The construct of 'resource system' as an analytic tool in understanding the work of teaching

Kenneth Ruthven

Abstract

This lecture examines professionally situated notions of 'resource system' relevant to the work of teaching, giving specific attention to mathematics teaching. Two historically significant exemplars are examined in the form of Euclid's *Elements* as a systematic logical organisation of resources and Durell's *A New Geometry* as a systematic didactical organisation of resources. Noting a subsequent shift towards the use of multi-sourced collections of resources the lecture examines how teachers create organised systems, considering the evolving notions of 'resource system' in two contemporary theoretical frames: Structuring Features of Classroom Practice (Ruthven 2009) and the Documentational Approach (Gueudet and Trouche 2009). Different perspectives situate 'resource system' in contrasting ways: as adhering to a particular type of agent – teacher, student, designer – or as intervening between such agents; as relating to a specific educational entity - especially the classroom, the course or the lesson – or as ranging across and beyond these. Professionals and researchers have clearly found each of these variations useful for some purpose: an implication is that we could benefit from an expanded notion of 'resource system' which acknowledges all these dimensions and encourages users of the term to take more explicit account of them.

Keywords

Documentational approach, Durell's *A New Geometry*, Euclid's *Elements*, Mathematics teaching, Resource system, Structuring Features of Classroom Practice

Notions of 'resource' and 'system'

When researchers use notions of 'resource' and 'system' – and join them together to form 'resource system' – they appeal to ideas already established in ordinary language and in the professional discourse of teaching. In due course this lecture will consider particular notions that educational researchers have developed in recent years. First, however, it is important to examine the everyday and professional usages of these terms, in which – by virtue of such choice of words – researchers ground their own specialised meanings.

In established everyday usage, a resource is an asset – typically monetary, material or human – capable of providing some form of support. In the field of education (as the Oxford English Dictionary records) a specialised usage of 'resource' developed during the 1960s, referring specifically to curriculum-related materials intended to support learning or teaching activity. This specialised usage remains predominant in the professional field, and it provides the focus for this lecture. Nevertheless, in theorising the notion within the research field there has been some reversion towards the more general usage in recognising a much wider range of human and cultural – as well as material – assets as resources for teaching and learning about a topic (Adler 2000), as this lecture will acknowledge where appropriate.

The professionally specific usage of 'resources' to refer to curriculum-related materials arose in response to technological changes – notably increasing provision of audio-visual and reprographic facilities – which broadened the range of media in which such materials could be created and facilitated their local production and reproduction. Indeed the institutionalisation of this trend was marked by a key educational site being renamed: as its functionality was reconceived, the traditional library became the modern 'resource centre' (Beswick 1974). This space now accommodated resources in a more diverse range of media – notably audio-visual materials as well as printed texts. Moreover, it catered for an expanded pedagogical repertoire. In particular, by allowing a user to select from – and make copies of – a varied stock of 'curricular', 'learning' or 'teaching' resources, it made possible forms of 'resource-based' learning and teaching involving more active curriculum design by teachers and more independent study by pupils (Graystone 1978). Over recent years, the advance of computer-based information and communication technologies has produced a further shift towards accessing such resources online in digital form, with the role of web portals and internet repositories growing correspondingly (Recker et al. 2004; de los Arcos 2016).

The central idea of a 'system' is one of organisation: the term may refer to some structure resulting from multiple entities being organised to form a functioning whole, or to some scheme or method which provides a basis for such organisation. Within the professional field, two corresponding notions of 'resource system' have developed. One usage – expanding the traditional notion of textbook – refers to a systematic curriculum scheme created through combining diverse resources to form a comprehensive programme (Gillespie and Humphreys 1970). Another usage – expanding the traditional notion of library – refers to organising and cataloguing a resource repository systematically so as to make its contents readily searchable and usable (Zhao et al. 1996).

This lecture will make reference to both of these notions of 'resource system'. Nowadays, indeed, the distinction between the two has become blurred. In particular, there is a growing tendency to regard any text as just one source amongst many, providing a collection of smaller resource units to be raided and combined with others. But this is to disregard the systematic way in which a text seeks to organise these many resource units to provide a coherent whole. To illustrate this point, I will examine two illuminating historical exemplars of the text as resource system *avant la lettre*.

