epiSTEMe

Teaching Notes

Probability

May 2011 version

OVERVIEW

INTRODUCTION

This module on Probability is one of four topic-specific modules that have been developed as part of the *epiSTEMe* project (Effecting Principled Improvement in STEM Education): it employs the principled teaching model developed by the project (outlined in Appendix 1), and has been informed by a range of research which has identified aspects of probabilistic thinking that challenge many students (and indeed many adults). It introduces the topic in line with the requirements of the English National Curriculum in Mathematics (outlined in Appendix 2).

The module has been designed to take account of students' informal knowledge and thinking about probability. It focuses initially on properties of everyday randomisation devices – notably coins and dice – so as to make connections with familiar experiences that are widely shared by students. The module then examines a range of chance situations with the aim of building strong conceptual foundations for the topic. In particular it incorporates activities for challenging common misconceptions that many students display in reasoning about probability. The module also seeks to show the human interest and social relevance of the topic. It starts by using news headlines to trigger recognition of probabilistic language, and goes on to show how probability has significant scientific applications.

The sequencing of activities within the module is designed to allow new ideas to be examined in a dialogic style in which different points of view are explored and students are given opportunity and time to reflect on them. At a later authoritative stage students are shown how the accepted mathematical point of view codifies such ideas: at this stage, key mathematical terms and representations are introduced as tools for thinking. Students are then given an opportunity to consolidate their knowledge and understanding of the accepted mathematical ideas and associated tools. As appropriate, a range of whole-class, small-group and student-pair discussion formats are used to support interactive discussion. Individual-student work (including homework) provides checks on understanding and opportunities to follow up with appropriate personal feedback.

Structure of the module

The main sequence of activities is set out in these Teaching Notes, and is supported by a Study Booklet for students and a set of Projection Slides for classroom use. The sequence has been organised into Lessons, notionally of one hour, but with at most 50 minutes of activity actually scheduled, leaving at least 10 minutes slack. To help in planning how to fit Lessons into sessions of a different length, or in adapting to unplanned circumstances, each Lesson has been divided into shorter Parts: for these purposes Appendix 3 shows the whole sequence of activities and indicative timings for them (in minutes).

Implementing the module

You will need to translate these Lessons into a form that will work for your particular class. It is important, though, that you make any adaptations in a way that seeks to maximise students' understanding of key concepts. Use your discretion to decide whether certain activities require more or less emphasis and more or less time than suggested. Consequently, a Lesson may not fit neatly into a single class session. You may wish to begin each session with a review of ideas covered in the previous session and an overview of the new ideas to follow.

There are aspects of the Lessons, which we suggest that you do not alter. A key feature of the module is that it includes a large amount of student thinking and talking. Try to ensure that, as far as possible, time for this thinking and talking is preserved. While the module should guide students in developing a 'mathematical' view, we ask you (and any assistants working with you) to help students to test and refine their own ideas during discussion-based activities rather than giving them ideas yourself. The point is to lead or support the articulation and refinement of student thinking rather than to proceed immediately to 'correct solutions'.

The tasks that have been designed for collaborative activity in small groups should be effective regardless of the ability or gender composition of these groups. Within limits, they should also be effective regardless of group size. However, when groups become too large, students can experience difficulties with managing the dialogue. For instance, some students can get left out or groups can split. For that reason, we suggest that students normally work in groups of two, three or four.

For schools following the English National Curriculum in Mathematics, this module covers the probability requirements of the Programme of Study for Key Stage 3 up to level 6. A detailed mapping of NC learning objectives to particular lessons in the sequence is provided in Appendix 2. The full module comprises seven Lessons. The core material appears in Lessons 1-4 and 6-7. The optional Lesson 5 has proved popular with students and effective in developing 'functional' mathematical thinking.

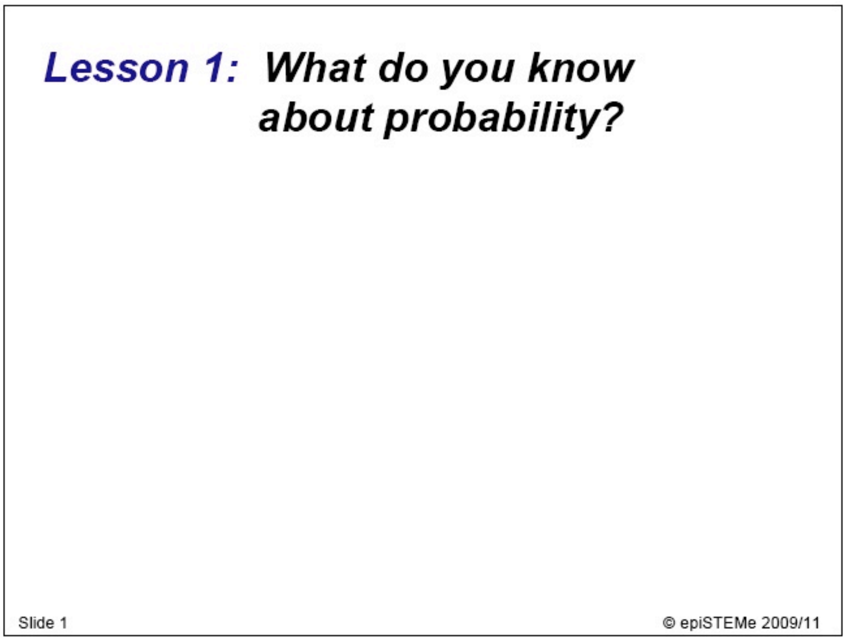
LESSON 1

OVERVIEW

This lesson introduces the topic of probability in a way that seeks to draw on students' existing knowledge and wider experiences.

RESOURCES

- Slides 1-4.
- Study Booklet pp. 3–5.
- Sufficient dice for each pair of students to inspect and use one.



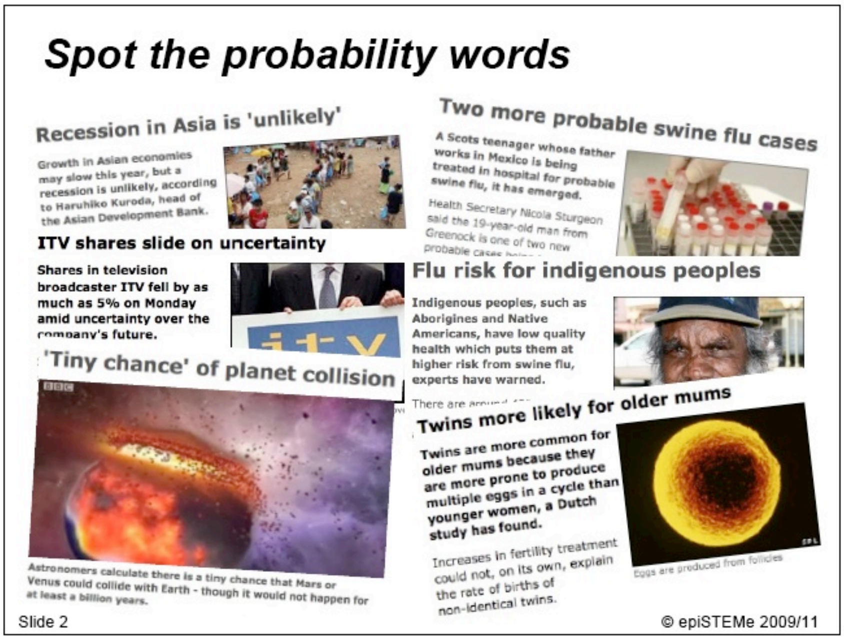
AIMS

- To elicit informal student knowledge about probability.
- To informally develop basic ideas about representing and quantifying probability.

STRUCTURE

- Part 1 *(Spot the probability words):* The class spots words associated with probability in a set of news headlines.
- Part 2 *(Share what you know about dice):* The class shares what students already know about the properties of an important type of "chance maker" – dice.
- Part 3 *(Rate the chance of dice events):* Student pairs tackle problems that involve marking dice probabilities on a scale, followed by whole-class review of their conclusions.

LESSON 1: PART 1



OBJECTIVE

To highlight familiar words and ideas to do with probability.

TIME: about 10 minutes.

RESOURCES

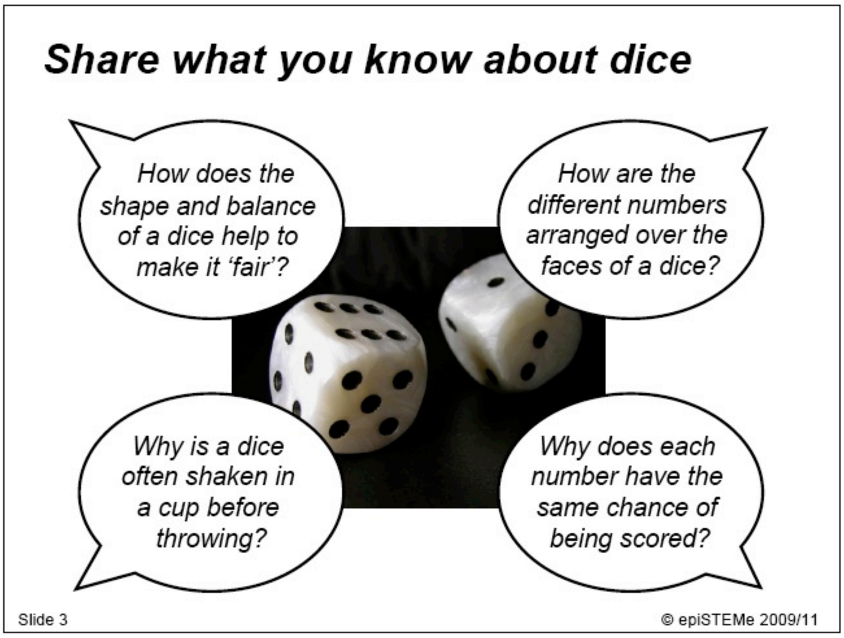
- Slide 2 (as shown).
- Study Booklet p. 3.

ACTIVITY

Whole class

- Explain that this module will be about the topic of probability.
- Ask students to identify words or phrases in the headlines shown on Slide 2 that have something to do with probability.
- Rather than directly evaluating student responses as right or wrong, ask students to explain why they think that a particular word or phrase has something to do with probability.
- Finally, ask students to underline all the probability words or phrases that the class has identified on the copy of Slide 2 in their own Study Booklet.

LESSON 1: PART 2



OBJECTIVE

To highlight special properties of dice which make it possible to reason about their probabilities.

TIME: about 15 minutes.

RESOURCES

- Slide 3 (as shown).
- Study Booklet p. 4.
- Some dice for purposes of demonstration.

ACTIVITY

Whole-class

- Start by asking for ideas about answering the question on Slide 3 about 'shape and balance'. Rather than directly evaluating responses as right or wrong, ask students to explain why they think that and encourage exchange of views by students.
- Bursts of pair discussion may be useful to generate ideas to feed into the whole-class discussion.
- Leave the question on 'the same chance' until last. Prompt students to explain how their answers to this question relate to their answers to earlier ones.

KEYPOINTS

How does the shape and balance of a dice help to make it 'fair'?

This is intended to elicit ideas of symmetry and regularity. If students do use words such as 'symmetric' or 'regular' or 'balanced', ask them to explain what they mean. Ideally, ideas should flow from students themselves, but, if necessary, invite comparison between 'fair' dice and 'biased' or 'crooked' dice, and pose questions.

Why is a dice often shaken in a cup before throwing?

This is intended to elicit the idea that using a cup prevents handling of the dice by the thrower, making sure that the dice is shaken around and out at random.

How are the different numbers arranged over the faces of a dice?

This is intended to elicit the idea that each of the six faces of the dice shows one of the numbers 1 to 6. A further idea that students may volunteer is that the numbers on opposite faces add up to 7.

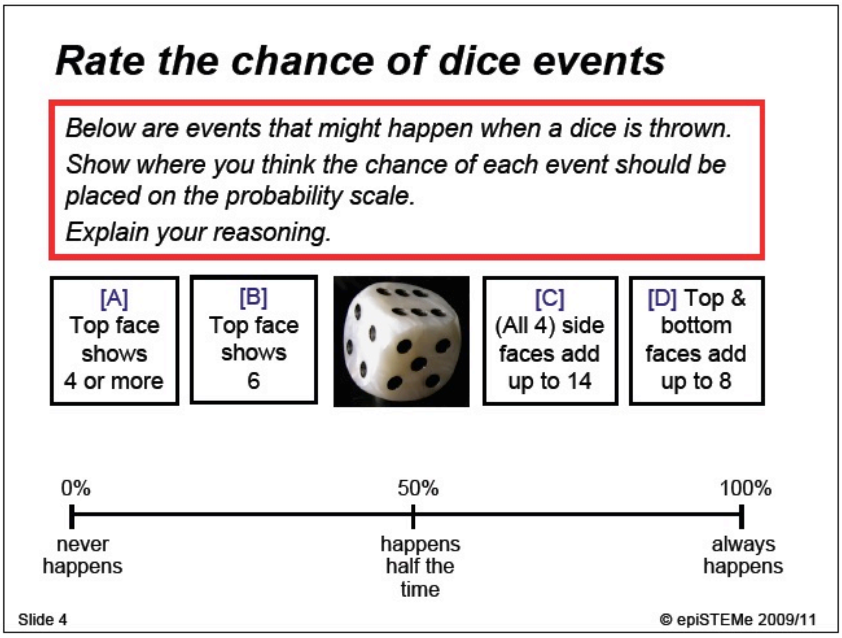
Why does each number have the same chance of being scored?

This is intended to elicit explanations drawing on the information provided in response to the preceding questions, and to focus attention on the key property of equally-likely outcomes.

Students may volunteer other interesting information about dice.

A survey article about dice can be found at http://en.wikipedia.org/wiki/Dice.

LESSON 1: PART 3



OBJECTIVES

To informally develop the equally-likely outcomes approach to quantifying probability.

To informally introduce the probability scale as a means of displaying probability values.

TIME: about 25 minutes.

RESOURCES:

- Slide 4 (as shown).
- Study Booklet p. 5.
- One dice for each student pair.

ACTIVITY

Student-pair, leading into whole-class

- Ask students to work in pairs. Emphasise that they should try to agree not just on answers but on the reasoning behind them.
- If many pairs are not developing sound responses to A and B, then bring the class together to exchange ideas before resuming work in pairs.
- Initiate a final whole-class discussion by choosing a suitable item and asking one pair to report what they agreed together. Ask them to back up their ideas with *reasons*. Then ask if any pair arrived at a different answer, or found different reasons.
- Discuss further items if time permits.

