeracy involves exercising number sense as well as effecting numeric calculation. Here, the curriculum was 'calculator-aware' in the further sense of exploiting use of calculators to stimulate and support children's exploration of properties of number (as will be illustrated below). Finally, these varied concerns were drawn together –as the middle principle indicates– around a curricular strand which focused on comparing and refining children's strategies of mental –and informal written– calculation, seen as a means by which number concepts could be actively developed. This, then, was a 'calculator-aware' number curriculum, but not a 'calculator-based' one. Indeed, in some senses, the calculator could be said to have acted more as catalyst than central agent: underwriting the shift in emphasis from written to mental calculation, and assisting the shift from a pedagogy of instruction towards a pedagogy of investigation.

This type of approach through supported investigation was not without its critics:

> For example, 8-9 year-old pupils are to 'explore number patterns' (sometimes as simple as … 91, 81, 71, 61, …) When using calculators, pupils contribute to the process of finding the result hardly more than if it were dictated to them by the teacher. The underlying process is totally concealed from them, and the calculator does not help pupils to understand why the result is true (Bierhoff, 1996: 38-39).

However, what is crucial in such activities is the way in which the calculator is used to structure the task, so shaping the strategies available to pupils so as to match learning objectives. Pupils could extend the pattern 91, 81, 71, 61 either by visualising and following a symbol pattern: *9*1, *8*1, *7*1, *6*1; or by verbalising and following a name pattern: *ninety* one, *eighty* one, *seventy* one, *sixty* one. Tasked to set the constant function on their calculator to produce the pattern, however, pupils are obliged to conceptualise it in terms of a repeated number operation. Framed in this way, such a task can help the teacher to diagnose the extent to which pupils have grasped the mathematical principles underpinning name and symbol patterns. If necessary, the task then provides a context in which such principles can be explained. Finally, further tasks of similar type can be used to reinforce ideas, or diagnose where further teaching is needed. This exemplifies a *Diagnose-Explain-Reinforce* sequence of calculator use.

Another appropriate way of using the calculator is through an *Observe-Predict-Surpass* sequence. Suppose we wish to move pupils on from the inefficient and unreliable process of using counting-on as a method of adding 10 to a number. First pupils analyse what happens when they use the calculator to add 10 to different numbers (with the machine employed to bypass their use of the count-on procedure, so avoiding rehearsing and reinforcing it); they then predict the results of machine calculations (so shifting the function of the calculator towards that of checking their mental calculations); and finally they try to surpass the calculator, through developing sufficient speed and accuracy in the curtailed mental strategy to be able to beat someone using the machine.

Used in such ways, the calculator can be the key to structuring an activity to produce effective learning. Analysis of the range of experiences reported from the CAN project suggests that the contribution made by calculators can be understood in terms of four ideal types of use, which will be illustrated and examined in turn

Computation-Implementing is using a calculator to carry out a calculation which has already been formulated. Such use of calculators enabled children in the CAN project to carry out –and to envisage carrying out– calculations which would not otherwise have been feasible for them; for example, single calculations involving large numbers, or multiple calculations, notably those arising from real problem situations or in investigating number patterns.

Result-Checking is using a calculator to review a calculation which has already been carried out. Where the calculation has been done mentally, the calculator provides a means by which a pupil can gain rapid feedback, on the direct result, but also indirectly on their mental strategy. Second, where a calculation has been executed on the calculator, there are important alternatives to simply repeating the same calculation. One is to reverse the calculation, working back from the result to the original data –for example, when a multiplication has been carried out, applying the inverse division. Another alternative is to reformulate the calculation –for example, repeated addition as multiplication. Checking, then, has the potential to support learning about the equivalence of variant calculations, so developing ideas about the structure of number operations.

By reducing the ‘effort’ of calculation, calculators encourage more spontaneous and speculative calculation. *Trial-Improving* is using the calculator to solve a problem by experimenting with some calculation scheme until a solution is found. Indeed, a general strategy widely used in the CAN project involved proposing a speculative solution to a problem –by guessing or estimating– then testing it –through some appropriate calculation– against the condition it must satisfy. Then, in the light of the information that the user is able to extract from this feedback –about the nature of the ‘gap’ between the actual result and the one sought– the proposed solution is revised and retested. This cycle continues until a satisfactory solution is reached. An example from the early primary years is of a child trying to find what number to subtract from 67 in order to arrive at 18; estimating 40 and computing 67 – 40 on the calculator to get 27; then revising the estimate upwards (Shuard et al., 1991: p. 71).