Euclid's *Elements* as a systematic logical organisation of resources

In the history of teaching and learning mathematics, one resource towers above all others in terms of its longevity and influence. The *Elements* of Euclid was created around 300 BCE and was subsequently annotated and adapted by others in more than a thousand editions. In one form or another, The *Elements* was widely studied until the early years of the twentieth century. Euclid's achievement was to combine and adapt mathematical sources already available to produce what was taken to be a comprehensive and coherent text providing a logically systematic exposition of a core of classical mathematics. Thanks to the remarkable Library of Alexandria, Euclid was able to draw on disparate mathematical texts from across the ancient world in compiling the *Elements*; texts created variously by Pythagoras, Hippocrates, Eudoxus, Theaetetus – father and son – and many others (Rouse Ball 1908). The result provides a prime example of Hilbert's nostrum that the importance of a great book is determined "by the number of previous publications it makes superfluous to read" (Brock 1975).

Nevertheless, there is a sense in which there are as many *Elements* as there are editions. Not only, in the early days, did copyists introduce inadvertent changes, but – much more significantly – later translators and editors created *Elements* which accorded variously with their conception of rigorous argument, their favoured didactical approach, or their image of Greek mathematics (Barrow-Green 2006; Chemla 2012). Amongst other things, they selected from different source editions, reorganised the sequencing of material, filled out perceived gaps in argument, modified the presentation of figures, and introduced new diagrammatic conventions. Thus, what is taken as constituting the *Elements* has been shaped not just by Euclid's original selection and organisation of resources, but by a continuing process of interpretation and adaptation.

However, despite this ongoing recreation of the *Elements*, it is possible to pick out some key features generally regarded as forming its core. The overarching organising principle of the *Elements* is one of logical deduction. From a base of 'definitions' of geometrical entities and of axioms taken to be self-evident – either in the form of 'postulates' about geometrical entities or of 'common notions' about magnitudes – the *Elements* derives a succession of 'propositions'. These propositions are numbered in sequence and organised thematically into 'books', providing a global structure for the text. Equally, the *Elements* employs a consistent local structure to present each proposition (Heath 1908, following Proclus). First, an 'enunciation' states what situation is given and what new result is sought. If required, a 'setting-out' then provides a labelled figure exemplifying the given situation; a 'definition' or 'specification' relates that figure to the enunciation; and finally a 'construction' or 'mechanism' elaborates the figure to support reasoning to produce the desired result. A step-by-step deductive 'proof' follows, in which the warrant for each step is indicated by indexing the relevant definition, postulate, common notion or prior proposition. Finally, a 'conclusion' relates the demonstration back to the original enunciation. This provides the standard template through which the *Elements* presents its propositions.

Thus, as a text for study, the *Elements* provides a means of gaining access not just to the substantive mathematical content of the disparate sources that Euclid drew from but to the logical method that he employed to organise these sources systematically, enabling him to present their content in a consistent and coherent manner. Over time, then, the *Elements* came to fulfil an important sociocognitive function as a canonical text, providing a shared framework – both of substantive knowledge and argumentative forms – supporting and shaping the diffusion and development of mathematical knowledge. In particular, the *Elements* found favour within an approach to liberal education based on familiarising students with the classical models of thought displayed in 'great books'. Indeed, the *Elements* was studied less for its content than for the habits of mind that such study was thought to inculcate. Induction into the Euclidean system through (what I cannot resist calling) exposure to the *Elements* was intended to teach students to reason in an abstract realm removed from sensory perception (Howsam et al. 2007).

Yet actual practice could be very different, so that study of the *Elements* often became associated with a reductive mnemonic pedagogy. For example, in England at the start of the 20th century, we can find the reformer Perry criticising the requirement – in order to gain a pass degree at Oxford University – to memorise two books of Euclid, even down to the lettering of figures, with no original exercises being required (Cajori 1910). As reformers gained the upper hand, then, the 'great book' gave way to the 'school book'; the *Elements* was replaced by texts written specifically to introduce school pupils to geometry; texts which gave a place to practical experimentation and took a less restrictive approach to modes of reasoning – in line with the didactical precepts of the reform movement.

Durell's *A New Geometry* as a systematic didactical organisation of resources

To characterise the 'school books' which took the educational place of Euclid's 'great book', I will use the example of Durell's *A New Geometry for Schools*. This text was first published in 1939, and – according to my 1945 copy – reprinted no less than once and often twice in each of the following years. Indeed, C. V. Durell[1] has been described by Quadling – in his review of English mathematics textbooks of the 20th century – as "the most prolific author of the century" so that his "name was for many pupils almost synonymous with mathematics" (Quadling 1996, p. 121).