KEYPOINTS

[A] top face shows 4 or more and *[B] top face shows 6*

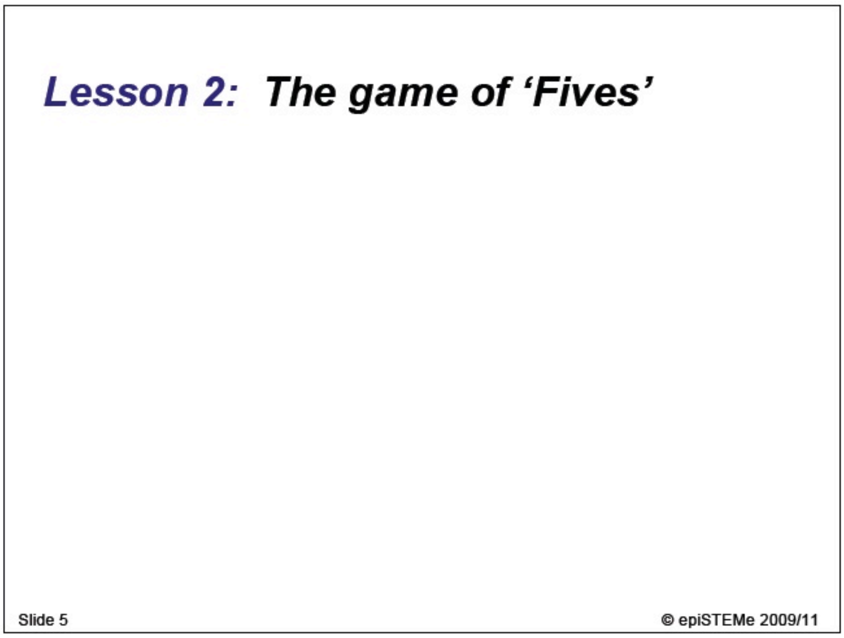
One line of argument on A is that scores 4, 5 or 6 represent half the (equally-likely) possibilities, balancing 1, 2 or 3, so that this event ‘happens half the time’. Another line of argument on both A and B is that any score represents one of six equally-likely possibilities, and so each score has a 1 in 6 chance of occurring. If the connections between such arguments are not clear to students then they are worth discussing further. This may provide an opportunity to discuss and debug the common misuse of ‘50-50 chance’ to refer very broadly to any uncertain result, or to any outcome for which there are other equally-likely ones.

[C] all four side faces add up to 14 and *[D] top and bottom faces add up to 8*

If they are unsure where to start, encourage students to make a guess, and then to test that guess against the results from several trial throws. They are likely to be surprised. The event C ‘always happens’, D never. Encourage students to discuss why this might be. Essentially, the result is driven by the property that a dice has three pairs of opposite faces, with each pair having a total score of 7. However the dice lands, one of these pairs occupies the top and bottom positions, and the other two pairs the side positions.

An extension task is to try to change the position of numbers on a dice so that event C ‘never happens’ or ‘happens half the time’.

LESSON 2



OVERVIEW

In this lesson, the game of 'Fives' is used as a context for examining chance patterns, and for deconstructing some common probability misconceptions.

TIME: 50 minutes.

RESOURCES

- Slides 5-12.
- Study Booklet pp. 6-9.
- 6 (normal) dice (and ideally cups to shake them in).

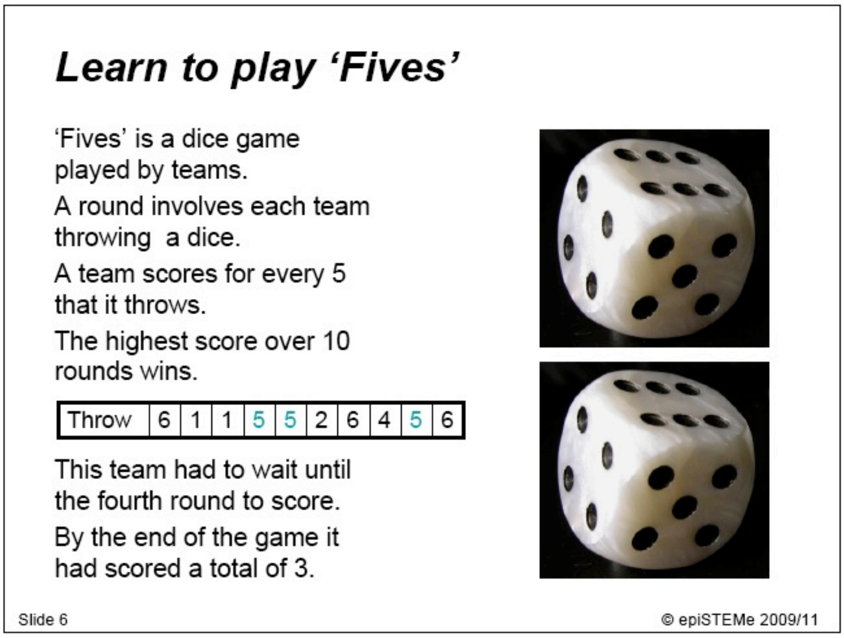
AIMS

- To form ideas about aggregate patterns of chance variation.
- To deconstruct some common probability misconceptions.

STRUCTURE

- Part 1 *(Learn to play 'Fives'):* The class learns how to play a dice game that will be used to stimulate analysis of chance patterns.
- Part 2 *(Play a game of 'Fives'):* The class plays a game of 'Fives' in six teams, with the results being systematically recorded.
- Part 3 *(Review 'Fives' results):* The class analyses the round-by-round pattern of results from the 'Fives' game.
- Part 4 *(Think about 'Fives' probabilities):* Small groups examine some (misconceived) statements about probabilities relating to the 'Fives' game, followed by whole-class review of their conclusions.

LESSON 2: PART 1



OBJECTIVE

To explain the game of 'Fives' and organise the class to play it.

TIME: about 5 minutes.

RESOURCES

- Slide 6 (as shown).
- Study Booklet p. 6.
- A dice for each team (and ideally a cup to shake it).

ACTIVITY

Whole-class

- Use Slide 6 to introduce the game of 'Fives' and its rules to the class.
- Explain that six teams will take part in a game of 'Fives'.
- If you decide that you would like one or two students to act as **recorder** of the results, appoint them to do so. Otherwise you will fulfil this role yourself.
- Organise the remaining students into six **teams**.
- Give each team a dice (and ideally a cup to shake it in).
- Explain that members should take turns to throw for their team.
- Explain that the game will be played one round at a time, and that you will tell each team in turn when to throw.
- Emphasise the importance of playing fair and not cheating.

KEY POINTS

It is important to have exactly six teams: some of the later questions depend on having this number of throws in a round; this number has been set so as to facilitate some of the thinking to be stimulated. In particular, having six throws per round means that each score can be expected to occur (on average) once per round.

LESSON 2: PART 2

OBJECTIVE

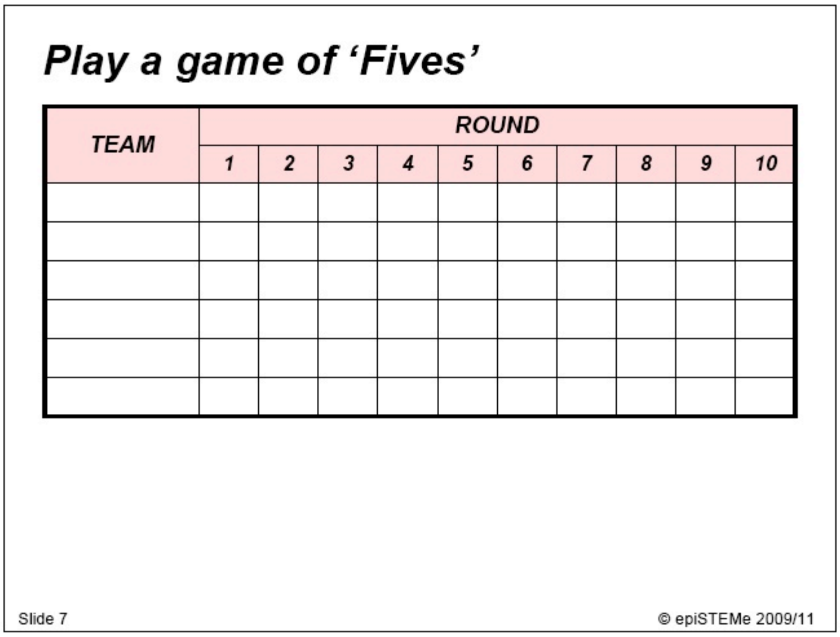
To examine whether 'Fives' is a fair game and a game of pure chance.

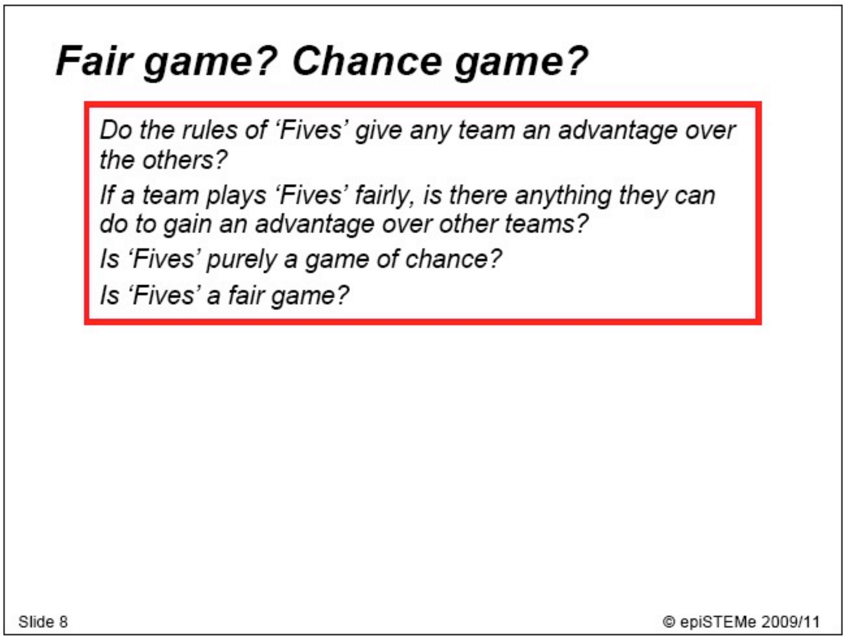
To compile a systematic record of a full game of 'Fives'.

TIME: about 10 minutes.

RESOURCES

- Slides 7 & 8 (as shown).
- Study Booklet pp. 6-7.
- A dice for each team (and ideally a cup to shake it).
- A way of recording results. Either use a separate board, or Slide 7 [which can be used in <Slide View> rather than <Slide Show> mode for this purpose; don't forget to save the results].





ACTIVITY

Whole-class

- Play the first round of the game rather deliberately, talking through what is happening at each turn, so as to establish good understanding of it.
- At the end of the first round, use the questions on Slide 8 to lead a discussion about the game. Try to elicit student contributions rather than commenting directly yourself. Bursts of pair discussion may be useful to generate ideas to feed into whole-class discussion.
- Play the remaining rounds of the game more swiftly.
- Nip any cheating in the bud. If cheating does take place, you might decide to announce that 2 rather than 5 will be taken as the target number in deciding the winner.

LESSON 2: PART 3

OBJECTIVE

To form ideas about aggregate patterns of chance variation.

TIME: about 10 minutes.

RESOURCES

- A display showing the record of results from the game, e.g. Slide 7.
- Slide 9 (as shown).
- The embedded spread-sheet on Slide 10 can be used to simulate a game. [Enter <Slide view> mode, select and open the spreadsheet, give the recalculate command (PC <ctrl> = or Mac <> =), and then close the sheet.]
- Study Booklet p. 8.

ACTIVITY

Whole-class

- Lead review and discussion of the two sets of questions on Slide 9.
- Try to elicit student contributions rather than commenting directly yourself.

Review 'Fives' results

Look at the class results from your game of 'Fives'.
How many 5s were thrown in each round of the game?
Did all the possible numbers of 5s in a round occur?
What were typical numbers of 5s for a round?
What was the average number of 5s for a round?
Are the results surprising? Why (not)?

Suppose your class was going to play lots and lots more games of 'Fives'.
What do you predict would be the average number of 5s in a round?
What do you predict would be the least common number of 5s in a round?
Explain the reasoning behind your answers.

Slide 9 © epiSTEMe 2009/11

Review 'Fives' results (spreadsheet)

TEAM	ROUND									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Team A	2	4	2	5	5	5	5	4	6	4
Team B	6	5	6	3	1	3	4	1	3	3
Team C	2	5	6	5	4	6	6	2	6	1
Team D	3	5	2	2	2	5	2	2	5	2
Team E	6	5	6	2	1	3	3	2	3	1
Team F	2	4	1	6	4	5	2	1	3	3

Slide 10 © epiSTEMe 2009/11

- The questions are designed to highlight variability in the number of 5s in a particular round, and the likelihood of that number being small (0, 1 or 2). The aim is that students see this number as varying in line with a long-term average of one in six throws producing a 5.
- The spreadsheet on Slide 10 can be used to generate more examples of the results from a game. This can be particularly useful if it turns out that the distribution of 5s per round in the class's own results differs markedly from statistical expectations.
- Following discussion, summarise conclusions along the following lines:
  - On average, we expect one sixth of the throws to produce a 5, but in a random pattern.
  - On average, then, we expect one team to throw a 5 over the course of a round. However, because the pattern is random, only in some rounds will exactly one team throw a 5; in other rounds more than one team will throw a 5, or no team at all.
  - It is possible to have a round where all six teams throw a 5, but very unlikely.

LESSON 2: PART 4

OBJECTIVE

To deconstruct some common probability misconceptions.

TIME: about 25 minutes.

RESOURCES

- Slides 11 & 12 (as shown).
- Study Booklet p. 9.