Structure-Modelling is using the calculator to carry out calculations so as to exemplify some aspect of the operation of number and calculation and support learning about it. For example, in response to an early primary pupil who has written two thousand and ten as 200010, a teacher uses a calculator to show her 2000 + 6 = 2006 and 2000 + 13 = 2013; the pupil herself then carries out further addition calculations –such as 2000 + 17 = 2017– before progressing to entering numbers directly –such as 2039 (Shuard et al., 1991: p. 13). Similarly, in response to the calculation 1 ÷ 4 = 0.25, a pupil speculates that ‘a quarter is 0.25’, and then follows this up with 0.25 + 0.25 + 0.25 + 0.25 = 1 and then 0.25 x 4 = 1 (Shuard et al., 1991: p. 21). There is a sense, of course, in which this is a form of checking. However, what distinguishes modelling from the other ideal types is the guiding concern with seeking meaning in, and establishing knowledge of, number and calculation. It is not the calculation itself which is of primary interest, but the mathematical ideas, principles and processes underlying it, as interpretable in the operation of the calculator.

SYSTEMATISING THE CURRICULUM DESIGN

The tangible outcomes of the CAN project were recorded in a text and video prepared by the project team (Shuard et al., 1991). These illustrate the curriculum principles presented above through a collage of classroom activities and accounts. The project team were able to draw on examples from the earlier years of primary school, but not the later years since the cohorts of children involved in the project had not yet reached that stage. A more structured curriculum plan was not developed. This reflected the exploratory and investigative pedagogical approach embraced within the project.

The CAN project drew to a close as a national curriculum came into force for the first time, soon followed by a system of national assessment. According to teachers in former project schools, interviewed after the first full cycle of this new system, the major influence of the reforms was threefold (Ruthven, 2001). First, some of the expansiveness of investigative work had gone, and there was a stronger tendency to structure and foreclose an activity than in the past. Second, although calculators continued to be readily available in the classroom, there were occasions when their use was challenged or proscribed. Third, standard written methods of recording and calculating had been reintroduced. Generally, however, the tenor of teachers' accounts was of seeking to retain valued CAN principles and activities, to establish the legitimacy of these within the new order, and to tighten aspects of their implementation.

Nevertheless, the teachers also pointed to pedagogical tensions arising within CAN. In supporting pupils' development of methods of calculation, they were conscious of having to manage an important tension between pupils' personal insight and authenticity on the one hand, and computational accuracy and efficiency on the other. Another issue was the uncertainty and effort arising from the abandonment of a conventional mathematics scheme, with limited alternative means of support. In these circumstances, it was difficult to plan for continuity and progression in children's learning, both from lesson to lesson and from year to year. One important effect of the curriculum and assessment reforms, then, was to press schools to develop a more systematic approach to number, building on the national frameworks.

ASSESSING LONG-TERM INFLUENCE ON PUPILS

Following the CAN project, favourable findings were reported from one of the participating clusters of schools, in which the mathematical achievement of the first cohort of project pupils was compared with that of peers in other schools (Shuard et al., 1991: pp. 59-60). The following year, the second cohort was involved in a similar comparison, with results again favourable to the project pupils, but less markedly so (Foxman, 1996: p. 47). Of course, the tests used did not seek to assess facility with the standard written methods of calculation which had been a major focus of the mathematics curriculum in the comparison schools. Indeed, as the CAN project team scrupulously pointed out, qualifying their reporting of favourable findings, 'it would not be possible to equate the conditions in project schools and control schools, and this kind of quantitative evaluation might be misleading' (Shuard et al., 1991: p. 55).

Our later study (Ruthven, Rousham & Chaplin, 1997) investigated the long-term influence of a calculator-aware number curriculum on a cohort of pupils entering schools after the end of the CAN project, once the exceptional conditions of project support had been removed. The study compared the attitude and attainment of pupils in the post-project schools with matched non-project schools. National assessment levels awarded at ages 7 and 11 were analysed, to determine whether the odds of high or low attainment in mathematics differed between schools, after taking account of the general scholastic attainment of pupils. At age 7, the odds of high mathematics attainment were found to be significantly greater in the post-project schools, as *also* were the odds of low mathematics attainment. In the post-project schools, then, pupils were more likely to be found at either extreme of the attainment distribution. Comments from teachers in post-project schools suggested that a plausible explanation was that their emphasis on investigative and problem-solving tasks produced greater differentiation of experience between pupils, creating higher expectations of, and greater challenges for, successful pupils, but providing less systematic teacher intervention to structure and support the learning of pupils who were making poor progress. However, this differential pattern did not persist through to the results at age 11, where no substantial differences were found between post-project and non-project schools, either on national assessments or on specially devised measures focusing on a range of number concepts. Similarly, no differences were found in reported enjoyment of number work. However, pupils in the post-project schools tended to rate mental calculation more positively than pupils in non-project schools.