Durell published his first geometry textbook in 1909, and others had followed before he embarked on writing *A New Geometry*. In his preface, Durell acknowledges what we might now describe as a process of documentational genesis:

> "It is now almost fourteen years since the author's *Elementary Geometry* was published and, in writing this entirely new book, he has taken the opportunity to recast his treatment of the subject in the light of experience gained, and the suggestions received since *Elementary Geometry* appeared. He has been able also, as will be seen later, to make use of the Second Report of the Mathematical Association on the Teaching of Geometry. (Durell 1939, pp. *iv*)

This testifies, then, to a range of prior resources – both personal and institutional, both human and material; encountered as much as teacher as textbook author – on which Durell drew in developing the approach taken by the new text.

Durell's approach was aligned with the contemporary reform movement, and particularly the recommendations of the Mathematical Association report, as the book's subtitle – *Stage A and Stage B* – signals. Thus *A New Geometry* opens with a (Stage A) section in which more practical, experimental methods are employed with the intention of building geometrical intuition and developing informal geometrical reasoning. The following (Stage B) sections proceed to a more expository and deductive approach. Durell sets out the systematic didactical organisation around which this main part of the book is designed:

> "The plan adopted throughout is to develop each group of geometrical facts by the following successive stages:–
> (i) *Examples for oral discussion.* ... This oral work gives the pupil a clear understanding of the relevant facts, familiarises him with the arguments which will be used later in the formal proofs of theorems, and trains him in methods for solving riders. ...
> (ii) *An exercise of numerical examples.* This gives practice in applying the facts deduced from oral discussion and ensures a firm grasp of these facts.
> (iii) *Formal proofs of the corresponding theorems.* The preliminary work makes it possible to deal with these proofs rapidly. Practice in writing out theorems is essential for examination purposes, but it will often be found sufficient to confine this to the key-theorem of each group, regarding the others as simple riders.
> (iv) *An exercise of riders.* The early examples in each exercise are direct and very simple applications of the properties of the group. Some assistance is supplied for the harder examples..." (Durell 1939, pp. *v-vi*)

---

[1] Initials of forenames included to distinguish C.V. Durell from his American contemporary author of mathematics textbooks, F. Durell.

This didactically inspired organisational scheme, then, provides for the systematic sequencing of activity within each topical unit of the text into four stages: each stage is linked to particular types of learning goal and a corresponding form of classroom activity. Consistent use of this scheme throughout the main part of the textbook accustoms teacher and pupils to conceiving and conducting their activity in terms of these stages, enabling them to focus on the mathematical tasks and learning goals in play. Stages (i) and (ii) provide a more informal introduction to the topic under consideration followed by simple reinforcement of key points. Stage (iii) provides a degree of continuity with the logical approach of the *Elements*, but is more selective in its attention to formal proofs and employs a simplified local template. Grouping geometrical facts so as to organise them conceptually around a key theorem incorporates a powerful learning principle. Equally this focus on a key theorem (rather than on a compete cluster) produces a balance, over the course of stages (iii) and (iv), between experience of formal proof and of solving riders.

The core of *A New Geometry* is the sequence of topic-specific resource units forming the chapters of the book, each employing the standard staged organisation outlined above. This core corresponds to the first sense of resource system noted earlier: that of a systematic curricular sequence of resource units forming a coherent programme. Equally, *A New Geometry* recognises the need for certain auxiliary resources beyond this core curricular sequence. For example, it includes a lengthy early chapter introducing students to the geometrical instruments that practical, experimental methods call into play, and covering the main usages of this tool system in constructing and measuring geometrical figures. Likewise, the book makes provision not just for the exposition of new material but for periodic review – marked by the inclusion of revision exercises. To support this review function, the text provides a systematic organisation and cataloguing of its contents so as to be readily searchable. As well as the table of contents at the beginning and the index at the end, there are appendices summarising, respectively, the constructions and the theorems covered by the book, and indicating where they are treated more fully in the main body of the text. Such a text, then, is designed with the creation of a comprehensive resource system in mind; one meeting the various needs of teachers and pupils over the course as a whole. It is this explicit and systematic didactical organisation which makes Durell's text identifiably a textbook.

From multi-sourced collections of resources to organised systems

As resource-based approaches to teaching and learning have become increasingly influential, there has been a shift away from the traditional model of a single course text. In his review of a century of mathematics textbooks, Quadling (1996, p. 125) reported that:

> "whilst the majority of teachers still felt the need for the security of a course textbook, by the 1970s an alternative style of mathematics teaching was emerging. Declaring that 'there is no right textbook for my needs', some teachers chose to equip their classrooms with small numbers of copies of several books, and to supplement these with self-produced materials."

As Quadling noted:

> "This new concept of the textbook, as a guidebook rather than a package tour, needed the support of a well thought out curriculum for pupils to retain a sense of purpose and achievement. Not surprisingly, it was most effective when implemented collaboratively in a group of schools with advisory support, such as the SMILE programme of individualised learning which originated in the Inner London Education Authority."