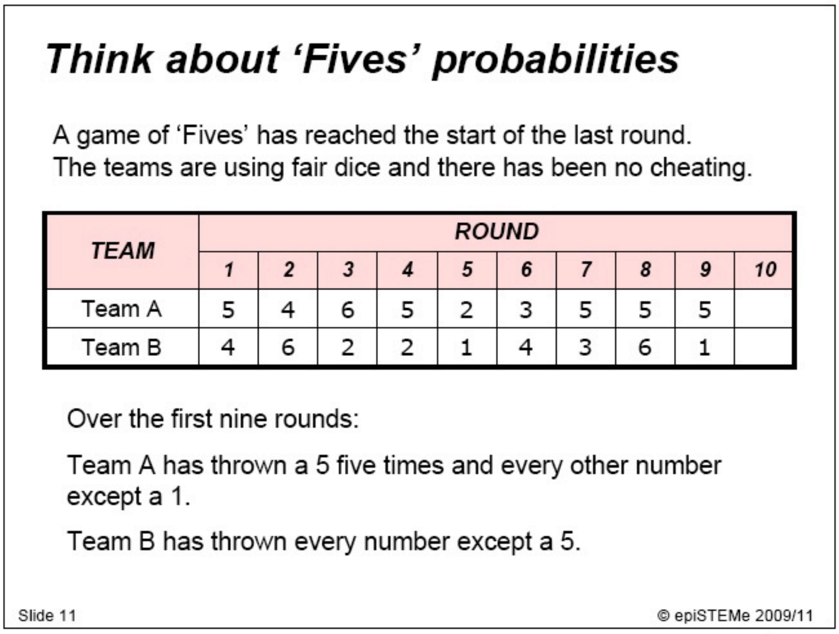
ACTIVITY

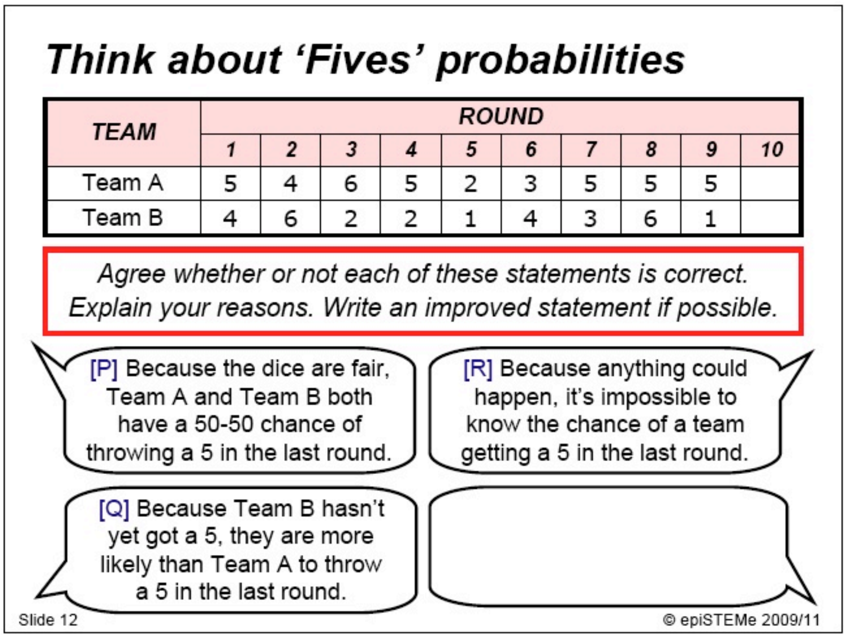
Whole-class

- Use Slide 11 to introduce the problem situation.
- Ask students to mark in their Study Booklet whether they personally agree/disagree with each statement on Slide 12.
- Organise small groups and remind students about discussion ground rules.
- Emphasise that each group should reach an agreed conclusion and make sure that any member will be able to give the reasons for it.

Small-group

- Circulate to monitor activity in the groups.
- Praise contributions that take different views into account or show thinking about reasons.
- Help students to exchange and examine ideas, and work towards their own agreed position.

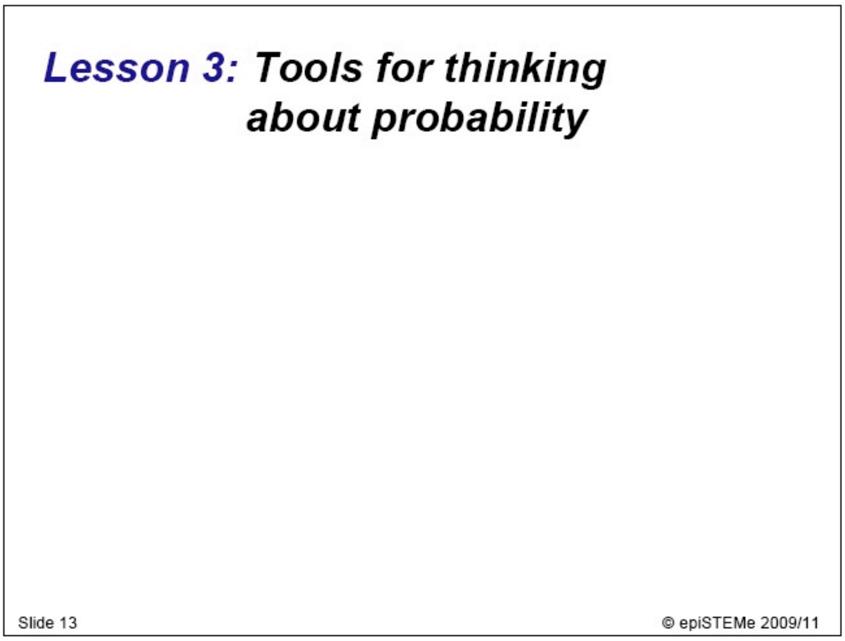




Whole-class

- Take each of the statements in turn. Ask one group to present its conclusion on the statement and their reasons for it, then structure collective discussion of any differences as follows.
- First, ask whether anyone has a direct comment to make, or question to ask, about that conclusion or the reasoning behind it. If so support exchange and development of ideas.
- Then ask whether any other group arrived at a different conclusion. If so ask such a group to present its conclusion and the reasoning behind it. Again invite comments and questions, and support exchange and development of ideas.
- Summarise issues for the class and identify key differences.
- Ask whether anyone in any group has changed their initial views on the statement. Invite such persons to say how their thinking changed and why.

LESSON 3



OVERVIEW

In this lesson, the key ideas introduced in the previous lessons are treated more formally.

TIME: 50 minutes.

RESOURCES

- Slides 13-20.
- Study Booklet pp. 10-14.

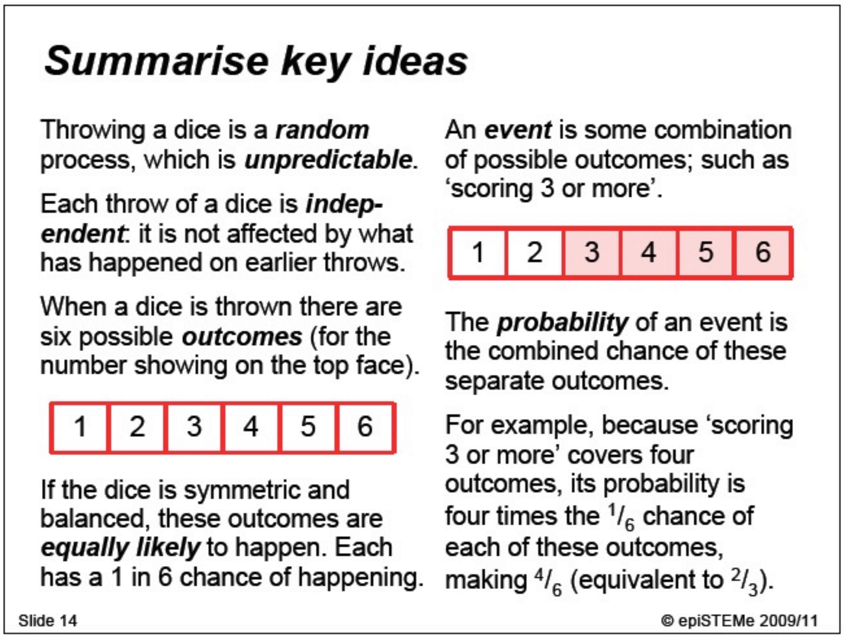
AIMS

- To introduce the use of key terms and diagrams that help to make probability ideas explicit.
- To make explicit relationships between the spatial, numeric and verbal representation systems used in measuring probability.

STRUCTURE

- Part 1 *(Summarise key ideas):* The class is introduced to important words and ideas that can help them with probability reasoning, and to explain it clearly.
- Part 2 *(Adapt key ideas):* Student pairs adapt the summary used in Part 1 to a new example as a way of practicing the use of these key words and ideas for themselves.
- Part 3 *(Dodecahedral die):* Students individually investigate probabilities involving a die which takes a different shape from the normal cubical one, followed by whole-class review of their conclusions.
- Part 4 *(Design your own probability scale):* Small groups compare different ways of labelling a probability scale, and come up with their preferred design.

LESSON 3: PART 1



OBJECTIVE

To introduce key terms and diagrams as tools for making explicit the key ideas that have already been explored informally through the example of throwing dice.

TIME: about 5 minutes.

RESOURCES

- Slide 14 (as shown).
- Study Booklet p. 10.

ACTIVITY

Whole-class

- With this activity there is an important shift to teacher-led presentation of accepted mathematical ideas and terms. Of course, it remains important to make connections to students’ own language and ways of thinking, and to review the material on the slide in an interactive way.
- This material is organised into short paragraphs designed to be read and discussed, one at a time. An individual student can, for example, be asked to read a paragraph, and then another student to put it into their own words. Likewise, a student can be asked to talk about what features of the outcome diagram are important.
- If desired, the slide can be adapted so that the paragraphs appear in a staged way rather than all being presented at once.

LESSON 3: PART 2

OBJECTIVE

To reinforce explicit use of key probability ideas, terms and diagrams.

TIME: about 10 minutes.

RESOURCES

- Slides 15-17 (as shown).
- Study Booklet pp. 10-11.

ACTIVITY

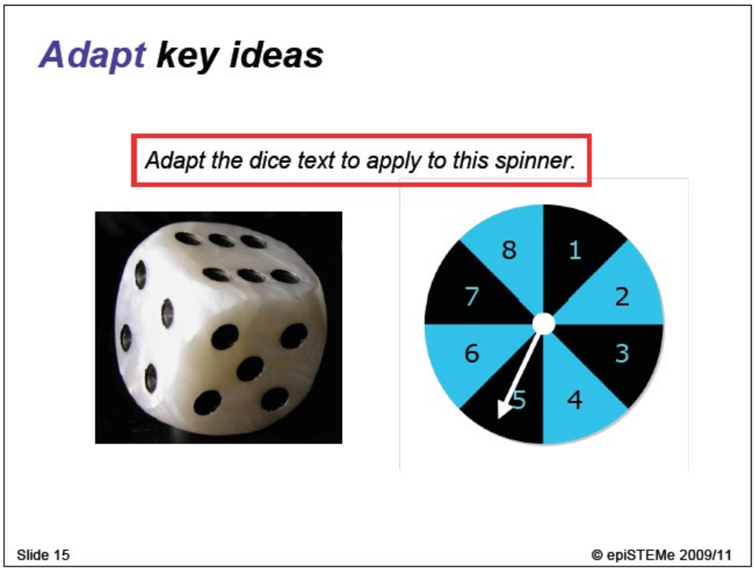
Student-pair leading into whole-class

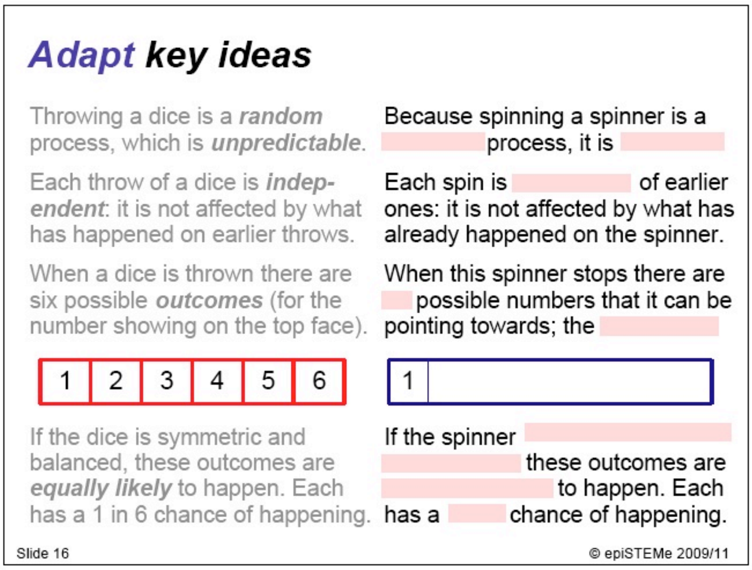
- Get students to work in pairs to adapt the information presented earlier on Slide 14, to provide equivalent information for the spinner shown on Slide 15.
- Ask them to record in their Study Booklets their decisions about filling out the blank spaces on Slides 16 and 17.
- Review answers with the class.

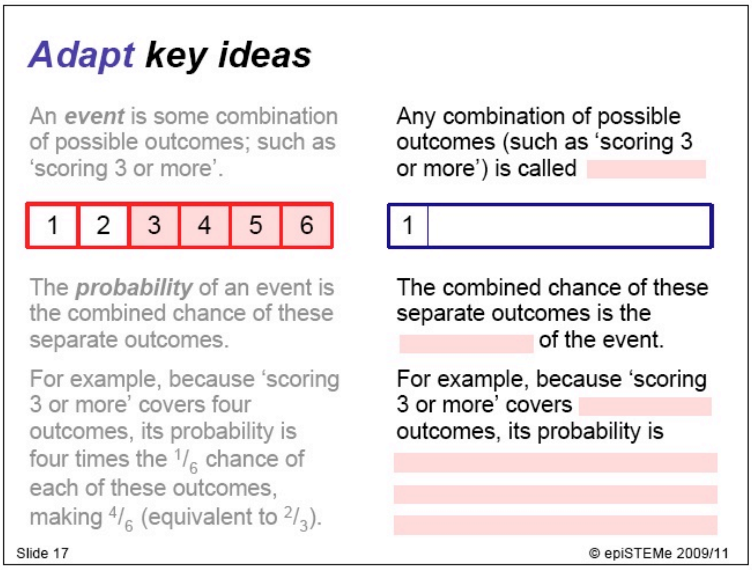
KEYPOINTS

It is particularly important to discuss how to adapt the part describing the conditions under which outcomes are equally likely as this is where the cases of dice and spinner are most different (with the key condition being that the spinner has sectors with angles of equal size).

The other things needing particular attention are the paragraphs and associated diagrams that deal with the number of outcomes, and with calculating the probability of 'scoring 3 or more'.







LESSON 3: PART 3

OBJECTIVES

To provide a checkpoint on individual understanding.

To focus attention on how to write up a solution.

TIME: about 25 minutes.