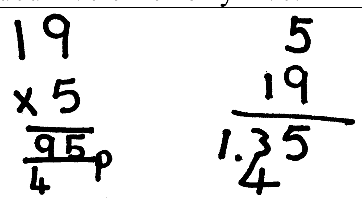
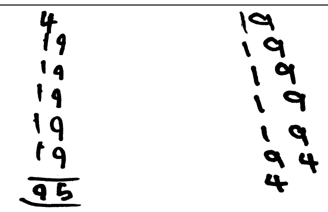
Analysis of the strategies used by a structured sample of pupils in tackling a set of number problems strengthened these findings (Ruthven, 1998). Pupils in post-project schools proved more prone to calculate mentally. Whereas 38% of pupils in post-project schools tackled all the problems mentally without any use of written or calculator computation, only 19% of pupils in non-project schools did so. Whereas only 24% of pupils in post-project schools used written or calculator computation on more than one occasion, 52% of pupils in non-project schools did so.

Pupils in post-project schools also proved more liable to adopt relatively powerful and efficient strategies of mental calculation. For example, in response to the problem of calculating the cost of five 19p stamps (Table 1), the more powerful mental strategies involved distribution and sometimes compensation. Whereas 55% of pupils in the post-project schools used a mental strategy of this type to tackle the problem, only 22% of those in the non-project schools did so.

It is plausible to see these outcomes as reflecting the contrasting numeracy cultures of the two groups of schools. In the post-project schools, pupils had been encouraged to develop and refine informal methods of mental calculation from an early age; they had been explicitly taught mental methods based on ‘smashing up’ or ‘breaking down’ numbers into component parts; and they had been expected to behave responsibly in regulating their use of calculators to complement these mental methods. In the non-project schools, daily experience of ‘quickfire calculation’ had offered pupils a model of mental calculation as something to be done quickly or abandoned; explicit teaching of calculation had emphasised approved written methods; and pupils had little experience of regulating their own use of calculators.

Table 1

Incidence of strategies for the Stamps *problem: Number of pupils in structured sample using specified strategy*

Strategy	**Examples**	**Post-project**	**Non-project**
Mental Distribution	<u>Distribution as 10+9</u>: "Times ten is fifty. Times nine is forty five. Add them together." <u>Distribution as 10+10-1</u>: "Five tens are fifty. Count nine as ten so you've got a hundred. Then take away five." <u>Distribution as 20-1</u>: "Five twenties which was one pound. Then I took away five."	16	6
Other Mental	<u>Cumulation by 5</u>: "Nineteen times five. Five, ten, fifteen like that." <u>Cumulation by 19</u>: Counts on using fingers. Records 19, 38, 52, 66, 80. <u>Digit Multiplication</u>: "If you have nine fives that would be forty five. Add the one so I added five onto forty five."	3	5
Written Column Multiplication		1	7
Written Column Addition		3	1
Calculator Multiplication	Keys [19] [x] [5] [=] 95 Keys [5] [x] [19] [=] 95	7	9
Pupils in group	*One pupil in each group used more than one strategy*	29	27

ANALYSING CALCULATOR-MEDIATED STRATEGIES

The CAN project put into action the idea that: 'With mental methods… as the principal means for doing simple calculations… calculators… are the sensible tool for difficult calculations, the ideal complement to mental arithmetic' (Plunkett, 1979: p. 5). However, as noted earlier, national reforms led to a weakening of this position in post-project schools, particularly once national assessment framed problems in terms of standard written methods, or required pupils to show working, or barred use of calculators. Consequently, the pupils in this follow-up study had not experienced such a strong emphasis on developing expertise in using calculators. This was illuminated by analysis of pupils' responses to a further number problem (Ruthven & Chaplin, 1997).