A key feature of early resource-based initiatives such as SMILE (Gibbons 1975; Povey 2014) was the development of a curriculum map into which carefully chosen (or specially devised) resources from different sources could be inserted. In the case of SMILE, this curriculum map came to be paralleled by a graded assessment system, GAIM, based on criteria describing specific cognitively-based strategies representing significant steps in mathematical development (Brown 1989). Here, then, was a resource system which combined a curricular model of domains of mathematical knowledge with a cognitive model of progression in mathematical thinking.

Nowadays, the ready availability of digital resources online, combined with their provisionality, facilitates the curation and adaptation of resources by teachers, but the same issue of coherence remains (Pepin et al. 2017). A modern equivalent to the integrated SMILE/GAIM map of curriculum and assessment – is the Math-Mapper digital learning system (Confrey et al. 2017). Indeed, the choice to designate Math-Mapper as a 'digital learning system' rather than as an 'e-textbook' reflects a concern to organise the system around learning trajectories intended to reflect progression in student thinking. The intention of this subject-specific learning 'shell' is to guide the learning efforts of students (and teacher support for these efforts) by first setting appropriate learning targets (according to the recognised learning trajectories), then identifying corresponding learning opportunities (through digital curriculum resources mapped to those targets), and eventually (through the assessment functionality of the system) providing diagnostic feedback on the success of these efforts and analysing progress so as to inform the next cycle.

Nevertheless, initiatives such as SMILE developed into comprehensive curriculum programmes – comprising a full set of curricular resources as well as the organising framework – distributed well beyond the contributing schools and teachers, and sustained by a group of core participants responsible for 'minding the system' (Gueudet, Pepin and Trouche 2013). Likewise, recognising that the local insertion of resources into a digital learning shell makes considerable demands on teachers, Math-Mapper comes pre-populated with suitable curricular resources, so taking on a form closer to the contemporary e-textbook or digital curriculum programme. These trends indicate the continuing importance of externally developed systems of resources in supporting mathematics education in mainstream schools. In particular, it seems that, given the conditions under which many schools operate and teachers work, such systems are necessary to support efficient, coherent and comprehensive provision, while admitting a degree of substitution or supplementation according to local concerns and capacities. Equally, current developments indicate an important degree of innovation and diversification in the form that such resource systems take and in the character of the systematic organisation of resources that they provide.

Choppin et al. (2014) have found that contemporary digital curriculum programmes are broadly of two types. Major educational publishers have developed what the researchers termed 'digitized versions of traditional textbooks': these have structure and content similar to existing textbooks but in a digitized rather than printed form; and they are intended to be used in much the same way as traditional textbooks, under the direction of a teacher. Another type consists of what the researchers termed 'individual learning designs': these are devised to be used more directly by students as individualised study programmes, largely independent of the teacher, often with built-in assessments used to adjust the pacing and sequencing of content to the individual student user. This second type of digital curriculum programme can be seen as extending the type of approach pioneered by earlier traditions of programmed learning, individualized instruction, intelligent tutoring systems, and integrated learning systems (Means 2007). In practice, however, it seems that teachers often appropriate such individualised digital programmes to create classroom resource systems which allow them to retain aspects of their role in which they are particularly invested, so that such systems prove complementary to teacher-led forms of instruction rather than a replacement for them (Ruthven 2018).

While curriculum programmes are, in principle, designed to be organised resource systems, it is now well recognised that, in practice, such designs can be re- or dis-organised as they are appropriated – and often repurposed to a degree – by users. Thus, recent research has given attention to the operational resource systems that teachers create for themselves and their classes, looking in particular at the associated development of their professional knowledge. Thus, I will now examine two of the main exemplars of this approach within recent research on mathematics education.

The evolving notion of 'resource system' in the Structuring Features of Classroom Practice framework

My own thinking about resource systems developed in the course of investigating the integration of digital tools and resources into everyday classroom practice. I recall, for example, the head of one school mathematics department commenting on the proliferation of computer-based resources being trialled in his department, and expressing concern about effectively incorporating such a range into departmental curriculum schemes and familiarising staff and students with their varying operating principles. This reminded me of much earlier research contrasting the way in which expert and novice mathematics teachers made use of representations (Leinhardt 1989): whereas novices tended to introduce new representations for each new topic, expert teachers were more sparing in the range of representational devices that they employed, and took pains to familiarise their pupils carefully with these devices as well as using them more intensively across a range of situations. Here, then, we see an economy of resource use emerging, whereby expert teachers attend to returns on the overheads of introducing a new resource. Indeed, in our own research, we found teachers embracing such economistic reasoning. For example, one teacher justified his decision to reserve dynamic geometry software only for teacher demonstration rather than having pupils use it for themselves in terms of it being "a difficult program for the students to master... [and t]he return from the time investment... would be fairly small" (teacher quoted in Ruthven et al. 2008, p. 307).