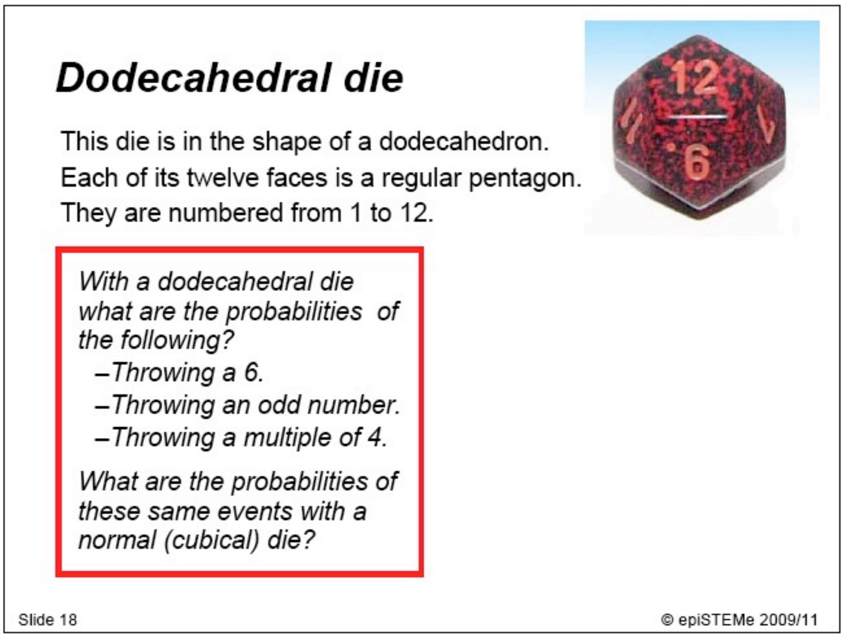
RESOURCES

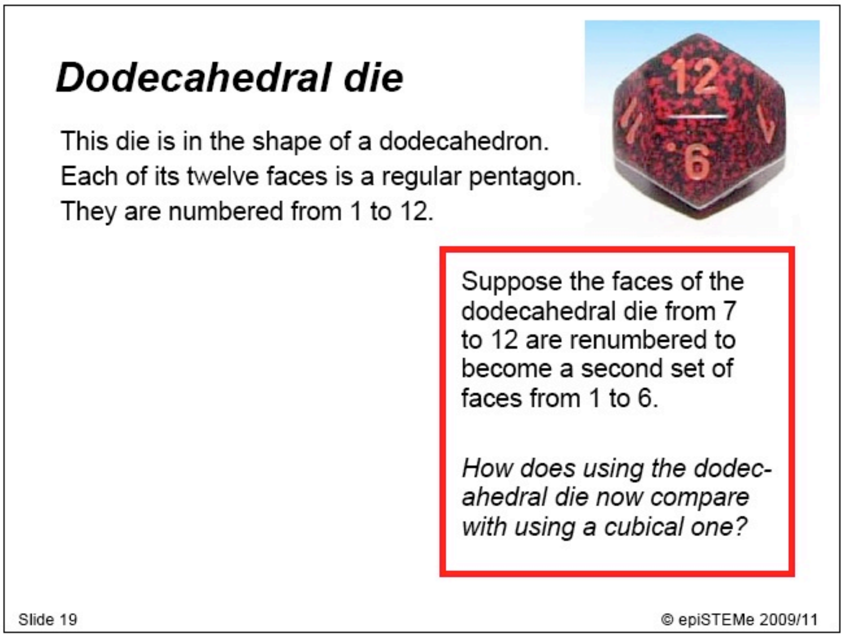
- Slides 18-19 (as shown).
- Study Booklet pp. 12-13.

ACTIVITIES

Whole-class

- Explain to students that ‘die’ refers to one ‘dice’. Originally ‘dice’ was the plural word, and ‘die’ the singular, but nowadays people tend to also use ‘dice’ as the singular.
- Ask students to try to solve the set of problems on Slide 18 and then Slide 19 on their own.
- Emphasise that they should write out their solutions carefully so that someone else can follow their reasoning. Using some of the special words highlighted earlier in the lesson may be helpful.





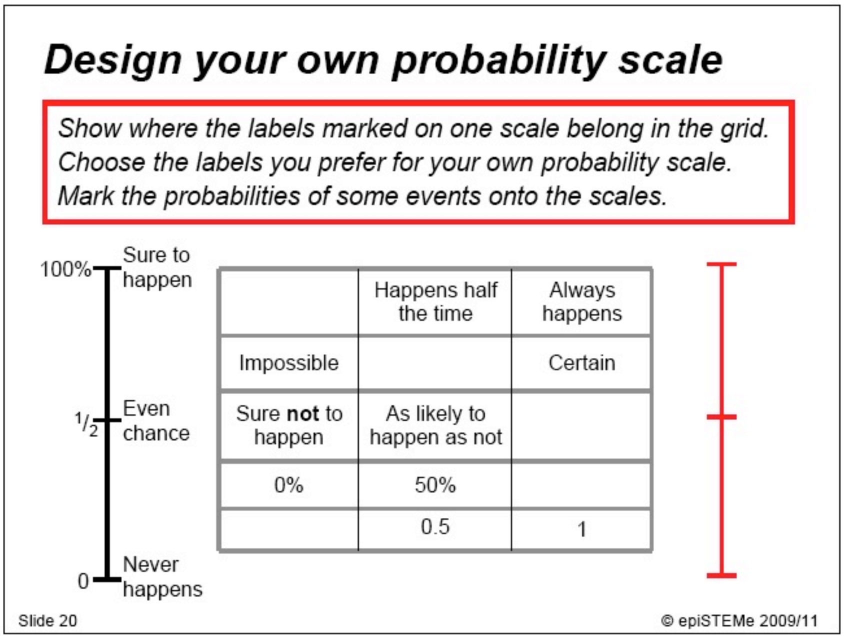
Individual-student

- Circulate to support students and monitor their writing up.
- For students who produce a good written solution quickly, an extension task is to try to find ways of relabelling the die so as to make the probability of throwing a 6: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or $\frac{1}{5}$.

Whole-class

- Choose one student to present their solution to the set of problems on Slide 18.
- Choose another student to comment on their reasoning and say whether the answer is correct or not (giving reasons).
- Invite questions and suggestions from the class as a whole. At this stage, do not correct their ideas. Allow several students to contribute, and to comment on each others’ ideas.
- Then take on a more authoritative stance to summarise, clarifying as necessary, and providing guidance on written recording of solutions.
- Repeat this same sequence for the set of problems on Slide 19.

LESSON 3: PART 4



OBJECTIVE

To highlight relationships between the graphic, numeric and verbal representation systems used in measuring probability.

TIME: about 10 minutes.

RESOURCES

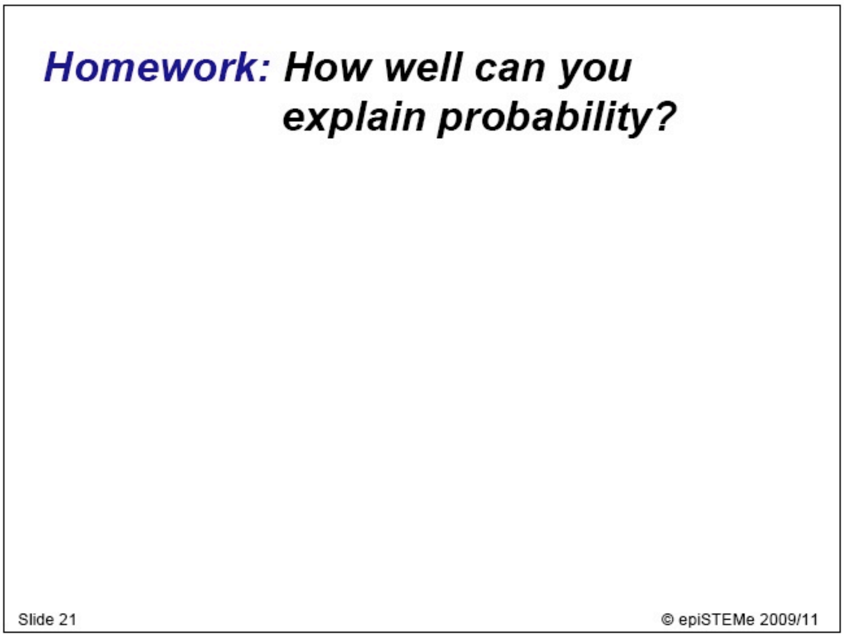
- Slide 20 (as shown).
- Study Booklet p. 14.

ACTIVITY

Small-group discussion concluding with a brief period of whole-class review

- Organise the students to work in groups.
- Ask groups to look at the example of a probability scale on the left-hand side of Slide 20, and to agree where each of the labels used there belongs in the grid in the centre of Slide 20.
- Ask groups next to agree what labels to put on their own preferred probability scale, waiting on the right of Slide 20.
- Ask groups finally to mark some probabilities on the scales, to check that they agree how they operate. Examples of probabilities should be taken from earlier in the lesson, and from their own suggestions.
- This activity does not require a long whole-class debriefing – but you might ask one or two groups to report back to the whole class on how they have designed their scale and (crucially) their reasons for choosing the labels involved.

HOMEWORK



OVERVIEW

This homework poses problems devised to reinforce and extend understanding of the use of the probability ideas introduced to date. The problems allow students to put into practice what they have learned from class debriefing of *Dodecahedral die*.

RESOURCES

- Slides 21-24.
- Study Booklet pp. 15-17.

AIMS

- To reinforce and extend understanding of use of the probability ideas introduced to date.
- To reinforce and extend understanding of how to write up a solution, including using appropriate technical language.

STRUCTURE

- Select from the three problems to make up an appropriate homework for a particular class.
- Lessons 5, 6 and 7 have been timed to provide scope for some later class review of these problems.

Setting the homework

- Suggest that before they tackle a homework problem, students may want to read it through with someone at home and discuss it.

Reviewing the homework in a later lesson

- Take an authoritative stance in questioning the class to develop a model written solution for each problem.
- Ask if anyone is puzzled about anything, and check that everyone has understood.
- If relevant, conclude with a more dialogic discussion of the controversial statement from the National Lottery website on the *Lucky Numbers?* slide. Does anyone have a convincing case in support of it being reasonable or unreasonable?

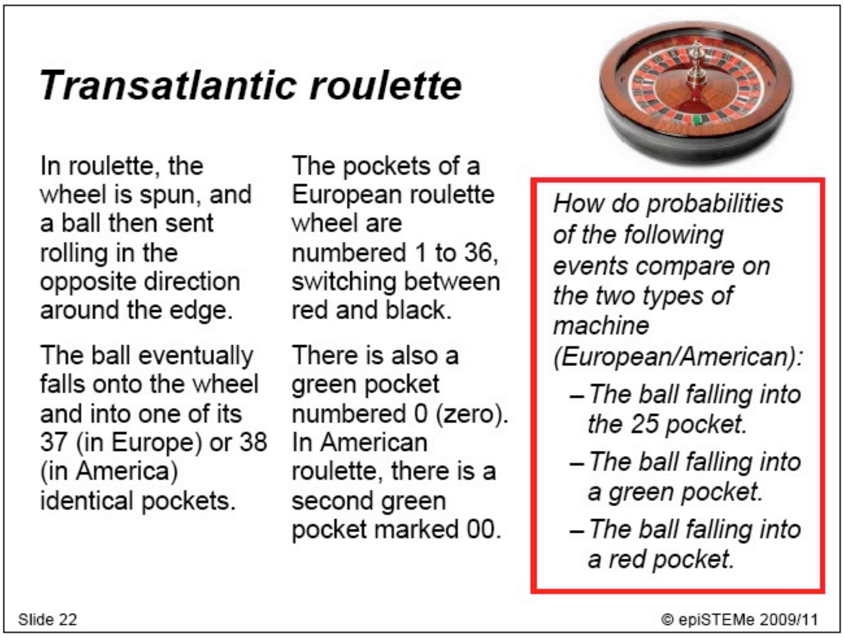
HOMEWORK: PROBLEM 1

OBJECTIVE

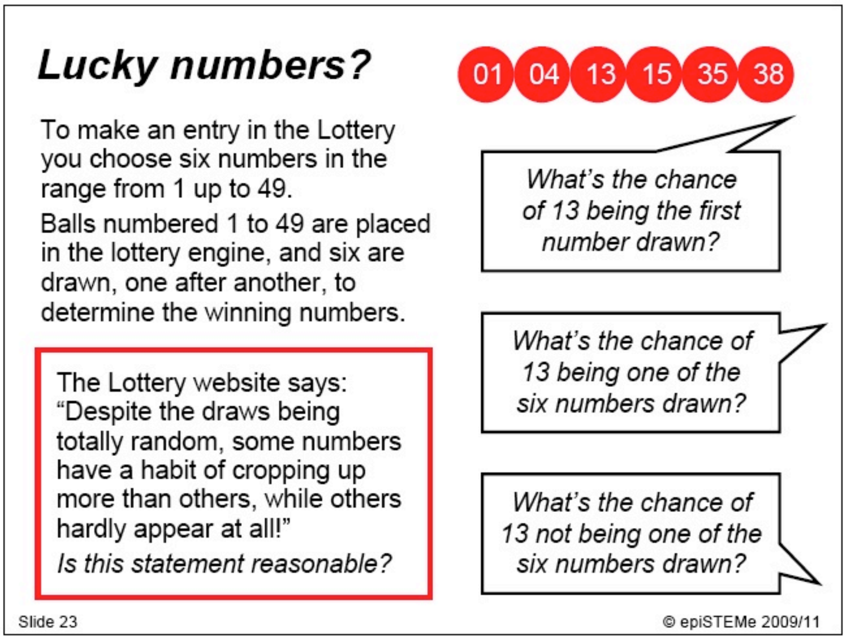
To develop understanding of appropriate ways of expressing reasoning about chance situations.

RESOURCES

- Slide 22 (as shown).
- Study Booklet p. 15.



HOMEWORK: PROBLEM 2



OBJECTIVE

To develop understanding of appropriate ways of expressing reasoning about chance situations.

RESOURCES

- Slide 23 (as shown).
- Study Booklet p. 16.

KEYPOINTS

Chances of events in the right-hand column

Experience has shown that people may be tempted to think about the second and third events in terms of a sequence of balls being drawn so that what happens on each draw then affects the subsequent probabilities. While such an approach is possible (but more complex because it depends on use of conditional probability) it is not necessary to take this approach to solve these problems. Perhaps the best way of thinking about them (that is accessible to lower secondary students) is to imagine that the draw continues until all the balls have been placed in order from first out to last out. Each of these 49 positions is equally likely to be the one in which ball 13 is drawn, and from this the requested probabilities can be formulated directly.

Reasonableness of statement in the left-hand column

Questions that might be asked to support discussion:

Are all the numbers equally likely to be drawn?

Does that mean that all the numbers will actually be drawn the same number of times?