The 'coach problem' was a close variant of an example in the national curriculum: *313 people are going on a coach trip. Each coach can carry up to 42 passengers. How many coaches will be needed? How many spare places will be left on the coaches.* Pupils were told that they could work out the problem however they liked; in their head, using pen and paper, or calculator, or a mixture of these. Around 60% of pupils made some use of a calculator, and three broad types of calculator-mediated strategy were found: direct-division, repeated-addition and trial-multiplication. Each of these gave insights into forms of expertise which pupils need to develop in order to use calculators effectively.

Table 2

Calculator-mediated direct-division strategies for the Coach *problem*

KAREN'S RESPONSE	**DAMON'S RESPONSE**
Karen keys [313][÷][42][=]7.452380952 Karen: Whoopsee! Interviewer: What have you got? Karen: I've got loads of numbers. Interviewer: Are they any good to you? Karen: No Interviewer: Why? Karen: I don't know Interviewer: Can you understand what they say? Karen shakes her head Interviewer: Okay. [pause] Karen rekeys [313][÷][42][=]7.452380952 [pause] Karen keys [42][÷][313][=]0.1341853035	Damon keys [313][÷][42][=]7.452380952 Interviewer: What have you got? Any good? Damon: About seven coaches. Interviewer: About seven coaches. [pause] Damon: I think it's four. Interviewer: Four. Damon: Yeah. Interviewer: Spare places? Damon: Yeah. Interviewer: How did you work that bit out? Damon: Because it's seven point four.

The most common use of a calculator was for direct-division. The responses of Karen and Damon (Table 2) exemplify widespread features of such responses. It seems that Karen's initial interpretation of the string of digits on the calculator display is that she has miskeyed; so she checks by rekeying. When this produces the same result, it appears that her next interpretation is that she has entered the numbers in the wrong order within the calculation; her checking shifts towards trialling. Such responses reflected an expectation that the result of a division should be a whole number. It is not just that the commonsense of the problem points in this direction. For pupils, mental and written division were processes within the whole numbers, yielding a quotient and possibly a remainder; whereas the calculator treats division as a process within the extended number system incorporating decimals. Equally, pupils' contact with decimals had been predominantly in terms of money and measures. Karen did not recognise the string of digits as a decimal. And Damon interpreted it as a remainder. These examples highlight the special character of calculator division. Indeed, carefully designed calculator-mediated tasks can support development of pupils' understanding of relationships between division, fraction and decimals; for example, by investigating which division calculations give the same decimal part (van den Brink, 1993).

Table 3

Calculator-mediated repeated-addition strategies for the Coach *problem*

LIAM'S RESPONSE	**KATH'S RESPONSE**
Liam: So you need to add up how many forty twos go into. I'll do that. I'm sure you could do it a quicker way but, well. Liam keys [42][+] [42][+] [42][+] [42][+] [42][+] [42][+] monitoring intermediate totals Liam keys [252][+] Liam: Oh no! Interviewer: Where have you got to? What's happened? Liam: Hmmm. Don't know.	Kath: 42 times Kath keys [42][x][=]1764 Kath rekeys [42][x][=]1764 Kath: I thought if you could do forty two times and then equals, it should keep going, forty two, eighty four like that and say how many forty twos to get up to that.

Another use of the calculator was for repeated-addition. Liam's example (Table 3) is typical, both in its keying pattern and in its eventual breakdown. The calculator leaves no trace of intermediate results, making any extended calculation incorporating a parallel mental computation extremely vulnerable to failure through miskeying or losing track of where the calculation has reached. Pupils who tried to compute mentally without recording had similar difficulties. In both cases, maintaining some form of written record provides an important means of augmenting working memory.

Alternatively, use of the calculator constant function offers a way of effecting repeated computations of this type. Kath was the only pupil who attempted this (Table 3). She knew that she wanted to repeat an operation, she knew how to get the calculator to do that, she knew that she wanted the multiples of 42, but misconstrued this as a matter of repeated multiplication rather than repeated addition.

Table 4

Calculator-mediated trial-and-improvement strategies for the Coach *problem*

JOANNE'S RESPONSE	**NICKI'S RESPONSE**
Joanne keys [42][x][12][=]504 Interviewer: Why did you do that? Joanne: Forty two times any number, but it was a bit too high. Joanne keys [42][x][10][=]420 Joanne: Forty two times ten, that's too high so.. Joanne keys [42][x][8][=]336 [pause] Joanne: They'd need eight coaches, and they'd have.. [pause] Joanne: Twenty three places left over.	Nicki keys [313][÷][5][=]62.6 Nicki: Fifty two. Nicki keys [313][÷][7][=]44.71428571 Interviewer: Tell me why you're choosing these numbers. Why did you just do five and now you've just done seven. Nicki: Well, five there were fifty two and that was too many, and so I tried seven. Interviewer: Why? What are the five and the seven about? Nicki: How many coaches. Nicki: Eight now. Nicki keys [313][÷][8][=]39.125 Nicki: Eight and lots of seats left over.