In the Structuring Features of Classroom Practice (SFCP) framework, then, 'resource system' refers to the various mathematical tools and curriculum materials in use in the classroom and to the way in which their use – individually and collectively – is organised and made functional. The fundamental hypothesis is that this is one of several structuring features of classroom practice which mediate the process through which teachers adapt their practice and develop associated professional knowledge. In the main paper outlining SFCP (Ruthven 2009), this framework was illustrated by an example – developed from an earlier study (Ruthven et al. 2008) – of the evolving classroom practice of a mathematics teacher in the process of incorporating use of dynamic geometry software. His intention was to complement established construction tasks which made use of classical tools by introducing new tasks employing dynamic software. The rationale for the double instrumentation involved in creating such a classroom resource system was twofold: first, to strengthen attention to the geometric ideas underpinning constructions through their mediation by the software in terms of its named and constrained geometrical operations; second, to give students experience of finding geometric rules and patterns through exploring a dynamic figure in ways impossible with static diagrams.

In many respects, the intentions behind this coordinated use of classical then digital tools were realised, producing a corresponding enhancement of the classroom resource system. Nevertheless, the teacher also experienced some discontinuities and diversions. First, he considered the correspondence between classical and digital techniques to be imperfect in some important respects, reducing the desired congruence between old and new tools: he had not yet found an effective resolution of this tension. Equally, the teacher experienced other issues which needed to be taken in hand if the classroom resource system was to function effectively. In resolving these, he developed new techniques and norms, extending his professional knowledge accordingly. For example, he developed knowledge not just of how the nuances of software operation might derail students' attempts at construction, but also of how such difficulties might be turned to advantage in reinforcing the mathematical focus of the task. Equally, recognising that students might not appreciate the geometrical significance of the invariant properties of a figure, the teacher was developing strategies for addressing this, notably through exploiting the distinctive affordances of dragging a dynamic figure. In both these respects, then, the teacher was building professional knowledge contributing to a more effective functioning of the expanded classroom resource system. More prosaically too, the teacher was finding that students could be deflected from the mathematical focus of a task by the ease of experimenting with the presentational options provided by the software: he sought to manage this by showing students mathematically appropriate use of differing fonts and colour coding; an example of securing a more satisfactory functioning of the classroom resource system by establishing norms and techniques for the use of new tools.

In a later study, the SCFP framework was applied more directly to investigation of teaching practices involving use of dynamic geometry software (Bozkurt 2016; Bozkurt and Ruthven 2017). Here, I will give a comparative sketch of the classroom resource systems established to teach the topic of transformations by two teachers, both with around 20 years of teaching experience, but differing markedly in their experience of using technology. First some similarities. Both teachers chose to have students make use of the software to tackle assigned tasks (in contrast to the example mentioned earlier of its use being restricted to teacher demonstration to the whole class). Equally, both teachers took a just-in-time approach to developing students' technical skills, introducing them to any unfamiliar features of the software immediately prior to tasks requiring their use. In both cases too, the resources in play comprised prepared dynamic files accompanied by printed worksheets giving students instructions on how to use the files and prompting them to record predictions and report findings. Finally, as this structuring of worksheets indicates, in both cases the resource system was designed to support processes through which the worksheet instructions prompted students to make a mathematical prediction and then guided them in using the dynamic file to test their prediction and generate feedback on it. In these respects, then, the two teachers followed similar approaches to making the digital tool part of a functioning classroom resource system.

However, there were also some important differences in the classroom resource systems that the teachers established. A first difference was in the provenance of the resources used. Whereas the most experienced teacher used his own file/worksheet duos, refined over a lengthy period, the least experienced teacher adopted a collection of duos found online, using them initially without modification. However, in the light of her experience of working with these borrowed duos, the least experienced teacher then adapted the worksheet for future use.