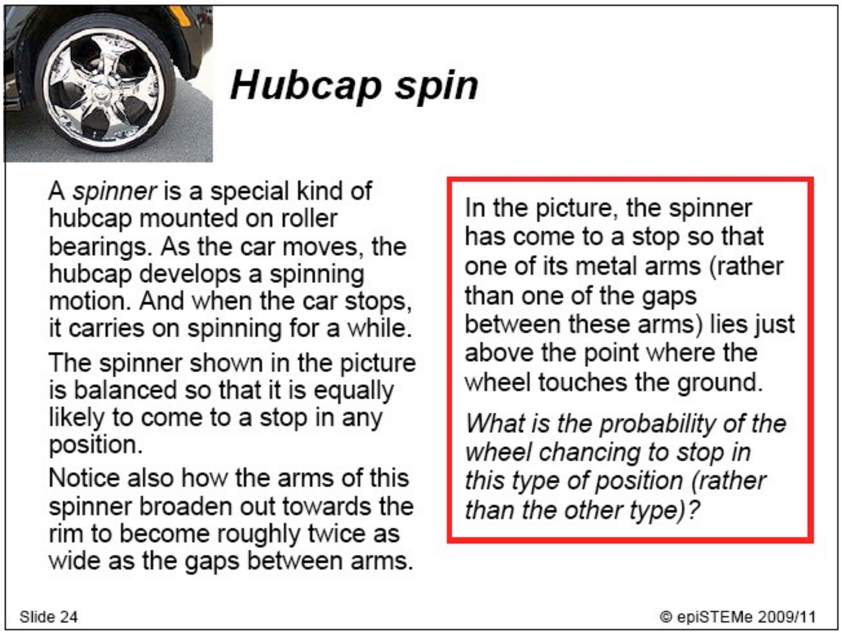
[To answers of Yes] So does that mean that if a number is one of the six drawn this week, it can't be one of the six drawn next week?

[To answers of No] So does that mean that if a number is one of the six drawn this week, it can also be one of the six drawn next week?

Is it reasonable to talk about numbers having a "habit of cropping up"?

Is it reasonable to suggest that some numbers "hardly appear at all"?

HOMEWORK: PROBLEM 3



OBJECTIVE

To provide a more challenging example of equally-likely outcomes.

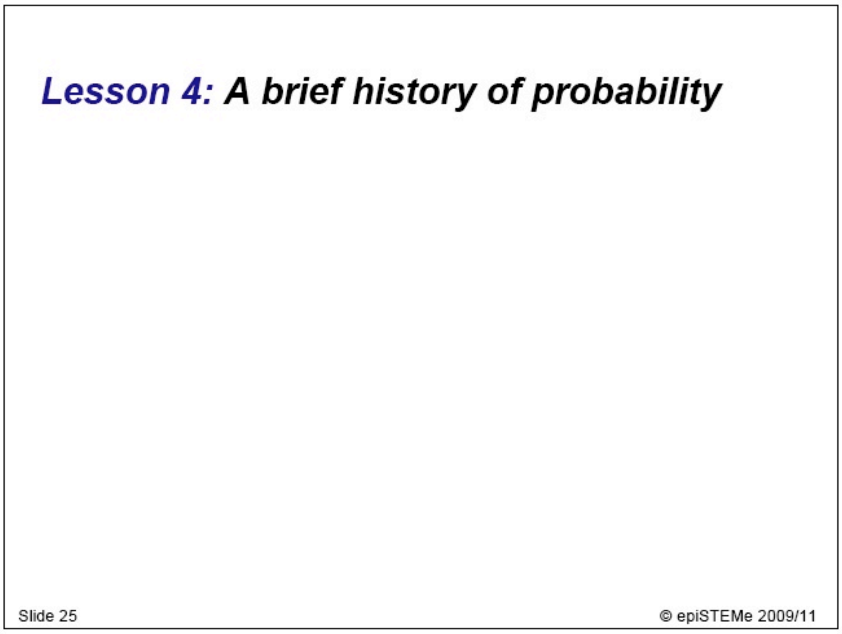
RESOURCES

- Slide 24 (as shown).
- Study Booklet p. 17.

KEYPOINTS

This task calls for more sophisticated use of equally-likely outcomes (rim of wheel divided into 5 ‘single’ sectors for gaps, 5 ‘double’ sectors for arms, to give a probability of 2/3), but is also open to misconceived use of the idea of equally-likely outcomes (not distinguishing between ‘single’ and ‘double’ sectors, to give a probability of 1/2).

LESSON 4



OVERVIEW

This lesson develops a historical theme while introducing the ideas of estimating probabilities from the results of many trials, and analysing the combination of two trials.

TIME: 50 minutes.

RESOURCES

- Slides 25-31.
- Study Booklet pp. 18-21.

AIMS

- To highlight that mathematical ideas develop over time.
- To introduce the idea of estimating probabilities from the results of many trials.
- To introduce the idea of systematically identifying combined outcomes in a repeated trial.

STRUCTURE

- This is a ‘time-travel’ lesson in which the class will go back to tackle mathematical problems that have puzzled scientists and mathematicians in the past. Luckily for them, they have more versatile mathematical tools to use than the ancient Romans did.
- Part 1 *(Estimate knucklebone chances* followed by *Summarise knucklebone chances):* Small groups find a way of calculating the chances of winning at knucklebones, followed by whole-class review of their conclusions.
- Part 2 *(The beginnings of probability):* The class learns when, where and why people first got serious about analysing chance and established the science of probability.
- Part 3 *(… the probability of at least one Head in two flips of a coin).* Small groups examine proposed solutions to a problem that mathematicians found puzzling in these early stages of developing the science of probability, followed by whole-class review of their conclusions.
- Part 4 *(… the probability of two Heads in two flips of a coin).* Students individually tackle a related problem on their own, followed by whole-class review of their conclusions. Experience indicates that students who still embrace a misconceived view at this stage often revise it by the start of the next session without any intervention on the part of the teacher. However, an authoritative summary should be provided either at the end of the session where this Part is tackled or at the start of the following one.

LESSON 4: PART 1

OBJECTIVE

To introduce the idea of estimating probabilities from the proportionate results of many trials.

TIME: about 20 minutes.

RESOURCES

- Slides 26-27 (as shown).
- Study Booklet
  p. 18 (Slide 26)
  p. 30 (Slide 27)

Estimate knucklebone chances

In ancient times people often used animal "knucklebones" (actually bones from the ankle) to play games of chance.

A knucklebone can fall in four positions which the Romans called *supinum*, *pronum*, *planum* and *tortuosum*.

By recording the results of throwing a knucklebone many times, people found a way of estimating the chances of it landing in each of the four different positions.

What do you estimate as the probabilities of the four different positions?

planum	64 times
tortuosum	63 times
supinum	197 times
pronum	176 times

Slide 26 © epiSTEMe 2009/11

Summarise knucklebone chances

We can estimate the ***probability*** of each position from the ***proportion*** of times it happens.

planum	tortuosum	supinum	pronum
64/500	63/500	197/500	176/500
0.128	0.126	0.394	0.352
12.8%	12.6%	39.4%	35.2%

Nowadays, of course, we are able to use mathematical ideas that the Romans didn't have.

We write numbers using the Hindu-Arabic (1, 2… 9, 10) system rather than the Roman one (I, II… IX, X).

The Romans did have ideas of ratio and percentage, but not of fractions and decimals as we know them.

And the science of probability had not been invented.

Slide 27 © epiSTEMe 2009/11

ACTIVITY

Whole-class

- Preview the task on Slide 26 with the class.

Small-group

- Circulate to find out about thinking within groups.
- Intervene where necessary to ensure that appropriate discussion is taking place and that all students are involved.
- Help students to exchange and examine their ideas, and work towards agreeing their own conclusion and the reasoning behind it.

Whole-class

- Experience suggests that some groups may take an equal-likelihood perspective rather than a proportionate one. This provides an excellent basis for whole-class discussion.
- Invite one group to indicate how they estimated the probabilities. Ask them to explain why they used this approach.
- Then invite other groups to ask questions or suggest alternatives. Provide a summary of the main ideas offered.
- Then, use Slide 27 to provide a more explicit presentation of fundamental ideas, taking an authoritative stance to summarise and clarify.
- Ask whether we can conclude that tortuosum is the least likely result, and related questions.
- Highlight how the mathematical tools available to tackle this problem have changed since Roman times, as explained in the lower part of the *Summarise* slide.

LESSON 4: PART 2

OBJECTIVE
To convey an idea of the historical origins of probability.

TIME: a few minutes.

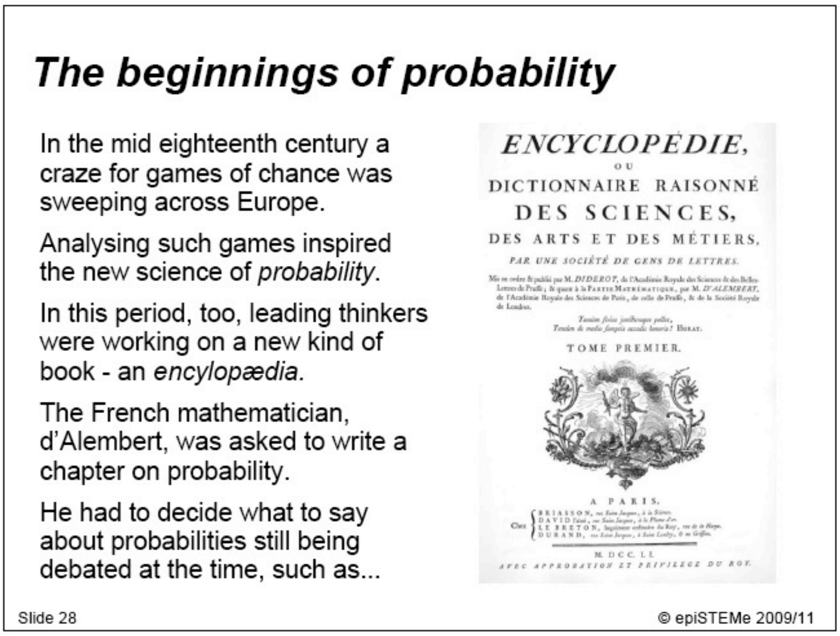
RESOURCES

- Slide 28 (as shown).
- Study Booklet p. 19.

ACTIVITY

Whole-class

- Discuss the information on Slide 28 with the class.



KEYPOINTS
Information about d'Alembert can be found at http://en.wikipedia.org/wiki/D'Alembert, and about the Encyclopedia at http://en.wikipedia.org/wiki/Encyclopédie.

LESSON 4: PART 3

OBJECTIVES

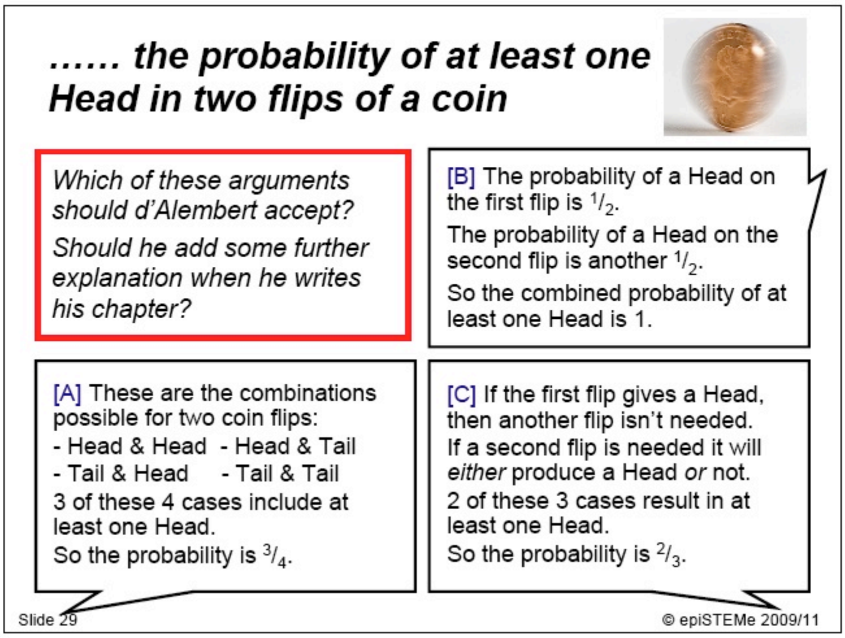
To deconstruct common misconceptions about combined trials.

To introduce the idea of systematically identifying combined outcomes.

TIME: about 20 minutes.

RESOURCES

- Slide 29 (as shown).
- Study Booklet p. 20.



ACTIVITY

Whole-class

- Preview the problem on Slide 29 with the class.
- Organise the students into groups. Remind them of the ground rules for discussion.
- Tell them they must decide and record whether they agree or disagree with each solution.

Small-group

- Circulate to gain information about the thinking within different groups.

Whole-class

- Invite one group to speak in support of the solution they have agreed to accept.
- Invite further groups to speak in support of other solutions. Structure collective exploration of differences. If there continues to be disagreement, identify any students who have changed their mind, and ask them to explain the reasons why.
- Should all the groups have agreed on the same solution, invite reasons for not choosing each of the other solutions.
- Conclude by telling students that d'Alembert found it very hard indeed to decide between two of the solutions. He knew that people tended to agree with A, but he was still attracted to C. So he included both in his encyclopedia chapter!

KEYPOINTS

Solution A identifies the combined outcomes of two flips of a coin in a systematic way (although it does not make it explicit that these outcomes are equally likely). Solution B implies that at least one head is a certainty. Solution C identifies three distinct outcomes or events in a systematic way but they are not equally likely: in particular, 'Head on first flip' has a probability of ½; which can be shown by comparing A and C as follows:

<table>
<tr><th>Solution A</th><th>Solution C</th></tr>
<tr><td>Head & Head</td><td rowspan="2">Head on first flip</td></tr>
<tr><td>Head & Tail</td></tr>
<tr><td>Tail & Head</td><td>Tail on first flip followed by Head on second flip</td></tr>
<tr><td>Tail & Tail</td><td>Tail on first flip followed by Tail on second flip</td></tr>
</table>

LESSON 4: PART 4

OBJECTIVE

To reinforce ideas about combined trials.

TIME: about 10 minutes.