A final use of the calculator was for trial multiplication, normally taking an estimate of 7 from direct-division and keying [42][x][7][=]294, and often then calculating –usually mentally– that 294 lay 19 short of 313. However, the typical interpretation of these findings was that 7 coaches were required with 19 spare places –reflecting a misconceived association between 'remainder' in the calculation and places 'left' in the problem. The only successful use of trial-multiplication, by Joanne (Table 4), took a rather different form, since she embarked on it immediately as her opening strategy, rather than following on from direct-division. Using the machine to carry out computations in a predictably routinised way, Joanne freed her attention to monitor her strategy and interpret results. And this devolution of computation was systematic, even extending to multiplying 42 by 10, something which Joanne was very capable of doing mentally; (earlier in the interview she had successfully mentally multiplied 24 by 10, answering within one second). Nicki (Table 4) also used a trial-and-improvement strategy from the start, similarly devolving calculation to the machine. This enabled her to work with an unusual representation of the problem, in which she focused on the shared number of passengers per coach, employing trial-division. This example also brings out another important feature of trialling strategies; that they are disposed to be self-correcting; Nicki's misreading of 62 is not critical because it is quickly superseded by the next trial.

As in all activities concerned with 'using and applying mathematics', pupils' work on this problem highlights mathematical topics which would benefit from more focused teaching. Indeed, discussion of the problem itself, and of strategies adopted by pupils, provides a good springboard for such work. For example, suitably recorded, Liam's repeated addition and then Joanne's trial multiplication provide the basis for developing a written technique of division. These examples also show that effective use of a calculator calls not only for mastery of operating procedures –such as use of the constant function– but a grasp of underlying mathematical ideas –such as the distinction between decimal part and remainder– and the development of distinctive calculator methods –such as that of integer division. These episodes also illustrate how access to a calculator can enable pupils to tackle a problem using direct strategies calling for computations beyond their current capabilities in mental and written calculation; and can support indirect strategies based on trialling or building up towards a solution.

FRAMING POLICY AND PRACTICE

Using a calculator is far from being the unthinking process of popular repute. It is a matter not simply of operating the machine, but of formulating computations and interpreting their results, as expressed in the specific terms of the device. In the case of division, this involves developing an understanding of the relationship of other forms of the operation to the particular form carried out by the machine, and of different forms of number representation. Moreover, when more complex sequences of computation are carried out, structuring these and recording their results may play an important part in effecting and interpreting the calculation with success. From this perspective, a calculator does more than simply execute computations; it mediates them. Consequently, a calculator-aware number curriculum needs to plan for the development of appropriate expertise in calculator use for specific mathematical purposes; not assume that little expertise is involved, or that pupils will pick it up informally.

A parallel can be drawn with the way in which long division is presented as a capstone of the traditional elementary number curriculum; not just as a crowning achievement in column arithmetic towards which pupils aim, but as a curricular organiser drawing on – and so having the potential to draw together– many important curricular strands. Both pedagogically and politically, one of the weaknesses of CAN may have been the absence of a corresponding calculator-mediated procedure to act as crowning achievement and curricular organiser. Elsewhere I have outlined how co-ordinated empty-number-lines provide a means of recording the whole range of informal strategies for quotient-and-remainder division –mental, calculator and hybrid (Ruthven 2001). This evolving scheme has the potential to support a learning trajectory in which informal strategies are structured, curtailed, reorganised and refined, leading eventually to an efficient and systematic calculator-mediated method of quotient-and-remainder division.

A broader lesson is that a calculator-aware number curriculum is much more than a conventional number curriculum with calculator use 'bolted on'. Nor is it a wholly 'calculator-based' one. While calculators replace standard written methods as the computational means of secure resort, children's strategies of mental and written calculation retain significance as means by which number concepts can be developed. Equally, various forms of calculator use are employed to stimulate and support children's exploration of properties of number. Here again, such an approach requires careful planning, particularly of curriculum sequences to underpin continuity and progression in children's learning; it cannot simply be improvised around a conventional curriculum.

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