There were also differences in the degree of task closure and direction that the teachers sought to achieve through their duos. When the least experienced teacher subsequently modified the worksheet part of her borrowed duos, she altered the wording of instructions so as to direct students more explicitly towards a particular solution envisaged in their design. By comparison, the task environments that the most experienced teacher provided were devised to permit a range of solution strategies: and while the dynamic files were tightly constrained, this served to reduce the need for direction in the worksheet while leaving open the possibility of different approaches. Finally, there were differences in the status that the teachers accorded to the two media. The least experienced teacher was concerned that students "did not have enough practice on paper to put into practice what they had actually seen on the computer" and this led her to add two further worksheets of solely pen-and-paper tasks for this purpose. For her, then, the dynamic software served simply as a pedagogical aid to introduce new mathematical ideas, whereas the experienced teacher treated it as a more central tool for students' mathematical work. In his lessons, students continued to work within the dynamic software environment, with the teacher projecting selected screens to support whole class discussion of the different strategies that they exemplified. Nevertheless, while differing in the balance between conventional and digital media, as in the degree of task closure and direction, both teachers clearly made refinements to the classroom resource system which were intended to make it function more closely in accord with their own didactical preferences. In both cases, then, we see evidence of a process of professional adaptation – albeit at different stages in incorporating the use of dynamic software.

The SFCP framework in its present form has used the idea of classroom resource system in a loosely defined manner. This has had the advantage of ensuring that the construct is well grounded empirically, through allowing flexibility in identifying relevant phenomena and accommodating them. However, as our knowledge of such phenomena grows, particularly across a wider range of educational contexts, it would be beneficial to demarcate the construct in a more precise manner, breaking it down into components and clarifying their interrelation.

The evolving notion of 'resource system' in the Documentational Approach framework

Other lectures at this conference give detailed consideration to the documentational approach (DA). Here, then, I will focus specifically on its notion of a resource system. Nevertheless, it is important to start by emphasising three broader points. The first is that the DA adopts an expansive notion of resource as comprising not just material but human non-material assets. The second point is that the primary concern of the DA to date has been with the resource systems of individual teachers (even if it acknowledges the part that other teachers and collectives play in shaping such systems) over the whole span of their professional activity (rather than only in the classroom or solely relating to a particular class or topic). The final important point is that – in the psychologically influenced DA – a crucial distinction is made between an artefactual *resource* and the result of its appropriation (often in combination with other resources) to form an instrumental *document.*

This specialised use of 'document' refers to the resource(s) in play plus an associated *utilization scheme*; this latter conceived as consisting of observable *usages* and not-directly-observable *operational invariants* governing these. Gueudet and Trouche illustrate their idea of a document with reference to a particular class of professional situations – delineated as ‘propose homework on the addition of positive and negative numbers’– as follows:

> "For this class of situations, a given teacher gathers resources: textbooks, her own course, a previously given sheet of exercises... She chooses among these resources to constitute a list of exercises, which is given to a class. It can then be modified, according to what happens with the students, before using it with another class during the same year, or the next year, or even later. The document develops throughout this variety of contexts. The operational invariants can be very general, like 'the homework must be extracted from the textbook', or more precisely linked with the mathematical content, like: 'the additions proposed must include the cases of mixed positive and negative numbers, and of only negative numbers,' etc. These operational invariants can be inferred from the observation of invariant behaviors of the teacher for the same class of situations across different contexts. They are teacher beliefs, and are both driving forces and outcomes of the teacher's activity, instrumented by a set of resources." (Gueudet and Trouche 2009, p. 205)

In particular, then, we should note that more generic operational invariants will be in play across multiple classes of professional situation. Thus the DA posits that the documents that a teacher establishes, in response to the range of classes of professional situations that s/he encounters, constitute a system structured by professional activity. This leads to the fundamental hypothesis of the DA that each teacher develops a structured *documentation system* which evolves over time with that teacher's professional practice.

Accordingly, early formulations of the DA avoided the term 'resource system', emphasising that:

> "each resource must be viewed as a part of a wider 'set of resources' (used here instead of 'resource system' which suggests an *a priori* structure of the resource sets)." (Gueudet and Trouche 2009, p. 200)

In due course, however, the DA embraced the term, while maintaining the crucial distinction between artefactual resource and instrumental document:

> "The resource system of the teacher constitutes the 'resource' part of her documentation system (i.e. without the scheme part of the documents)." (Gueudet and Trouche 2012, p. 27)

The rationale for considering this too to be a system lies in a wider structure made visible by the renewal of resources over time (leading to a reconfiguration of activity and to a renewal or abandonment of other resources):

> [E]ach 'renewing' of a resource impacts on other teacher resources, and may have different outcomes for what we name teacher resource system —the word 'system' is purposefully chosen to emphasize that this system is highly structured, the structure being linked, more or less explicitly, to teacher activity." (Gueudet et al. 2013, p. 1004)

Such structuring of the resource system may be attributable to the structure of the documentation system: for example, through the influence of generic operational invariants:

> "Identifying [the] documentation system allows, for example, understanding the adoption or rejection of resources by the teacher (a new resource is more likely to be integrated if it matches other resources already present in the teacher's resource system)." (Gueudet et al. 2014, p. 142)

However, one method characteristic of the DA suggests other types of structuring of a teacher's resource system. In this method the researcher asks the teacher to draw a schematic representation of the structure of the resources that s/he uses, so generating what that DA terms a *schematic representation of the resource system* (Gueudet and Trouche 2012, p. 28). Typically, it seems, the process of eliciting such representations brings out socio-spatio-temporal-material dimensions of the relatively immediate organisation of teachers' work. In one study, a teacher's representation of her resource system identifies four 'zones' with which resources are associated: her work at home; her work at school without students; her work at school in the classroom with her students; her work in in-service training collectives (Gueudet and Trouche 2012, p. 35). In another study a teacher's representation of her resource system is configured first by worksite – home or school, linked by USB key – then, within site, by the places where resources are kept – shelves, bedroom or computer, and cupboard or computer, respectively, and finally, at home, by resource form – emails, books, scientific journals, paper folders, digital folders (Gueudet et al. 2013, p. 1008). Another teacher groups resources, first, according to function – lesson preparation, or communication with pupils and parents; then, for lesson preparation, according to form – audiovisual and online resources, games and similar activities – and status – the adopted textbook, other textbooks – or provenance – her own, from her colleagues (Gueudet et al. 2013, p. 1010).

The DA framework, then, incorporates two perspectives on teacher resource systems: one deriving from a theorised notion of teacher documentation with a particular focus on utilization schemes; and another originating from teachers' own representations of the structure of their resources, evoking varied aspects including the socio-spatio-temporal organization of their work as well as the perceived form and function of the resources available to them. Clarifying the relationship between these perspectives represents one fruitful area for development of the DA. It would also be interesting to explore congruences, complementarities and conflicts between the DA and theories of distributed cognition and situated knowledge which offer alternative – but similarly socioculturally informed – accounts of the organisation and development of professional knowledge.

Conclusion

It is clear, then, that ideas of 'resource system' differ considerably in the ways in which they demarcate 'resources' and formulate 'system'. Equally, closer examination shows that different perspectives situate 'resource system' in contrasting ways: as adhering to a particular type of agent – teacher, student, designer – or as intervening between such agents; as relating to a specific educational entity - especially the classroom, the course or the lesson – or as ranging across and beyond these. Professionals and researchers have clearly found each of these variations useful for some purpose: an implication is that we could benefit from an expanded notion of 'resource system' which acknowledges all these dimensions and encourages users of the term to take more explicit account of them in framing their thinking about a particular issue, and in describing and justifying that framing.

It would be remiss, however, not to conclude by emphasising the value of existing constructs of resource system in analysing the work of teaching. Collectively they highlight the central role that resources play in such work, illuminating the dynamic between designers, teachers and students in developing and refining resources and the manner in which they are used. Within the research field, particular attention has been given to resource systems as they relate to teachers. Of course, the motivation for developing the structuring features (SFCP) framework was very explicitly to better understand the adaptation of teachers' professional knowledge. Equally, the intention of the established body of studies using the documentational approach (DA) has been to study resource systems as a phenomenon of teacher cognition. Accordingly, both approaches rely heavily on methods using teacher informants. There is scope, then, to develop approaches which take account of other perspectives[2] and introduce more comprehensive theoretical framings.

[2] As, for example, does recent research using the documentational approach to study the use of digital resources by students in higher education (Gueudet & Pepin, 2018).

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, *3*(3), 205-224.

Barrow-Green, J. (2006). 'Much necessary for all sortes of men': 450 years of Euclid's Elements in English. *BSHM Bulletin: Journal of the British Society for the History of Mathematics, 21*(1), 2-25.

Beswick, N. (1974). Library Resource Centres: a developing literature. *Journal of Librarianship*, *6*(1), 54-62.

Bozkurt, G. (2016). *Teaching with technology: a multiple-case study of secondary teachers' practices of GeoGebra use in mathematics teaching*. Unpublished PhD thesis, University of Cambridge.

Bozkurt, G., & Ruthven, K. (2017). Teaching with GeoGebra: resource systems of mathematics teachers. In G. Aldon & J. Trgalová (Eds.) *Proceedings of the 13th International Conference on Technology in Mathematics Teaching* [ICTMT 13], (pp. 216-223). Lyon: École Normale Supérieure de Lyon / Université Claude Bernard Lyon 1.

Brock, W. H. (1975). Geometry and the universities: Euclid and his modem rivals 1860–1901. *History of Education*, *4*(2), 21-35.