RESOURCES

- Slides 30 and 31 (as shown).
- Study Booklet
  p. 21 (Slide 30)
  p. 30 (Slide 31)

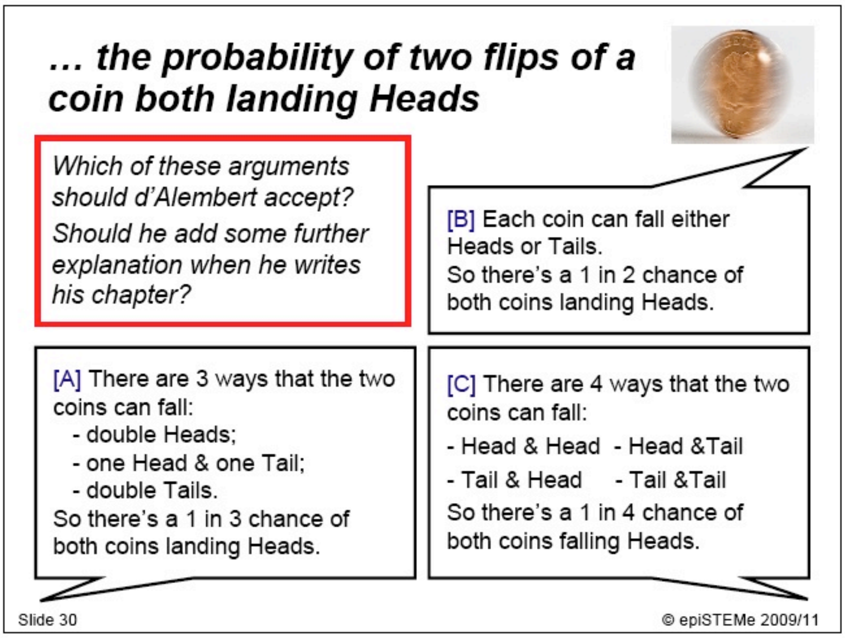
ACTIVITY

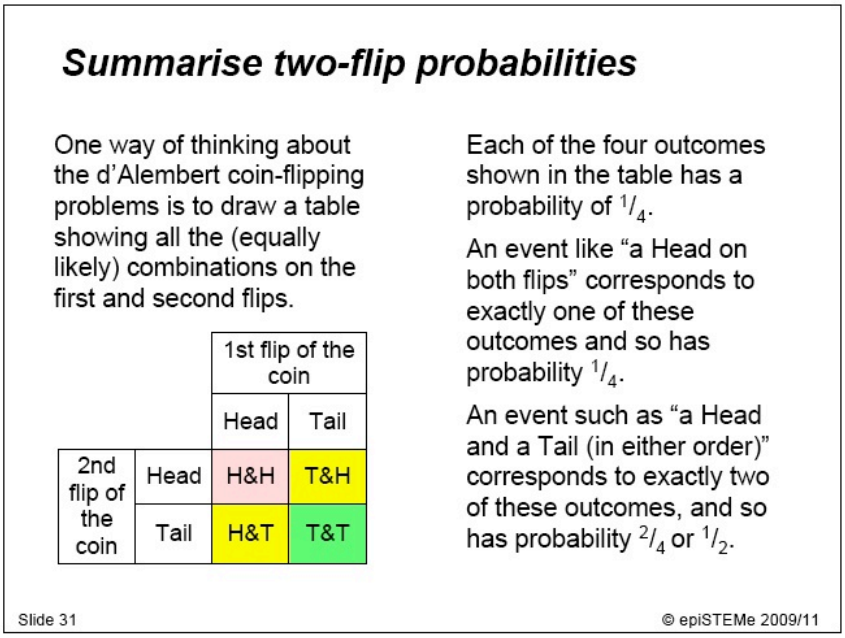
Individual-student

- Students work on their own on the problem of Slide 30 to decide which solution they agree with and why.

Whole-class

- Conduct a class poll of preferred solutions.
- Interactively examine each solution.
- Use Slide 31 to provide an authoritative summary either at the end of this session or the start of the next.





KEYPOINTS

Solution A identifies three events in a systematic way but these are not equally likely since there are two ways of getting 'one Head and one Tail': with a 1p and a 10p coin, these would be H on the 1p and T on the 10p, or T on the 1p and H on the 10p. Solution B implies, by symmetry, that there is also a ½ chance of both coins landing Tails, and so zero probability of Heads and Tails combined. Solution C identifies the combined outcomes in a systematic way (but fails to make it explicit that these outcomes are equally likely). A and C can (eventually) be compared as follows:

Solution A	**Solution C**
double Heads	Head & Head
one Head & one Tail	Head & Tail
	Tail & Head
double Tails	Tail & Tail

LESSON 5

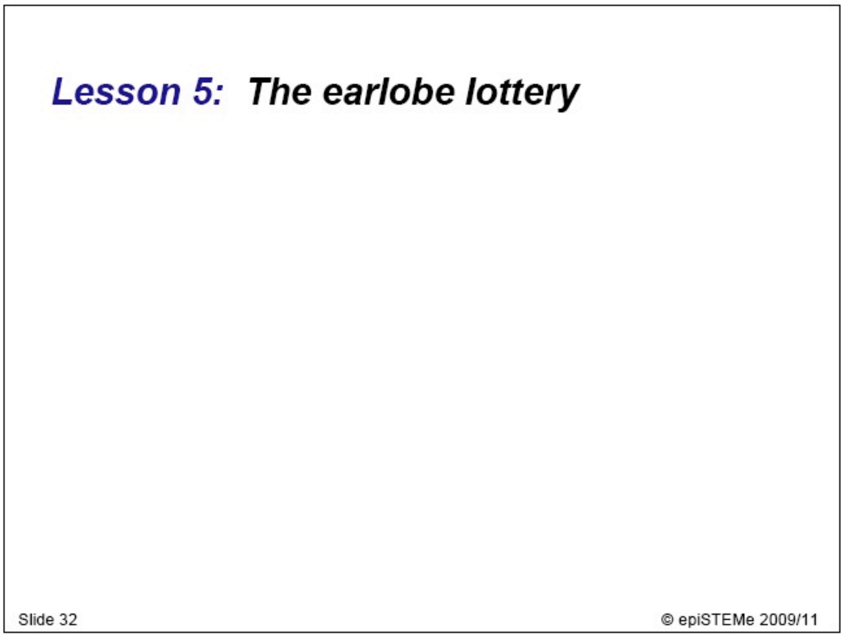
OVERVIEW

This lesson shows how the idea of combined trials as developed in the preceding lesson can be used to build a scientific model of genetic inheritance.

TIME: about 40 minutes.

RESOURCES

- Slides 32-36.
- Study Booklet pp. 22-24.



AIMS

- To examine in depth a scientific application of probability.
- To rehearse and extend the idea of the probability of combined trials.

STRUCTURE

- This is a lesson in which the class applies mathematical ideas about probability to the scientific problem of how genes control a simple human characteristic.
- Part 1 *(From coins to genes):* The class is set the context for the lesson.
- Part 2 *(The facts of earlobe life):* The class is introduced to some basic scientific ideas about how genes control the form of our earlobes.
- Part 3 *(The spin on earlobes):* The class tackles some simple mathematical problems related to this scientific idea.
- Part 4 *(Earlobe problems to pierce).* Small groups tackle some more complex problems, followed by whole-class review of their conclusions.
- The combined duration of these Parts is less than the normal 50 minutes to provide some flexibility at this stage of the module, perhaps to incorporate review of the earlier homework, perhaps to permit more wide-ranging discussion.

LESSON 5: PART 1

OBJECTIVE

To introduce the idea of the application of probability in building scientific models.

TIME: a few minutes.

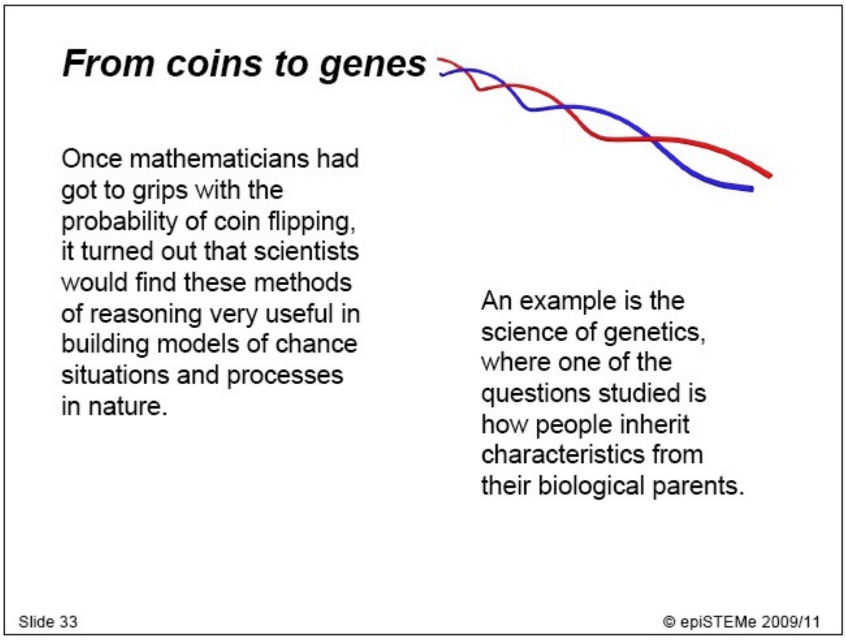
RESOURCES

- Slide 33 (as shown).
- Study Booklet p. 22.

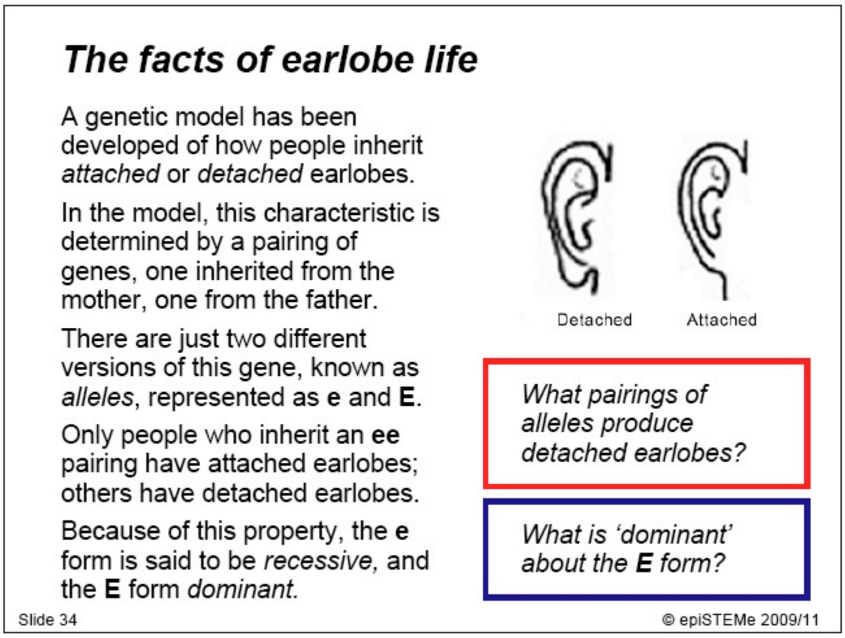
ACTIVITY

Whole-class

- Introduce the information on Slide 33.



LESSON 5: PART 2



OBJECTIVE

To introduce and explain the genetic model of earlobe characteristics.

TIME: about 5 minutes.

RESOURCES

- Slide 34 (as shown).
- Study Booklet p. 22.

ACTIVITY

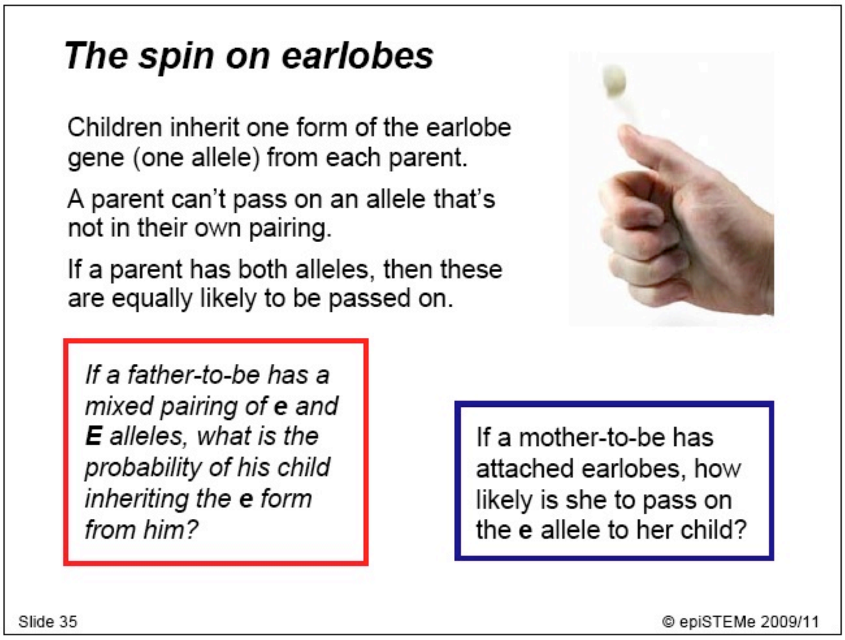
Whole-class

- Interactively examine and interpret the information shown on Slide 34.
- With some classes it may be helpful to read and discuss this paragraph by paragraph.
- Use questions such as those displayed on the slide, to check and develop understanding.

PRONUNCIATION

Allele is pronounced "AH-LEEL". Say **E** as "capital ee" and **e** as "small ee" to be able to tell them apart in speech. [Avoid "big ee" and "little ee" which may lead to double entendres!]

LESSON 5: PART 3



OBJECTIVE

To model the process of solving problems which call for knowledge about probability and about genes to be combined.

TIME: about 10 minutes.

RESOURCES

- Slide 35 (as shown).
- Study Booklet p. 23.

ACTIVITY

Whole-class

- Treat each problem on Slide 35 in turn.
- Give students a little time to think how they might tackle it (either on their own or through pair discussion) and then interactively tackle the problem with the whole class.

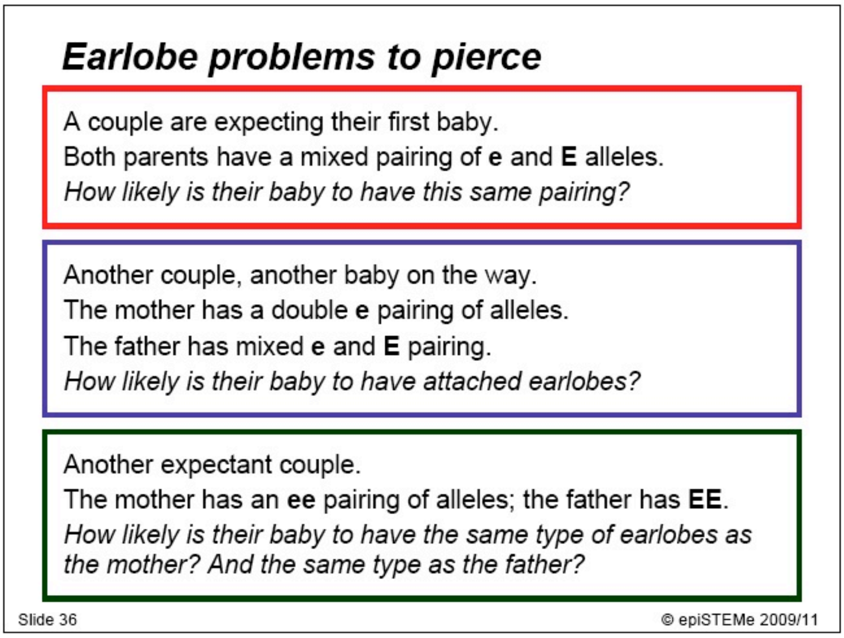
KEYPOINTS

It may be helpful to note that simply listing the pair of alleles possessed by the parent shows the possible outcomes for passing one on to a child.

Left problem: **e E** (same as **E e**) [requested probability is ½]. The analogy is with a coin that has **e** on one side and **E** on the other.

Right problem: **e e** (inferred from attached earlobes) [requested probability is 1; i.e. this is certain]. The analogy is with a coin that has **e** on both sides.

LESSON 5: PART 4



OBJECTIVE

To engage students more independently in the process of solving problems which call for knowledge about probability and about genes to be combined.

TIME: about 25 minutes.

RESOURCES

- Slide 36 (as shown).
- Study Booklet p. 24.

ACTIVITY

Whole-class

- Preview the first problem on Slide 36 with the class.
- Organise the class into groups and remind them of the ‘ground rules’ for discussion.

Small-group

- Circulate to gain information about the thinking within different groups. If groups are coping confidently with the first problem, encourage them to go on to tackle the others. Allow sufficient time for group discussion in some depth of at least the first problem before reconvening the class.
- Intervene where necessary to ensure that appropriate discussion is taking place and that all students are involved, helping students to exchange and examine their ideas, and work towards their own conclusion. Insist that they try to reach agreement, and to be able to give reasons for their decisions.

Whole-class

- From your observations, decide which problem would be most productive for this class to discuss.
- Invite one group to indicate how they solved that problem, explaining their reasoning.
- Invite other groups to ask questions or suggest alternatives.
- Provide a summary of the main ideas offered.
- If time permits, repeat this process for the other two problems.

KEYPOINTS

In tackling these problems students may be influenced by everyday thinking about heritability as well as different mathematics viewpoints. On the first problem, for example, experience suggests that groups may propose 100% (on the grounds that since both parents have the same allele pair and earlobes their child is bound to inherit these). This problem may also elicit proposals of 1/3 (where students have not made the analogy with coin flipping). These provide a good basis for whole-class discussion.

KEY POINTS

Problem 1:

		From father	
		E	e
From mother	E	EE	Ee
	e	eE	ee

So 50% chance of baby inheriting both **e** and **E** alleles.

Problem 2:

		From father	
		E	e
From mother	e	eE	ee
	e	eE	ee

OR, because mother is certain to pass on **e** allele:

		From father	
		E	e
From mother	e	eE	ee

So 50% chance of baby having attached earlobes (due to inheriting two **e** alleles).

Problem 3:

		From father	
		E	E
From mother	e	eE	eE
	e	eE	eE

OR, because mother is certain to pass on **e** allele, and father **E**:

		From father
		E
From mother	e	eE

So baby is certain to inherit **eE** pairing: i.e. to have detached earlobes

A more challenging extension question is whether attached earlobes "will die out", something that students often ask spontaneously in the course of this lesson.

LESSON 6

OVERVIEW

This lesson generalises the idea of combined trials through considering two further variant examples.

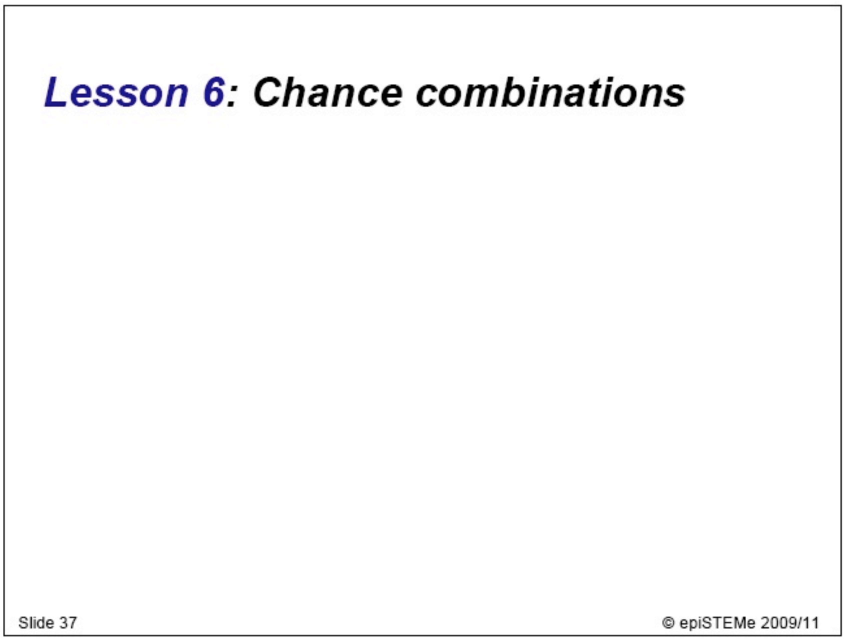
TIME: about 40 minutes.

RESOURCES

- Slides 37-40.
- Study Booklet pp. 25-27.

AIM

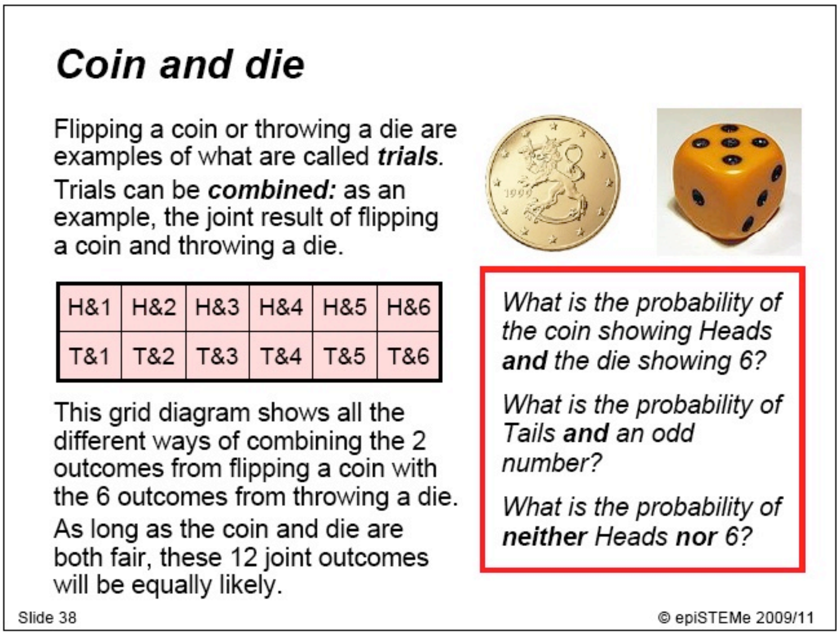
- To generalise the idea of combined trials.



STRUCTURE

- Part 1 *(Coin and die).* The class learns how two different types of trial can be combined and tackles a problem set on coin and die combinations.
- Part 2 *(Two dice).* Small groups tackle the problem of the combined scores from two dice.
- Part 3 *(Two-dice bingo).* Students use what they have learned to cunningly design a card for the game of two-dice bingo, with their choices reviewed and tested in whole-class activity.
- The combined duration of these Parts is less than the normal 50 minutes to provide some flexibility at this stage of the module, perhaps to incorporate review of the earlier homework, perhaps to permit more wide-ranging discussion.

LESSON 6: PART 1



OBJECTIVE

To generalise the idea of combining trials.

TIME: about 10 minutes.

RESOURCES

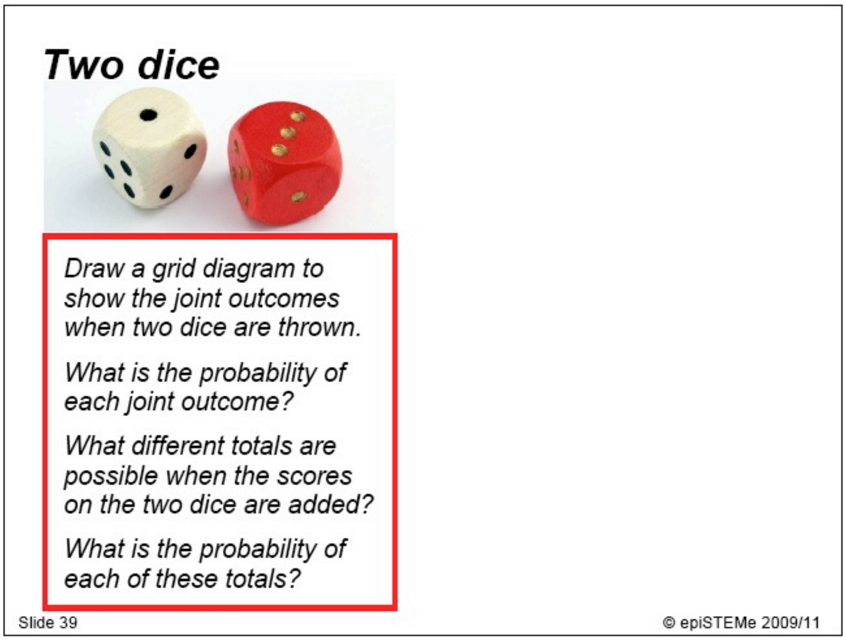
- Slide 38 (as shown).
- Study Booklet p. 25.

ACTIVITY

Whole-class

- Use the left-hand column of Slide 38 to introduce the class to the idea of combining different types of trial.
- Then interactively tackle the set of problems in the right-hand column of Slide 38 to develop the idea of the probability of a combined outcome or event.
- This could be organised as a whole-class activity in which students are directly invited to propose and discuss ideas about tackling each problem.
- Alternatively, students might be given a minute or two to tackle a problem individually or in pairs, followed by whole-class discussion of proposed solutions.

LESSON 6: PART 2



OBJECTIVE

To reinforce the generalised idea of combined trials.

To focus attention on how a structure of equally-likely outcomes can be paralleled by a structure of unequally-likely outcomes.

TIME: about 20 minutes.

RESOURCES

- Slide 39 (as shown).
- Study Booklet p. 26.

ACTIVITY

Whole-class

- Preview the problem on Slide 39 with the class.

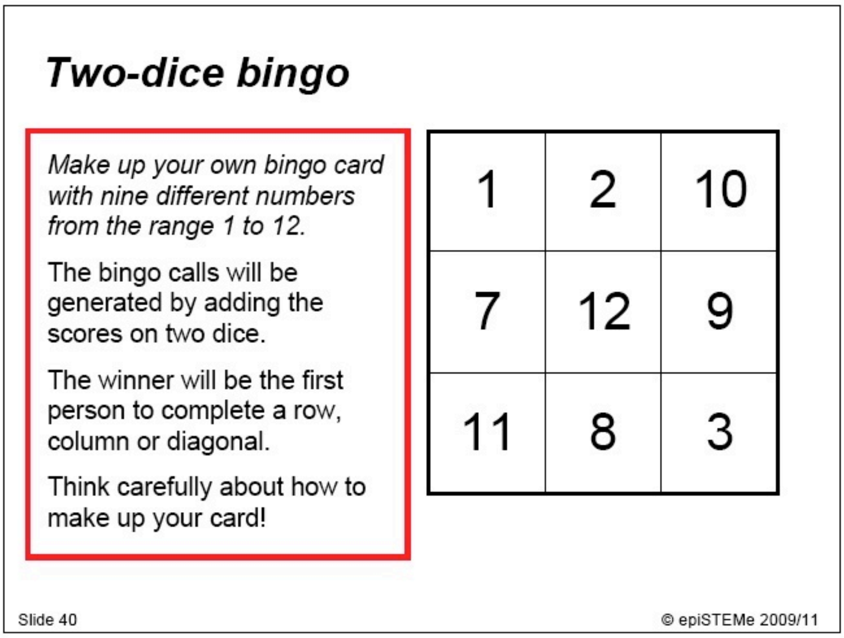
Individual-student or Student-pair

- Give students at least 10 minutes to work individually or in a pair on the problem set.
- Any student/pair that finishes very quickly can be asked to think about the combination of three coins.

Whole-class

- Draw the class back together for discussion of the set of problems. Consider the problems one at a time.
- Ask one student/pair to present their answer, then invite others to ask this student/pair questions. Remember to ask the students for reasons to back up their views.
- The teacher's main role should be to orchestrate a dialogue which will reveal different ideas amongst the students.
- The class should come to an agreement about what is acceptable as an answer.

LESSON 6: PART 3



OBJECTIVE

To apply ideas and results from Part 2 of the lesson.

TIME: about 10 minutes.

RESOURCES

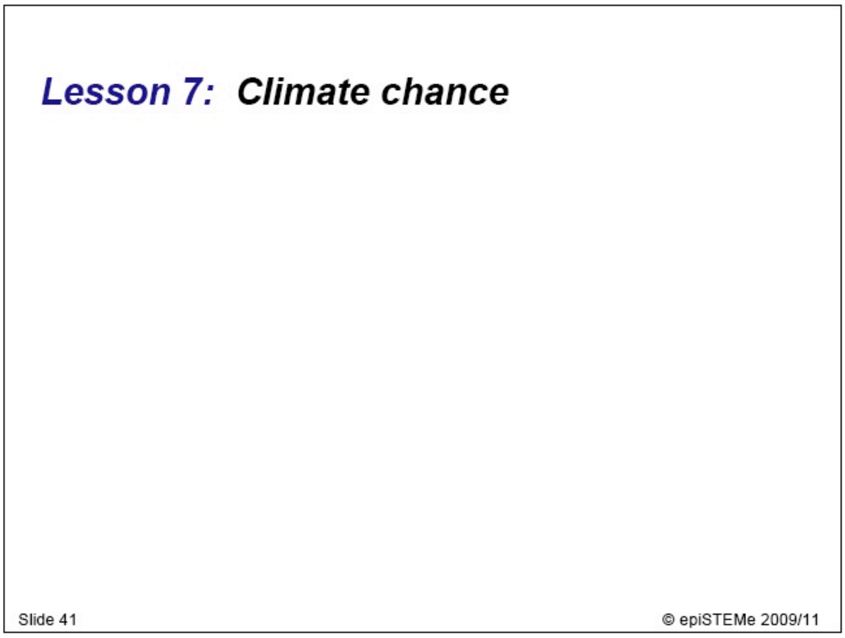
- 2 dice.
- Slide 40 (as shown). [To remove numbers from grid enter <Slide View> mode rather than <Slide Show>].
- Study Booklet p. 27.

ACTIVITY

Whole-class

- Use Slide 40 to introduce the activity.
- Give students a minute or two to make up their own bingo card.
- Review strategies for making up the card. Suitable questions might be:
    - Who has included 1 in their card? Good decision? Why, or why not?
    - Who has included 7 in their card? Good decision? Why, or why not?
    - Does it matter where 7 is placed? Why, or why not?
    - Invite more suggestions regarding good and bad choices and placements.
- Give students an opportunity to modify their bingo card in the light of the discussion.
- Finally, throw dice to generate calls, until a winner materialises.
- If time permits, explore whether there is an optimal grid. Suitable questions might be:
    - How many possible winning lines go through each square on the grid?
    - Where would it be best to place the totals that are most likely to be thrown?