Brown, M. (1989). Graded assessment and learning hierarchies in mathematics – an alternative view. *British Educational Research Journal*, *15*(2), 121-128.

Cajori, F. (1910). Attempts made during the eighteenth and nineteenth centuries to reform the teaching of geometry. *American Mathematical Monthly*, *17*(10), 181-201.

Chemla, K. (Ed.) (2012). *The history of mathematical proof in ancient traditions*. Cambridge: Cambridge University Press.

Choppin, J., Carson, C., Borys, Z., Cerosaletti, C., & Gillis, R. (2014). A typology for analyzing digital curricula in mathematics education. *International Journal of Education in Mathematics, Science, and Technology, 2*(1), 11–25.

Confrey, J., Gianopulos, G., McGowan, W., Shah, M., & Belcher, M. (2017). Scaffolding learner-centered curricular coherence using learning maps and diagnostic assessments designed around mathematics learning trajectories. *ZDM, 49*(5), 717-734.

de los Arcos, B., Farrow, R., Pitt, R., Weller, M., & McAndrew, P. (2016). Adapting the curriculum: how K-12 teachers perceive the role of open educational resources. *Journal of Online Learning Research*, *2*(1), 23-40.

Durell, C. V. (1939). *A new geometry for schools.* London: Bell.

Gibbons, R. (1975). An account of the Secondary Mathematics Individualized Learning Experiment. *Mathematics in School*, *4*(6), 14-16.

Gillespie, R. J., & Humphreys, D. A. (1970). The application of a learning resource system in teaching undergraduate chemistry. *Pure and Applied Chemistry*, *22*(1-2), 111-116.

Graystone, J. A. (1978). The role of the teacher in resource based learning: towards a conceptual framework. *British Educational Research Journal*, *4*(1), 27-35.

Gueudet, G., Buteau, C., Mesa, V., & Misfeldt, M. (2014). Instrumental and documentational approaches: from technology use to documentation systems in university mathematics education. *Research in Mathematics Education, 16*(2), 139-155

Gueudet, G., & Pepin, B. (2018). Didactic contract at university: a focus on resources and their use. *International Journal of Research in Undergraduate Mathematics Education* *4*(1), 56-73.

Gueudet, G., Pepin, B., & Trouche, L. (2013). Collective work with resources: an essential dimension for teacher documentation. *ZDM, 45*(7), 1003–1016.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218.

Gueudet, G., & Trouche, L. (2012). Teachers' work with resources: documentational geneses and professional geneses. In G. Gueudet, B. Pepin, B., & L. Trouche (Eds.) *From text to 'lived' resources: mathematics curriculum materials and teacher development.* New York: Springer.

Heath, T. L. (Ed.). (1908). *The thirteen books of Euclid's Elements*. Cambridge: Cambridge University Press.

Howsam, L., Stray, C. Jenkins, A., Secord, J. A., & Vaninskaya, A. (2007). What the Victorians learned: perspectives on nineteenth-century schoolbooks. *Journal of Victorian Culture, 12*(2), 262-285.

Leinhardt, G. (1989). Math lessons: a contrast of novice and expert competence. *Journal for Research in Mathematics Education*, *20*(1), 52-75.

Means, B. (2007). Technology's role in curriculum and instruction. In F. M Connelly (Ed.) *The Sage handbook of curriculum and instruction* (pp. 123-144). London: Sage.

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: Foundations for change. *ZDM, 49*(5), 645-661.

Povey, H. (2014). The origins and continued life of SMILE Mathematics. *Mathematics Teaching*, *241*, 5-6.

Quadling, D. (1996). A century of textbooks. *Mathematical Gazette*, *80*(487), 119-126.

Recker, M. M., Dorward, J., & Nelson, L.M. (2004). Discovery and use of online learning resources: case study findings. *Educational Technology & Society, 7*(2), 93-104.

Rouse Ball, W. W. (1908). *A short account of the history of mathematics* (4th ed.). London: Macmillan.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: the example of school mathematics. *Education & Didactique*, *3*(1), 131–149.

Ruthven, K. (2018). Instructional activity and student interaction with digital resources. In L. Fan, L. Trouche, C. Qi, S. Rezat & J. Visnovska (Eds.) *Research on Mathematics Textbooks and Teachers' Resources: Advances and issues* (pp. 261-275). New York: Springer.

Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: a study of the interpretative flexibility of educational software in classroom practice. *Computers & Education, 51*(1), 297-317.

Zhao, Z., Cook, J., & Higgen, N. (1996). Online learning for design students. *ALT-J*, *4*(1), 69-76.