LESSON 7



OVERVIEW

This lesson develops another scientific application theme while introducing the ideas of complementary, mutually exclusive and exhaustive events.

TIME: 40 minutes.

RESOURCES

- Slides 41-45.
- Internet access is necessary to show the online video clip about penguin population study.
- Study Booklet pp. 28-29.

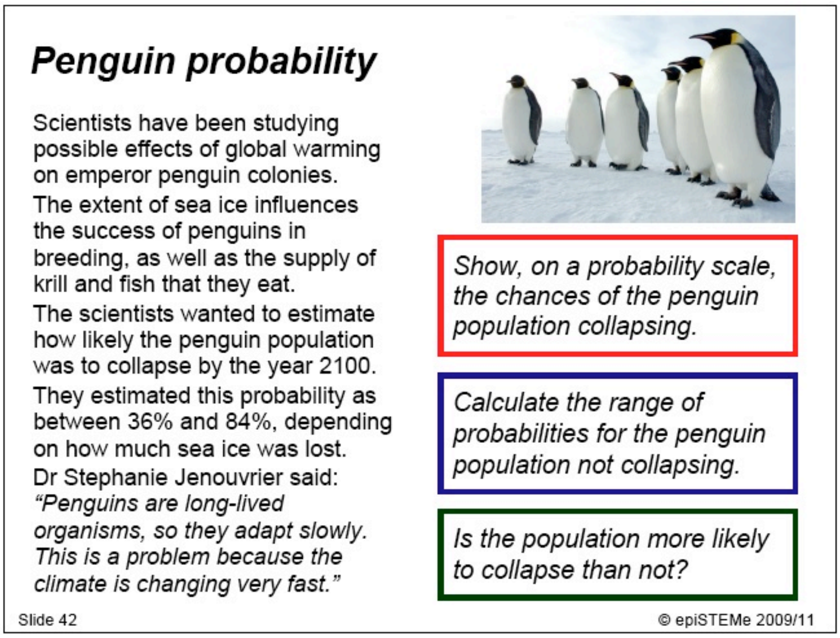
AIMS

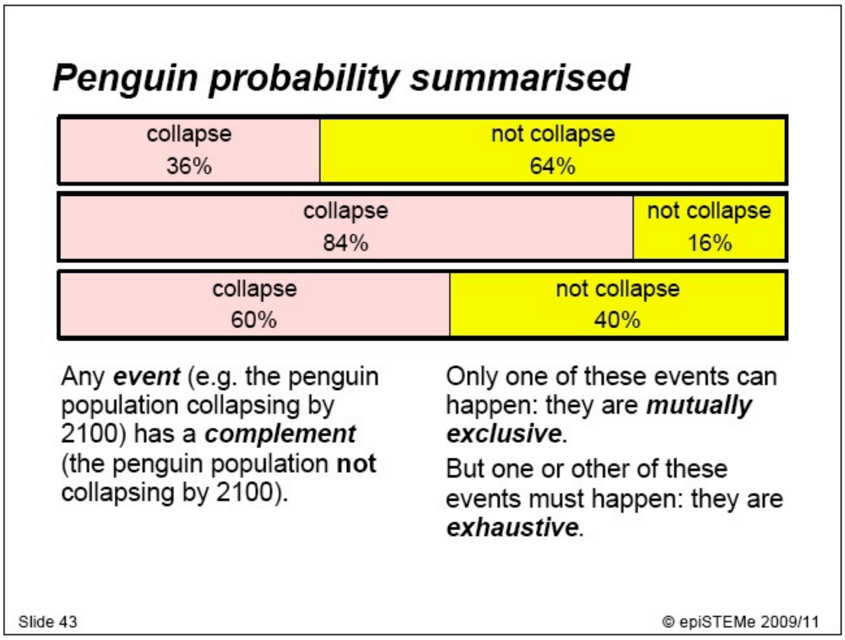
- To introduce the idea of working with given probabilities without (as previously) an explicit underlying structure of equally-likely outcomes or observed frequencies.
- To informally then formally introduce the ideas of complementary, mutually exclusive and exhaustive events.
- To demonstrate use of probability ideas in another scientific domain.

STRUCTURE

- Part 1 (*Penguin probability)*. The class looks at a problem on combining probability values in a context relating to scientific study of the potential impact of climate change.
- Part 2 *(Icesheet impact).* The class looks at another problem on combining probability values in a context relating to scientific study of the potential impact of climate change.
- The combined duration of these Parts is less than the normal 50 minutes to provide some flexibility at this stage of the module, perhaps to incorporate review of the earlier homework, perhaps to permit more wide-ranging discussion.

LESSON 7: PART 1





OBJECTIVE

To introduce the system of ideas of complementary, mutually exclusive and exhaustive events.

TIME: about 20 minutes.

RESOURCES

- Slides 42-43 (as shown).
- Internet access necessary to show online video clip about penguin population study at http://news.bbc.co.uk/1/hi/uk/7852731.stm.
- Further information about the penguin population study is at http://news.bbc.co.uk/1/hi/sci/tech/7851276.stm.
- Study Booklet
  p. 28 (Slide 42)
  p. 31 (Slide 43).

ACTIVITY

Whole-class

- Use the 1-minute video-clip to introduce the topic. [Click on the penguin picture on the slide to activate a hyperlink to this clip. On the BBC webpage, click on the bottom right corner of the video window to show the clip in full-screen mode.]
- Invite students to read out and comment on sections of the text on Slide 42.
- Preview the three problems and then get students to tackle them on their own or in pairs.

Student-pair or Individual-student

- Circulate to observe and help students develop their own solutions.

Whole-class

- Debrief each problem in turn by asking one student (or pair) to present their solution. Then invite questions or alternatives. Structure collective exploration of differences.
- Once the whole set of problems has been discussed, use Slide 43 to codify authoritatively.

LESSON 7: PART 2

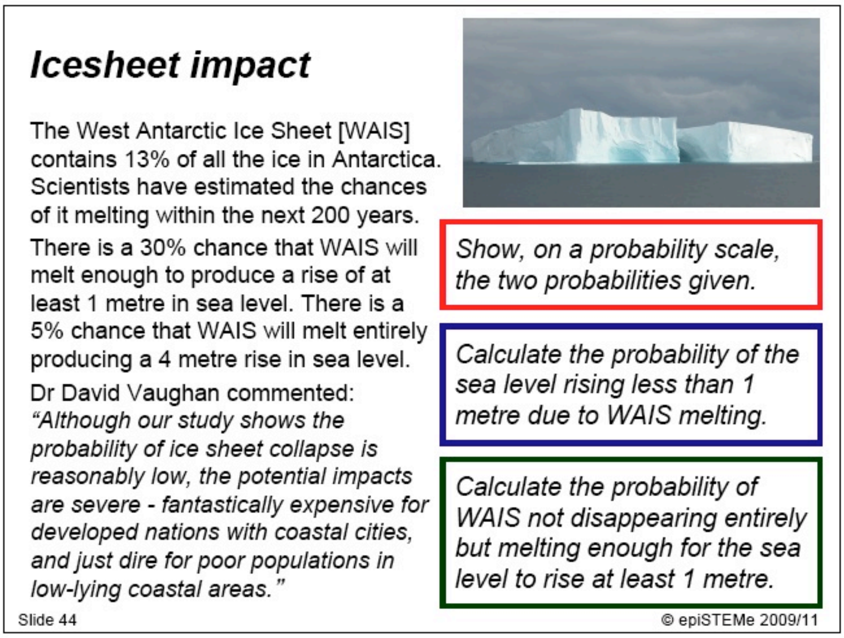
OBJECTIVE

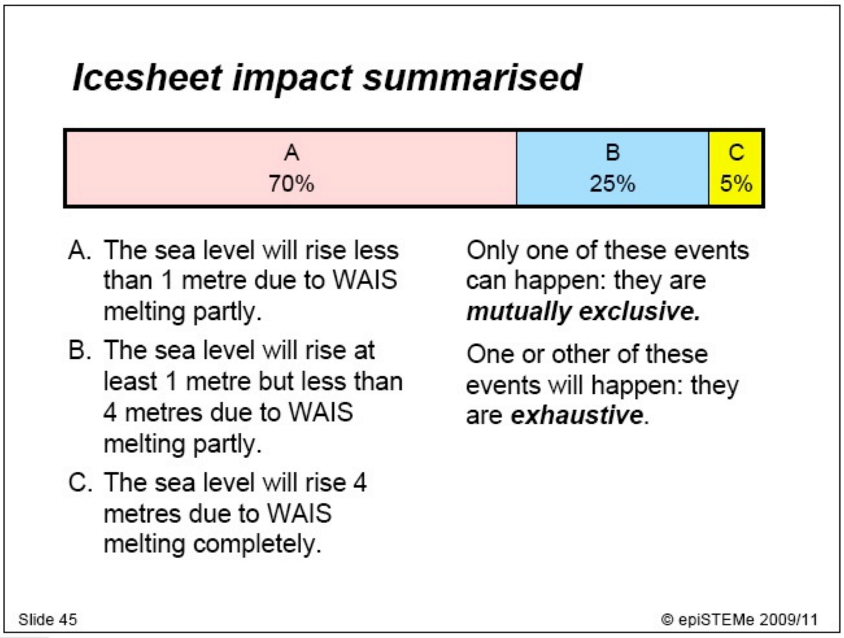
To consolidate and extend the system of ideas of complementary, mutually exclusive and exhaustive events.

TIME: about 20 minutes.

RESOURCES

- Slides 44-45 (as shown).
- Further information on the ice sheet melting study is at http://news.bbc.co.uk/1/hi/sci/tech/1731163.stm
- Study Booklet p. 29 (Slide 44) p. 31 (Slide 45).





ACTIVITY

Whole-class

- Invite students to read out and comment on sections of the text on Slide 44.
- Develop discussion of why scientists have been encouraged to study the relationship between polar icesheets and sea levels, and why they are using probabilities to report results.
- Preview the three problems and then get students to tackle them on their own or in pairs.

Student-pair or Individual-student

- Circulate to observe and help students develop their own solutions.

Whole class

- Debrief each problem in turn by asking one student (or pair) to present their solution. Then invite questions or alternatives. Structure collective exploration of differences.
- Once the whole set of problems has been discussed, use Slide 45 to codify authoritatively.

APPENDIX 1: *epiSTEMe*

The *epiSTEMe* project [Effecting Principled Improvement in STEM Education] is part of a national programme of research that aims to strengthen understanding of ways to increase young people's achievement in physical science and mathematics, and their participation in courses in these areas. Drawing on relevant theory and earlier research, the *epiSTEMe* project has developed a principled model of curriculum and pedagogy designed to enhance engagement and learning during a particularly influential phase in young people's development: the first year of secondary education. This module on Probability is one of four topic-specific modules that have been developed to operationalise that model and support its classroom implementation.

The teaching model

The *epiSTEMe* teaching model builds on current thinking in the field and on promising exemplars that have been extensively researched. These suggest that students' learning and engagement can be enhanced through classroom activity organised around carefully crafted problem situations designed to develop key disciplinary ideas. These situations are posed in ways that appeal to students' wider life experience, and draw them more deeply into mathematical and scientific thinking. Such an approach is intended not just to help students master challenging new ways of thinking, but also to help them develop a more positive identity in relation to mathematics and science.

An important feature of the teaching model is the way in which it makes explicit links between mathematics and science. Within mathematics modules, the primary rationale for this is that science represents a major area where an unusually wide range of mathematics is applied, often for a variety of purposes. Within science modules, the primary rationale is that understanding of scientific ideas is deepened by moving from expressing them in qualitative terms to representing them mathematically.

The teaching model also emphasises the contribution of dialogic processes in which students are encouraged to consider and debate different ways of reasoning about situations. These dialogic processes are designed to take place in the course of joint activity and collective reflection at two levels of classroom activity: student-led (and teacher-supported) collaborative activity within small groups, and teacher-led (and student-interactive) whole-class activity. Because of the importance of developing dialogic processes that support effective learning, these processes are the focus of a separate Introductory Module, which is additional to the four topic-specific modules.

The design of the topic-specific modules

The function of all four topic-specific modules is to provide examples of concrete teaching sequences that incorporate classroom tasks well suited to the teaching model. In particular, many tasks have been specifically designed to support dialogic processes, and student learning from these tasks will be supported by using these processes.

Each module has been designed to cover those aspects of the topic prescribed for the start of the Key Stage 3 curriculum in England, and to do so in a way that is suited to students across a wide range of achievement levels. Taking account of available theory and research on the development of students' thinking in the topic, the module 'fills out' the official prescriptions in ways intended to build strong conceptual foundations for the topic. This includes providing means of deconstructing common misconceptions related to the topic, and introducing appropriate technical terms and representations as tools for thinking.

The modules take account of students' informal knowledge and thinking related to a topic, and they make connections with widely shared student experiences relevant to a topic. Equally, with a view to helping students understand how mathematics and science play a part in their wider and future lives, the modules try to bring out the human interest, social relevance and scientific application of topics.

Finally, while the modules place a strong emphasis on exploratory dialogic talk, both in small groups and whole class, they also make provision later for authoritative codification and consolidation of key ideas, and build in individual checks on student understanding that can be used to provide developmental feedback.

APPENDIX 2: Learning objectives from English National Curriculum Attainment Targets mapped against Lessons

NC level	NC AT descriptor *(our addition)*	Lesson
4	[none stated]	
5	understand that different outcomes may result from repeating an experiment	LESSON 2: *Review the 'Fives' results*
5	understand and use the probability scale from 0 to 1, or *0% to 100%*	LESSON 1: *Rate some dice chances* (implicitly) LESSON 3: *Design your own probability scale* (explicitly) LESSON 7: *Climate chance*
5	find and justify probabilities by selecting and using methods based on equally likely outcomes	LESSON 1: *Rate some dice chances* (implicitly) LESSON 3: *Summarise key ideas* and *Adapt key ideas* (explicitly) then *Dodecahedral die* HOMEWORK: all problems
5	find and justify approximations to probabilities by selecting and using methods based on experimental evidence	LESSON 2: *Review the 'Fives' results* (implicitly) LESSON 4: *Knucklebone chances* (implicitly then explicitly)
6	when dealing with a combination of two experiments, identify all the outcomes *and calculate probabilities accordingly*	LESSON 4: *Two-flip probabilities* (implicitly then explicitly) LESSON 5: *The earlobe lottery* LESSON 6: *Chance combinations* (explicitly)
6	when solving problems, use knowledge that the total probability of all the mutually exclusive outcomes of an experiment *or an exhaustive set of mutually exclusive events* is 1	LESSON 1: *Rate some dice chances* (implicitly) HOMEWORK: *Lucky numbers* (implicitly) LESSON 4: *Knucklebone chances* (implicitly) LESSON 6: *Coin and die* (implicitly) LESSON 7: *Climate chance* (explicitly)